DESCRIPTION OF THE MAIN SCIENTIFIC WORK OF FABRIZIO CATANESE.

1. Presentation

Fabrizio Catanese was born in Firenze, March 16, 1950. In 1968 he was admitted at the Scuola Normale Superiore di Pisa, where he got his Diplom in 1972. At the same time he earned his Doctor degree at the University of Pisa, under the supervision of Alberto Tognoli: his Thesis and his first works (publications 1,2,3 in the enclosed list) were devoted to real analytic spaces.

In 1974 he started to work as assistant at the University of Pisa, and he started to work in Algebraic Geometry under the influence of Enrico Bombieri in the three academic years 1974-77, then in the year 1977-78 he was in contact with David Mumford when visiting Harvard as research associate.

He held a chair of Geometry in Pisa from 1980 to 1997, then he was appointed to the Gauss chair in Göttingen in 1997. Since the year 2001-2002 he held the chair for algebraic geometry in Bayreuth.

Quoting from the special issue of the Journal 'Science in China Mathematics', vol. 54/8 (2011), published by Springer Verlag, and dedicated to Fabrizio Catanese on the occasion of his 60th birthday:

'He has had more than 25 PhD students and has supervised many postdocs, most of whom are now themselves successful researchers. Beyond that he has surely influenced many more young mathematicians, since he is always willing to discuss with them and share his ideas.

Since the nineties he is a member of the Accademia Nazionale dei Lincei and of the Akademie der Wissenschaften zu Göttingen (added: in 2015 he was elected member of the Academia Europaea).

He has more than one hundred (154) published mathematical papers. While the main theme is algebraic surfaces, his mathematical interests and knowledge are very wide-ranging: he has authored papers in complex analysis, differential geometry and topology of algebraic varieties, singularities, commutative algebra methods, real algebraic geometry, group theory, etc.'

2. F. Catanese's contributions to the theory of algebraic surfaces, their classification and moduli

In 1914 Enriques and Castelnuovo concluded the 'rough' classification theorem of algebraic surfaces, also called the P_{12}-theorem, dividing the world of surfaces into the 'special surfaces' (those with $P_{12} = 0, 1$, which are explicitly described and belong to a few classes), the properly elliptic surfaces (those for which the 12th canonical system yields a fibration onto an algebraic curve, with fibres elliptic curves) and the surfaces of general type, which are the vast majority (for smooth hypersurfaces in 3-dimensional space, general type means that the degree $d \geq 5$).

The war interrupted this 23 year long collaboration, and after the war, when Enriques wanted to attack the hard task of studying the surfaces of general type, Castelnuovo told him that new methods were in his opinion needed, so Enriques worked until his death on many still open questions, which he proposed in his book 'Le superficie algebriche', which was published posthumously in 1949.

In the meantime new methods, topological ones developed by Lefschetz (and many others), algebraic ones developed for instance by Zariski, differential theoretic and analytical ones due to Kodaira, paved the way for a revival of interest on the subject of algebraic surfaces. The Russian school of Shafarevich, the Harvard school of Zariski and Mumford, Kodaira and his school, the work of Bombieri and Mumford, led to an extension of the rough classification theory to other algebraically closed fields, other than the complex numbers.

Bombieri gave moreover an answer to the questions raised by Enriques concerning the pluricanonical maps of surfaces of general type, and proposed explicit classifications, in the spirit of Enriques, of surfaces of general type with small invariants.

2.1. **Classification of surfaces of general type with low invariants.** Catanese and his school contributed substantially to this theme, which developed in the 70's, first as a rediscovery of many classical works of the Italian school by many authors which are too many to be cited here: especially Horikawa developed satisfactorily the effective use of deformation theory, a major novel theory introduced by Kodaira and Spencer in the 50's, and developed then by Kuranishi, Artin and Grauert.

New tools were available, especially sheaf theory and the new ideas of Grothendieck, which were concretely applied to geometry by Mumford.

The papers 5,6,7,9,10, and later 32, 60, 67, 37, 40, 78, 79, were devoted to classification. The problem usually starts with the data of some numerical invariants, the geometric genus p_g (number of linearly independent regular 2-forms, introduced by Noether), the irregularity q (half of the first Betti number), the selfintersection K^2 of the canonical divisor of the minimal model: the ambitious project is to classify all the surfaces with those invariants.

This was indeed achieved, for many values of the invariants, with the use of new algebraic tools, such as graded rings, methods from homological and commutative algebra , combined with geometric methods (as in the joint works with Ciliberto) and also topological ones. The proofs are extremely elaborate, and very interesting for the ingenuity of the arguments, and the combination of many techniques via a stringent strategy aiming at excluding many possible cases by deductive arguments, which eventually lead to the classification of the remaining cases. Deformation theory must be also used in order to exclude some cases, as depending on too few parameters.

The algebraic methods developed b Catanese and his many students were:

- the theory of abelian coverings of varieties, and their invariants and deformations (see the PhD thesis of Pardini, published on Crelle); the non abelian case was taken up again in 154, in joint work of Catanese and Perroni;
- a general algebraic method to write equations of regular surfaces (22) through the description of the ring structure of the canonical rings; this was later extended in 26 to higher dimensional varieties, and to the irregular case in joint work with Schreier in 80, with an extension to a derived categorical approach done in the thesis of Canonaco, published in the Memoirs of AMS.
- simple results concerning linear systems on non reduced curves and on surfaces, developed with his student Franciosi in 53 (extended in 59 to the non Gorenstein case). The statement for curves extends in a natural way the statement that a line bundle of degree 2g+1 on a smooth curve of genus g is very ample.
- the study of subvarieties of small codimension in projective space, and their defining ideals, developed in 64, 76, and later extended in the master thesis of Böhning, who solved a conjecture of Szpiro, inspired by paper 22.
- the explicit description of the relative canonical algebra of small genus (g=2,3) fibrations, done with Pignatelli in 79; this result is very short and elegant, sheaf theoretic, is global and supersedes drastically many previous long classifications of singular fibres done according to the method of Kodaira.

The construction and classification of surfaces with low invariants was taken up again via the use of new topological methods and the development of powerful deformation theoretic techniques in later works, mostly joint with Bauer or Pignatelli (113, 117, 118, 127, 130, 133, 136, 145).

2.2. **Moduli spaces of surfaces of general type, and their irreducible and connected components.** Probably this difficult problem concerning algebraic surfaces is the one to which Catanese devoted most of his efforts in a time span of 20 years.

The paper 17 was inspired by the question: how much the moduli theory of surfaces differs from the one of curves?

According to a statement of Riemann, the algebraic curves of genus g depend on 3g + 3 parameters, called moduli. The statement was confirmed by results of Ahlfors-Bers, Mumford and Gieseker, showing that the moduli space of curves is an irreducible quasi-projective normal variety of dimension 3g + 3.

For surfaces (notoriously created by the devil) most of the wishful thinking goes wrong. In 17 it was shown that the moduli spaces of surfaces with fixed numerical invariants have many different irreducible components, possibly of different dimensions.

The ground breaking results of Donaldson on the differentiable invariants of 4-manifolds led to the hope (appearing in the literature as a 'speculation' by Friedman and Morgan, calle def= diff) that, at least for simply connected algebraic surfaces, the (oriented) differentiable type would determine a single connected component of the moduli space. Catanese made the opposite conjecture, and it took a long series of papers to show that deformation type and differentiable type differ drastically: 17, 20, 24, 28, 31, 45, 74, 90, 107, 119.

Catanese' student Manetti gave the first examples of (non simply connected) surfaces which are diffeomorphic but not in the same connected components of the moduli space; Catanese gave the simplest examples in 74, as quotients of products of curves.

A real tour de force was achieved in joint work with Wajnryb, (paper 70). Here it was shown that the original examples found in 17 lead to simply connected surfaces, called abc surfaces, which

1) are not in the same connected component of the moduli space (this used the series of results accumulated in the previous 20 years through the study of moderate limits of algebraic surfaces, and the study of quotients of rational double points and their deformations)

2) are diffeomorphic for certain values $(a, b, c = d)$ of some integer parameters: this was done through a very difficult analysis of equivalence of factorizations in the mapping class group.

The story is not yet over. While Catanese showed in 104, via a new theory of symplectic smoothings, that Manetti's examples yield algebraic surfaces which are symplectomorphic for the canonical symplectic structure associated to the canonical class, there remains to find simply connected surfaces, which are canonically symplectomorphic but not deformation equivalent (ie, not in the same connected component of the moduli space). The case of abc surfaces, via a very ingenuous method due to Auroux and Katzarkov, leads however to practically impossible asymptotical calculations of braid group factorizations.

2.3. **Moduli spaces of surfaces and arithmetic.** For curves, which are simple topological objects, the moduli spaces can be defined in quite different ways: using analysis, differential geometry, or algebraic geometry. The result is the same.

For surfaces, Gieseker showed that, fixing the basic numerical invariants, one obtains a quasi projective variety defined over the integers. There remains the question about how to detect its different connected components, and it is clear that surfaces in the same connected component are canonically diffeomorphic (symplectomorphic).

If one studies the moduli spaces form the transcendental approach, than by definition the differentiable structure is fixed. But, in the Gieseker moduli space, even the topology of the surfaces can vary, as we pass from one connected component to another (this phenomenon was first discovered by Jean Pierre Serre).

A nice byproduct, answering a question posed by Grothendieck in his Exquisse du program, was found by Catanese with Bauer and Grunewald in 140 (completing a first attempt done in 109): the absolute Galois group (Galois group of the field of complex algebraic numbers over the field of rational numbers) acts faithfully on the set of components of the moduli spaces of surfaces of general type. In particular, it is shown in 140 that, for any automorphism σ of the complex numbers, not in the conjugacy class of complex conjugation, there is an algebraic surface S such that S and its conjugate surface S^σ have non isomorphic fundamental groups.

3. Long standing-important open questions solved

(1) Counterexamples to local and global Torelli theorems for simply connected algebraic surfaces. These were given in 9,10, answering negatively some questions by Griffiths, who had posed them in his theory of periods of algebraic integrals (extending classical work of Picard, Fuchs, Poincaré..). The results had impact, and were used in work of Andre' and others on motives.
(2) Counterexample to the conjecture of Babbage that if the canonical map of a surface maps to a surface but has strictly positive degree, then the image has geometric genus zero. This example was suggested by Mumford and Gallarati, and was independently obtained by van der Geer-Zagier, and by Beauville. The article 11 contains also a new theory of so called 'even' nodes of a surface in projective 3-space, which was essential in the further study of nodal surfaces, for instance in order to give some explicit upper bound for the number of nodal singularities that a surface of degree d can have. The paper also contains new

results on thetacharacteristics on plane curves, and on hypersurfaces defined by determinants of matrices of polynomials; also new constructions of surfaces, which were applied for instance by R. Barlow to disprove a conjecture by Severi about the non existence of simply connected surfaces of general type with geometric genus zero.

(3) Moduli spaces can be everywhere non reduced (33). These results were used by many authors, notably Vakil established later 'Murphy's law" that any singularity type (up to product with a smooth manifold) can occur.
(4) Together with his student Capocasa, Catanese solved in 35 a fundamental question of complex analysis in several variables, giving sufficient and necessary conditions on a discrete subgroup $\Gamma \subset \mathbb{C}^n$ for the existence of a meromorphic function whose group of periods is exactly Γ. The condition is an extension of the classical Riemann bilinear conditions which lead to the existence of theta functions of several variables. These conditions had been conjectured by Andreotti and Gherardelli on the basis of the solution of the problem given by Cousin in 1910 for the case of two variables (which can be conveniently reduced to the case of elliptic curves). The methods of solution are elementary, and bear resemblance with the case of small denominators appearing in KAM theory.
(5) Together with Casnati, in the paper 62, Catanese solved in the positive a question raised by Barth in 1979, that even sets of nodes on surfaces in 3-space, and more general higher dimensional analogues, are given by the determinantal loci of symmetric maps of vector bundles in projective space. The new technical tool is a clever adaption of new techniques of homological algebra introduced by Walter for the Horrocks correspondence.
(6) In 51, with Kollár, Catanese gave new examples of varieties with non residually finite fundamental groups (the first examples had just been established by Toledo, answering a question by Jean Pierre Serre).
(7) In joint work with Claude Le Brun in the article 65, Catanese gave a negative answer to a classical question posed by Besse: can there exist a differentiable manifold with two Einstein metrics, but one with negative curvature, the other with positive curvature? The answer is: yes, and the result is surprising. Via a clever idea of Le Brun, most of the hard technical work is algebraic geometry and it amounts to show that the Barlow surface has a deformation to a surface with ample canonical bundle, so that one can the apply the Aubin-Yau theorem.
(8) The article 73, joint with Catanese's student Frediani, achieves the rough classification of real algebraic surfaces, mostly done

by the Russian school. The case of tori was done by Silhol, Kharlamov and Itenberg did the Enriques surfaces, in the paper 73 the real hyperelliptic surfaces are classified, the connected components of the moduli space, and the topological types. The classification is long and technically complicated, but the underlying idea is very simple: since the complex variety is a classifying space, the deformation and differentiable type is determined by the real orbifold fundamental group.

(9) Catanese and Bauer, in the article 103, gave a negative answer to a long standing question posed by Enriques, suggesting that the canonical volumes of surfaces with geometric genus 4 should be bounded by 25; they found extremal examples, with canonical volume 45.

(10) Also the articles 106 and 11, joint with Mangolte, contribute to real algebraic geometry. Comessatti in 1916 classified the topology of real rational surfaces, thus giving, as observed by Kollár, a counterexample to one of the conjectures posed by Nash in the 50's. Kollár considered the Nash problem in higher dimension for algebraic varieties, in a remarkable series of papers. These left some open questions open, which were solved in the papers by Catanese and Mangolte through very explicit geometric models of Del Pezzo surfaces and complicated topological and combinatorial analysis.

(11) As already written, a positive answer to Grothendieck's question about faithful actions of the absolute Galois group on moduli spaces, was given in the article 140, joint with Bauer and Grunewald.

(12) In the papers 134, 142 and 148, Catanese and Dettweiler gave a negative answer to a question posed by Fujita in 1982, concerning Variation of Hodge Structures over curves: is the direct image sheaf V of the relative canonical sheaf ω semiample? The authors first, using a vast literature developed by Griffihts, Fujita, Zucker, Schmid, Deligne, Kollár and others , gave a complete proof of a result announced by Fujita and never fully proven: V is the direct sum of an ample and of an unitary flat bundle. After that, the main ingredients are on the one side a theorem stating that a unitary flat bundle is semiample if and only if its monodromy is finite, and then the study of cyclic covers of the line branched in 4 points. The first examples were found through the theory of hypergeometric integrals, later an infinite series of elementary examples (where the theory of Deligne and Mostow is no longer needed) were given in 148. The rigidity of these examples has been proven then by Bauer and Catanese in 155.

4. Seminal work

4.1. **Irregular varieties.** The paper 20 by Catanese contains several open problems which were then approached and solved by several authors. In particular, the question about the structure of the jumping cohomology loci in the Picard variety was refined by Beauville, and that these loci are translates of subtori was proven in remarkable work by Green and Lazarsfeld. In the paper 41 the dual approach, involving the Albanese variety, and proving generalized Castelnuovo de Franchis theorems for the cohomology algebra of Kähler manifolds, was effectively developed. The simplest outcome is the topological nature of certain fibrations, for example maps to curves of genus at least two (Catanese's theorem is a simple linear algebra statement, using easy Hodge theory, simpler than the previous approaches by Siu, Yau, Mok and others , which were using harmonic maps).

Further conjectures by Catanese and Beauville, then proven by Beauville and Simpson, lead to a vast progress on the topic of irregular varieties, and their role in classification; too many authors are here involved, among them the students of Lazarsfeld, Hacon, Pareschi and Popa, Barja, Schnell and many others.

4.2. **Topology, moduli spaces, isogenous varieties.** The results about the topological nature of fibrations developed by Catanese in several papers, lead to the seminal paper 66. It contains many results, about the topological and differentiable nature oh higher genus fibrations. In particular, the simplest theorem to state is that a variety homeomorphic to a product of curves of genus at least two is a product of curves. VIP, varieties isogenous to a product, are proven here to be quotients of a product of curves by the action of a finite group acting freely, and Beauville varieties are defined as those which are rigid: rigidity amounts to the fact that certain subgroups act on the factor curves with quotient the projective line, and branched on three points. From here stems the relation, via Belyi's theorem, to the absolute Galois group.

The article 94, joint with Bauer and Grunewald, translates the classification of Beauville surfaces to some problems in group theory, and raises some conjecture, which is proven in the paper for important classes of simple, groups. This problem has attracted a lot of attention by group theorists, as Lubotzky, Guralnick, Malle, and others, who proved that every simple group which is not cyclic or the alternating group in 5 letters admits a Beauville structure. The case of nilpotent groups as addressed by Boston and his collaborators. There are many interesting questions in group theory related to the paper 105. written by Catanese in collaboration with his student Rollenske, in order to answer a question by le Brun on the slope of Kodaira fibred surfaces.

The study of surfaces isogenous to a product, and of more general quotients, as in 113, has generated an explosion of activity in the classification of surfaces with geometric genus zero (surveyed in 124); these surfaces present particular interest since the long standing open conjecture by Bloch about algebraic cycles asserts that for them the Chow groups are finite dimensional groups.

4.3. **Deformation theory.** The lecture notes by Catanese on deformation theory and its developments, 31, 81, 107, 128 have had a deep influence; in 128 some new results and problems on the Teichmüller space for higher dimenional varieties are posed; several results are contained in 33, 41, 85, 127.

4.4. **Topological methods.** The long recent survey paper 146 addresses the use of classical topological methods for the study of moduli spaces of certain varieties, in particular the so called Inoue type varieties (so defined in honour of a beautiful construction by Masahisa Inoue), defined as quotients by the action of a finite group G of a hypersurface in a projective classifying space. The theory is yet to be more fully developed, and rests on the study of moduli spaces of varieties with a group action. For the case of curves (interesting for the sake of VIP) important results have been obtained in collaboration with Lönne and Perroni in 125, 132, and especially 139, where the new homological invariant for the action of a group on an algebraic curve, introduced in 132, is used for a stable classification of actions, extending a famous theorem by Dunfield and Thurston for the case of a free action.

5. More general influence

Catanese has had over 30 students who wrote their first research papers under his guidance, and who are now actively researching as professors in universities in Italy, France, Germany, UK and China. They were mostly working on quite different subjects, and with the idea that mathematics is more than a specialized subject.

Catanese has also been working on other concrete problems than just algebraic geometry, such as topological classification of level sets of polynomial and algebraic functions (the so called lemniscates of 39, 50, 57, 151), and such as the application of Lefschetz theory to algebraic statistics (93, joint work with Sturmfels and his collaborators), or the theory of foci and caustics 69 and 141; moreover he has also given contributions to the history of mathematics.