

# Advances in Mathematics and Theoretical Physics

Roma, Accademia Nazionale dei Lincei, September 19-22, 2017

*"La filosofia è scritta in questo grandissimo libro che continuamente ci sta aperto innanzi a gli occhi (io dico l'universo), ma non si può intendere se prima non s'impara a intender la lingua, e conoscer i caratteri, ne' quali è scritto. Egli è scritto in lingua matematica, e i caratteri son triangoli, cerchi, ed altre figure geometriche, senza i quali mezzi è impossibile a intenderne umanamente parola; senza questi è un aggirarsi vanamente per un oscuro laberinto."*  
(Galileo Galilei, *Il Saggiatore*, Cap. VI)

The aim of the conference is to present most recent aspects of today research in Mathematics and Theoretical Physics and of their interconnections through the talks by some of the most known and active international researchers. Two Public Lectures are planned to attract a broad audience.

## Speakers

<b>Dorothea Bahns</b> (Göttingen)	<b>Stefan Hollands</b> (Leipzig)
<b>Marcel Bischoff</b> (Athens USA)	<b>Yasuyuki Kawahigashi</b> (Tokyo)
<b>Romeo Brunetti</b> (Trento)	<b>Maxim Kontsevich</b> (IHES)
<b>Thibault Damour</b> (IHES)	<b>Antti Kupiainen</b> (Helsinki)
<b>David Evans</b> (Cardiff)	<b>Gandalf Lechner</b> (Cardiff)
<b>Jürg Fröhlich</b> (ETH Zürich)	<b>Carlangelo Liverani</b> (Roma 2)
<b>Giovanni Gallavotti</b> (INFN - Roma 1)	<b>Andrei Okounkov</b> (Columbia NY)
<b>Irene Giardina</b> (Roma 1)	<b>Giorgio Parisi</b> (Roma 1)
<b>Alice Guionnet</b> (Lyon)	<b>Slava Rychkov</b> (CERN and ENS Paris)

## Public Lectures

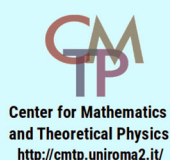
Auditorium Ara Pacis

**Giorgio Vallortigara** (CIMEC Trento)    **Luciano Maiani** (Roma 1 and CERN)

**Scientific Committee:** Enrico Arbarello, *Ciro Ciliberto*, *Corrado De Concini*, *Sergio Doplicher*, *Giovanni Gallavotti*, *Giovanni Jona Lasinio*, *Roberto Longo*, *Giorgio Parisi*.

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## Program

	Tuesday 19	Wednesday 20	Thursday 21	Friday 22
9:00 - 10:00	Registration			
9:30 - 10:20	Welcome address	<a href="#">I. Giardina</a>	<a href="#">M. Kontsevich</a>	<a href="#">C. Liverani</a>
10:25 - 10:55	coffee break			
10:55 - 11:45	<a href="#">G. Gallavotti</a>	<a href="#">T. Damour</a>	<a href="#">Y. Kawahigashi</a>	<a href="#">G. Parisi</a>
11:50 - 12:40	<a href="#">R. Brunetti</a>	<a href="#">G. Lechner</a>	<a href="#">A. Guionnet</a>	<a href="#">M. Bischoff</a>
	lunch break			
14:30 - 15:20	<a href="#">D. Bahns</a>	<a href="#">S. Hollands</a>		<a href="#">S. Rychkov</a>
15:25 - 15:55	coffee break		free afternoon	coffee break
15:55 - 16:45	<a href="#">A. Okounkov</a>	<a href="#">A. Kupiainen</a>		<a href="#">J. Fröhlich</a>
16:50 - 17:40	<a href="#">D. Evans</a>			
18:30 - 20:00		<a href="#">L. Maiani</a> Public lecture Auditorium Ara Pacis		<a href="#">G. Vallortigara</a> Public lecture Auditorium Ara Pacis
20:00 - 23:00			Work dinner LINCEI - Palazzo Corsini	

\*Click on the author's name to see title and abstract

# Table of contents

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<b>Talks: Title and Abstract</b>	<b>1</b>
<a href="#">Dorothea Bahns</a>	1
<a href="#">Marcel Bischoff</a>	2
<a href="#">Romeo Brunetti</a>	2
<a href="#">Thibault Damour</a>	2
<a href="#">David Evans</a>	3
<a href="#">Jürg Fröhlich</a>	3
<a href="#">Giovanni Gallavotti</a>	4
<a href="#">Giardina Irene</a>	4
<a href="#">Alice Guionnet</a>	5
<a href="#">Stefan Hollands</a>	5
<a href="#">Yasuyuki Kawahigashi</a>	6
<a href="#">Maxim Kontsevich</a>	7
<a href="#">Antti Kupiainen</a>	7
<a href="#">Gandalf Lechner</a>	8
<a href="#">Carlangelo Liverani</a>	8
<a href="#">Andrei Okounkov</a>	9
<a href="#">Giorgio Parisi</a>	9
<a href="#">Slava Rychkov</a>	9
<b>Public Lectures: Title and Abstract</b>	<b>10</b>
<a href="#">Luciano Maiani</a>	10
<a href="#">Giorgio Vallortigara</a>	12

[\\*Click on the author's name to see title and abstract](#)

# Talks: Title and Abstract

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## **Dorothea Bahns**

University of Göttingen

### ***Constructive QFT with Minkowski signature - The Sine Gordon model in pAQFT***

After recalling the formalism of perturbative Algebraic Quantum Field Theory (pAQFT), I will explain how it allows to prove finiteness of the S-matrix of the Sine Gordon model in different representations and how the net of local observables can be constructed.

## **Marcel Bischoff**

Ohio University, Athens

### ***Symmetries in Quantum Field Theory beyond Groups***

I will report on some recent progress in understanding generalized symmetries in quantum field theory, in particular, rational conformal field theory in low dimensions.

## **Romeo Brunetti**

University of Trento

### ***Positive, self-adjoint and additive Wick's polynomials of scalar fields***

I wish to present a possible solution to the old problem of self-adjointness of Wick's polynomials by enlarging the Wightman framework as to include additive non linear maps (work done in collaboration with Fredenhagen and Hollands)

## **Thibault Damour**

IHES

### ***Gravitational Waves and Binary Black Holes***

The recent discovery of several gravitational wave events by the two Laser Interferometer Gravitational-Wave Observatory (LIGO) interferometers has brought the first direct evidence for the existence of black holes, and has also been the first observation of gravitational waves in the wave-zone. The talk will review the theoretical developments on the motion and gravitational radiation of binary black holes that have been crucial in interpreting the LIGO events as being emitted by the coalescence of two black holes. In particular, we shall present the Effective One-Body (EOB) formalism which led to the first prediction for the gravitational-wave signal emitted by coalescing black holes. After a suitable Numerical-Relativity completion, the (analytical) EOB formalism has allowed one to compute the bank of 200 000 accurate templates that has been used to search coalescence signals, and to measure the masses and spins of the coalescing black holes.

## **David Evans**

Cardiff University

### ***K-theory and Conformal Field Theory***

I will discuss the programme to understand conformal field theory via twisted equivariant K-theory. In particular, studying module categories and modular invariants for the twisted doubles of finite groups through correspondences as bivariant Kasparov KK elements, and realising the twisted doubles of finite groups as conformal field theories. This has also resulted in a better understanding of the double of the Haagerup subfactor, which was originally thought to be exotic and un-related to known models.

## **Jürg Fröhlich**

ETH Zürich

### ***The Statistical Mechanics of the Universe***

A dilettante's view of some of the central enigmas of cosmology is presented. A proposal for the description of the state of the Early Universe is brought forward. A mechanism possibly explaining the generation of primordial magnetic fields is described and analyzed. The axion model of "Fuzzy Dark Matter" is described and criticized for its instabilities. A model of Dark Energy is introduced and discussed, and a conjectural application of this model to an explanation of the observed matter-antimatter asymmetry is sketched.

## **Giovanni Gallavotti**

INFN - Roma 1

### ***Friction, reversibility and nonequilibrium ensembles***

The Navier-Stokes equations are "irreversible" (are they?). I shall use them as an example of a general proposal for a theory of statistical ensembles, and their equivalence, describing the statistical properties of nonequilibrium stationary states.

## **Irene Giardina**

University of Roma Sapienza

### ***Dynamic scaling in natural swarms***

Collective behavior is widespread in biological systems across many different scales and organisms. As physicists, our hope is that the (complex) details of the individuals are not important when looking at collective properties, and that large scale behavior can be characterized in terms of general laws, much as we do in condensed matter. However, this assumption cannot be given for granted and must be experimentally justified.

In an attempt to improve on this situation, we present here experimental evidence of the emergence of dynamic scaling laws in natural swarms. We find that spatio-temporal correlation functions in different swarms can be rescaled by using a single characteristic time, which grows with the correlation length with a dynamical critical exponent  $z \sim 1$ . We run simulations of a model of self-propelled particles in its swarming phase and find  $z \sim 2$ , suggesting that natural swarms belong to a novel dynamic universality class. This conclusion is strengthened by experimental evidence of non-exponential relaxation and paramagnetic spin-wave remnants, indicating that previously overlooked inertial effects are needed to describe swarm dynamics. The absence of a purely relaxational regime suggests that natural swarms are subject to a near-critical censorship of hydrodynamics.

Authors: A. Cavagna, D. Conti, C. Creato, L. Del Castello, I. Giardina, T.S. Grigera, S. Melillo, L. Parisi, M. Viale

## **Alice Guionnet**

ENS de Lyon

### ***Fluctuations of Discrete Beta Ensembles and Random tilings***

## **Stefan Hollands**

University of Leipzig

### ***Entanglement Measures in Quantum Field Theory***

In quantum theory, there can exist correlations between subsystems of a new kind that are absent in classical systems. These correlations are nowadays called "entanglement".

An entanglement measure is a functional on states quantifying the amount of entanglement across two subsystems (i.e. causally disjoint regions in the context of quantum field theory).

A reasonable measure should satisfy certain general properties: for example, it should assign zero entanglement to separable states, and be monotonic under separable, completely positive maps ("LOCC-operations"). The v. Neumann entropy of the "reduced state" (to one of the subsystems) is one such measure if the state for the total system is pure. But for mixed states, it is not, and one has to consider other measures. In particular, one has to consider other measures if the subsystems have a finite non-zero distance.

In this talk I will present several good measures, and in particular analyze the "relative entanglement entropy",  $E_R$ , defined as the "distance" of the given state to the set of separable states, where "distance" is defined using Araki's relative entropy. I will show several features of this measure for instance: (i) charged states, where the relative entanglement entropy is related to the quantum dimension of the charge, (ii) vacuum states in 1+1 dimensional integrable models, (iii) general upper bounds for certain special regions in general CFTs in  $d$  dimensions, (iv) area law type bounds. I will also explain the relationship between  $E_R$  and other entanglement measures, such as distillable entropy.

[Based on joint work with Jacobus Sanders.]



## **Yasuyuki Kawahigashi**

Tokyo University

### ***Conformal field theory, operator algebras and vertex operator algebras***

We compare two mathematical axiomatizations of chiral conformal field theory. One is a conformal net based on operator algebras and the other is a vertex operator algebra which grew out from the Moonshine conjecture. We present various results on conformal nets and compare them with those on vertex operator algebras with emphasis on comparison of the representation theories. In particular, we show that we can pass from a vertex operator algebra to a conformal net and come back under a mild technical assumption.

## **Maxim Kontsevich**

IHES

### ***Complexified path integral***

I'll present the viewpoint on path integrals via totally real contours in the complexified path space.

## **Antti Kupiainen**

University of Helsinki

### ***Proof of the DOZZ Formula***

In 1994 Dorn and Otto and in 1996 independently Zamolodchikov and Zamolodchikov proposed a remarkable explicit expression, the so called DOZZ formula for the 3 point structure constants of the Liouville Conformal Field Theory (LCFT) which is expected to describe the scaling limit of large planar maps properly embedded in the sphere. I will review a rigorous construction of LCFT and sketch a recent proof of the DOZZ formula obtained together with R. Rhodes and V. Vargas.

## **Gandalf Lechner**

Cardiff University

### ***Yang-Baxter representations of the infinite symmetric group***

The Yang-Baxter equation (YBE) lies at the heart of many subjects, including quantum statistical mechanics, QFT, knot theory, braid groups, subfactors, quantum groups, quantum information ... . In this talk, I will consider mainly involutive solutions of the YBE ("R-matrices"). Any such R-matrix defines a representation and an extremal character of the infinite symmetric group as well as a corresponding tower of subfactors. Using these structures, I will describe how to find all R-matrices up to a natural notion of equivalence inherited from applications in QFT (given by the character and the dimension), how to completely parameterize the set of solutions, and how to decide efficiently whether two given R-matrices are equivalent. Examples include diagonal R-matrices as they appear in DHR theory, or Temperley-Lieb type R-matrices at parameter  $q=2$ .

## **Carlangelo Liverani**

University of Roma Tor Vergata

### ***Energy transport in Hamiltonian systems***

I will review some results on energy transport in weakly coupled classical Hamiltonian systems and discuss ongoing attempts to go beyond the weak coupling limit.

## **Andrei Okounkov**

Columbia University

### ***Gauge theories and Bethe eigenfunctions***

The talk will be based on a joint paper <https://arxiv.org/abs/1704.08746> with Mina Aganagic. In this paper, we essentially complete the program of Nekrasov and Shatashvili who explained the meaning of Bethe roots, Bethe equations, etc. of quantum integrable systems via their correspondence with supersymmetric gauge theories. We explain the meaning of off-shell Bethe eigenfunctions (which also give solutions of the quantum Knizhnik-Zamolodchikov equations and related difference equations). Our formulas may be seen from a geometric, representation-theoretic, combinatorial, and other angles.

## **Giorgio Parisi**

University of Roma Sapienza

### ***Random Matching Theory***

## **Slava Rychkov**

CERN and ENS Paris

### ***Constructing Quantum Field Theories Non-perturbatively with Hamiltonian Methods***

While path integral approach to quantum field theories has come to dominate the field, the Hamiltonian methods have been unjustly neglected. I will discuss some work, inspired by the Rayleigh-Ritz method in quantum mechanics, which uses the Hamiltonian approach to do approximate but precise non-perturbative computations in strongly coupled quantum field theory in 1+1 dimensions.

# Public Lectures: Title and Abstract

## Luciano Maiani

Università di Roma La Sapienza, Roma CERN, Ginevra

### *Elementary particles: the search for simplicity*

The search of the last constituents of matter, the “Elementary Particles”, has fascinated mankind since the beginning of scientific thinking. Modern science has made impressive progresses, discovering a new level of reality beyond atoms and atomic nuclei: quarks, which make now part of our everyday language and of the collective imaginary. The discovery of the “Higgs boson” has marked the success of a description of physical reality based on elements of simplicity, unthinkable only fifty years ago. Along the way, we have discovered fascinating connections between the physics of Microcosm and the large scale structures, galaxies and clusters of galaxies, that have been formed in the Universe since primordial Big-Bang.

There are still mysteries. Among them, the possible existence of new particles, to explain the non-luminous, dark matter gravitating around galaxies, that accounts for more than 90% of the matter present in the Universe. To clarify this and other mysteries, powerful particle accelerators are planned, to reach energies of one order of magnitude larger than the energy of the Large Hadron Collider now working at CERN.

## ***Particelle Elementari: la ricerca della semplicità***

La ricerca dei costituenti ultimi della materia, le “Particelle Elementari”, ha affascinato l’uomo fin dai primi passi del pensiero scientifico. La scienza moderna, nel Novecento, ha compiuto progressi straordinari scoprendo un nuovo livello del mondo reale, oltre l’atomo e il nucleo atomico: i quark, entrati ormai nel linguaggio di ogni giorno e nell’immaginario collettivo. La scoperta del “bosone di Higgs” ha sancito il successo di una descrizione della realtà fisica basata su elementi di semplicità impensabili solo cinquant’anni fa. Lungo questo percorso, si scoprono affascinanti relazioni tra la fisica del Microcosmo e le strutture su grande scala che si sono formate, nell’Universo, a partire dal Big Bang iniziale.

Ci sono ancora importanti misteri. Tra questi, la possibile esistenza di nuove particelle che potrebbero essere la chiave della materia non luminosa che gravita intorno alle Galassie e rende conto di più del 90% della materia esistente nell’Universo. Per chiarire i misteri, sono allo studio nuove macchine con energie superiori di un ordine di grandezza all’energia della macchina del CERN.

## **Giorgio Vallortigara**

Centre for Mind/Brain Science, University of Trento

### ***The Sense of Number***

#### *Evolutionary origins and neurobiology*

The abilities to deal with non-verbal and non-symbolic aspects of numerosity by an approximate number system (or “number sense”) are evolutionarily ancient, and they can be investigated from a comparative perspective. I shall discuss here evidence in non-human animals and human infants showing precocious abilities to represent the cardinal and ordinal aspects of numerical cognition, as well as arithmetic operations. Furthermore, non-human animals and human infants appear to associate smaller numbers with the left space and larger numbers with the right space, mapping numbers to space in a way that resembles adult human mental number line. These abilities are observed very early in development and in the absence (or with very reduced) experience, thus supporting a nativistic foundation of non-symbolic numerical cognition.

*Title and abstract in Italian*

### ***Il Senso del Numero***

#### *Origini evolutive e basi neurobiologiche*

Quali sono le basi neurobiologiche della nostra conoscenza del numero? Sottostanti le capacità matematiche simboliche e discrete che si sono sviluppate in alcune società umane, giacciono, incarnate nell'attività dei sistemi nervosi, le radici non-simboliche della rappresentazione della numerosità. Basate sulla stima della quantità e sul continuo, esse sono osservabili anche nelle creature prive di linguaggio simbolico, i bambini più piccoli e gli animali non umani.