Metal-Insulator Transition in disordered electron systems as a paradigmatic example of a Quantum Phase Transition

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Rome: Enrico Fermi - 100

$$\mathcal{F}_\varepsilon = \tanh\left(\frac{\varepsilon}{2T}\right)$$

At what moment do we need Fermi formula.
if fermionic variables are integrated out?

For example, in the case of the NLσM used for the description of disordered fermionic systems active degrees of freedom are bosonic.

Content of the talk

MIT in disordered electron liquids

Non-linear sigma model, NLσM; in fact, not a model but a minimal functional

Connection with the disordered Fermi-liquid theory

Role of the parameter z in the description of the dynamical properties; relation between ω-T scaling and z

Heat capacitance and heat conductivity

Generalization of NLσM for studying heat transport: gravitational potentials

Heat density - heat density correlation function

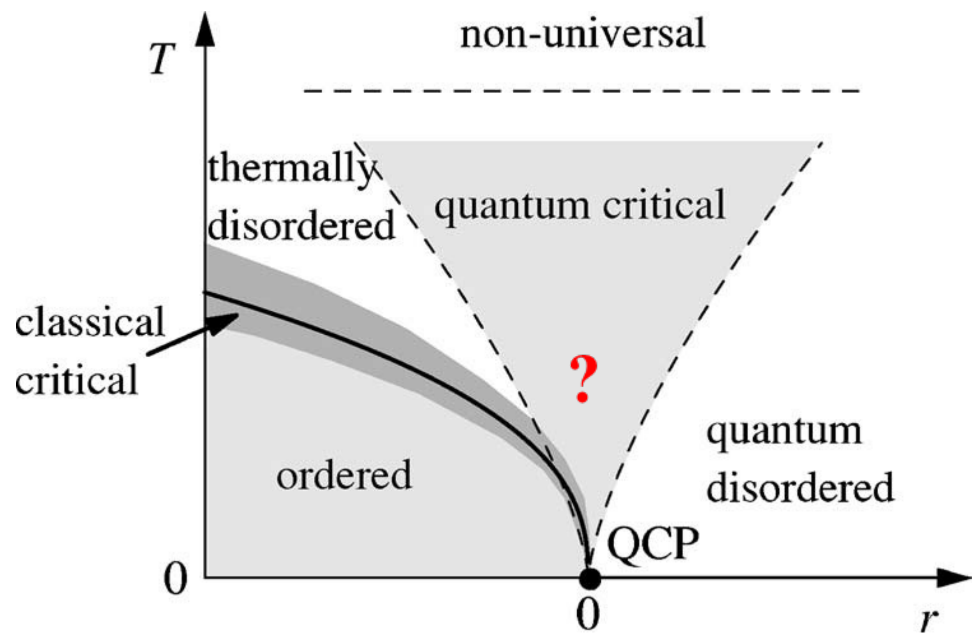
$$\mathcal{F}_\varepsilon = \tanh\left(\frac{\varepsilon}{2T}\right)$$

Sub-thermal corrections in the case of the Coulomb interaction

Why does my old argument based on the gauge invariance doesn't work in the case of the heat transport?

Perspectives of the theory

Quantum Phase Transition (physics of soft modes)

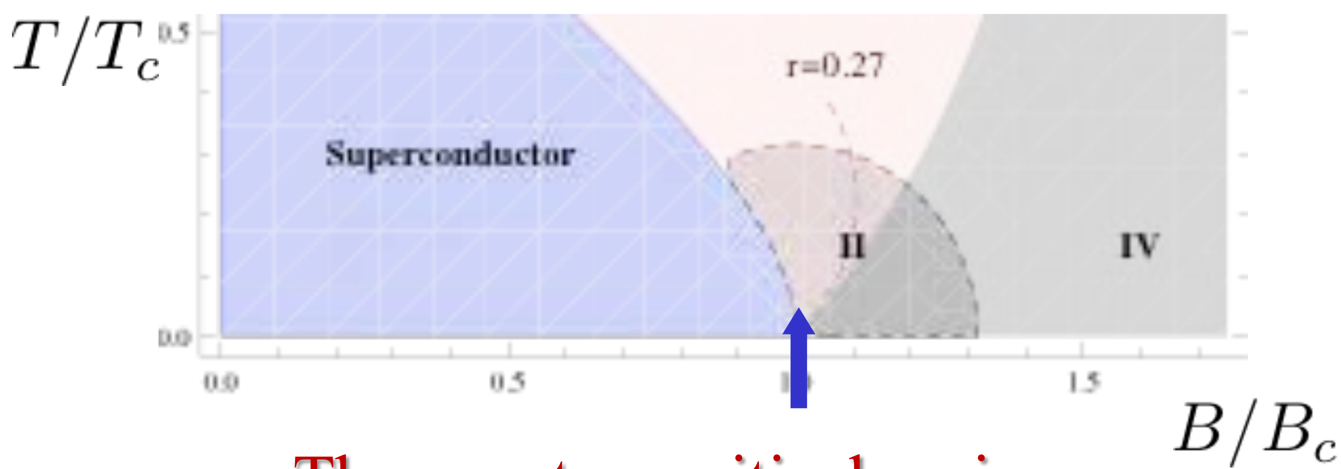


The Quantum Phase Transition (QPT), and the MIT in particular, are characterized by non-conventional behavior of matter in a broad region of the phase diagram which, paradoxically enough, is controlled by a single point at **T=0**, the quantum critical point, where radical changes occur in response a small variation of an external parameter.

“Fluctuation Conductivity in Disordered Superconducting Films”

KT,GS, and AF PRB 85, 174527 2012

Example of a QPT



$$h = \frac{B - B_c(T)}{B_c}$$

$$t = \frac{T}{T_c}$$

The quantum critical regime

There are two distinct regimes:

$$r = \frac{1}{2\gamma}\frac{h}{t}$$

Low temperature: $$\delta\sigma = -\frac{2e^2}{3\pi^2}\ln\frac{1}{h} \quad (t \ll h)$$

Classical regime: $$\delta\sigma = \frac{2e^2\gamma}{\pi^2}\frac{t}{h} \quad (t \gg h)$$

Sign changes!

$$\gamma \approx 1.78$$

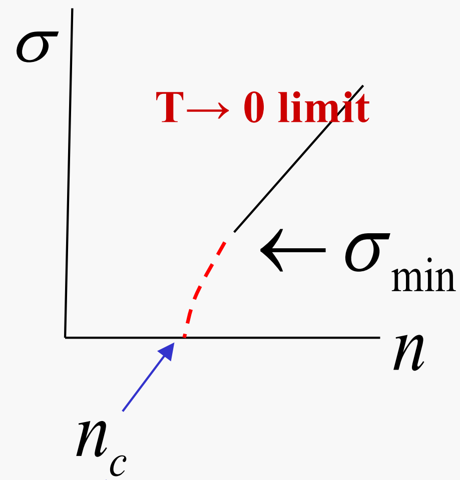
We recover the result obtained by Galitski, Larkin (2001)]

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

Nature of the M-I transition



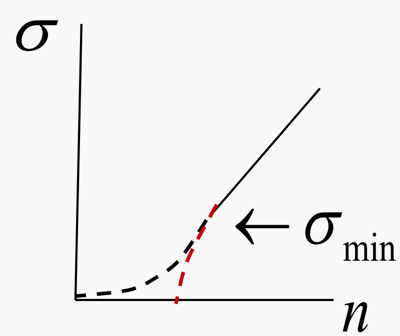
M-I Transition at zero temperature in 3d
(σ denotes conductivity, n – electron density)

Quantum Critical Point (not a jump):

$\xi_{loc} \rightarrow \infty$

Anderson localization:
importance of large scales

$\xi_{loc} >> l_{tr}$



no M - I Transition
(naïve quantum tunneling picture)

MIT in a 3D system as a full-fledged QPT

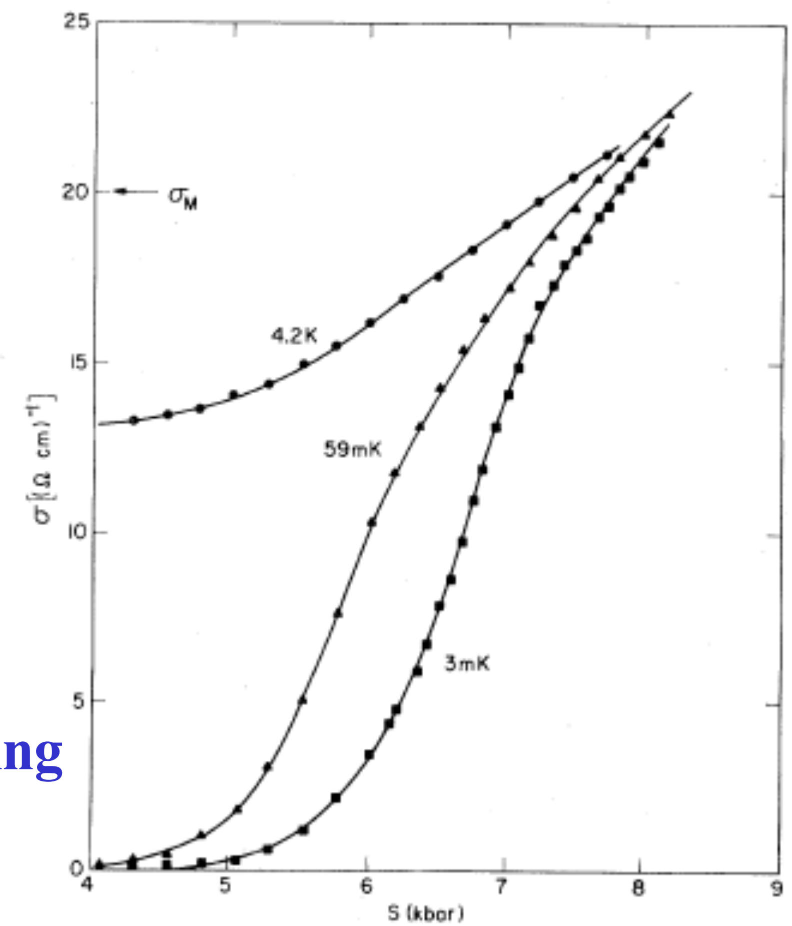
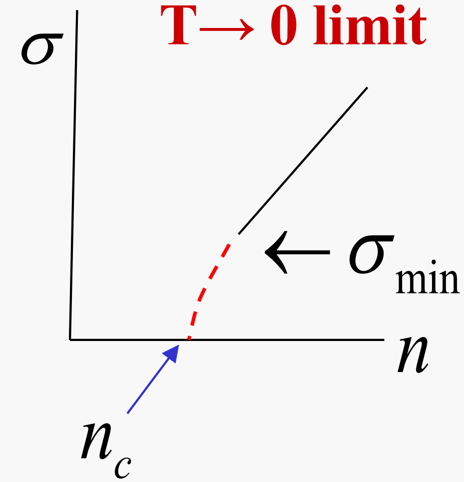
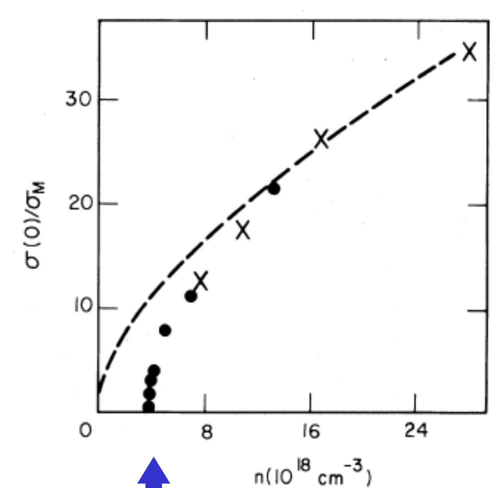


FIG. 3. Conductivity σ as a function of uniaxial stress s at three temperatures for a sample tuned through the metal-insulator transition. Low T is essential for determining the true $T=0$ K behavior.

Si:P
stress tuning



Quantum Critical Point:



Rosenbaum et al.
Phys. Rev. B, 1983

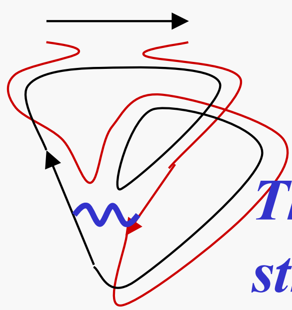
NLσM with *e-e* interaction is a unified scheme which includes disorder and e-e interactions;
charge, spin, and valleys

Two steps: 1) to perform the RG flow procedure, then
2) to link the RG-parameters with the physical quantities

*AF 1983-84 (Finkel'stein's "**model**");*
*G.Schwiete & A. F. 2014 re-derivation of all previous results in **Keldysh technique**;*
*Generalization of **NLσM** for heat transport G. Schwiete & A.F. 2016*

$$S[Q]=\frac{\pi\nu}{4}\int \mathrm{Tr}\left[D\left(\vec{\nabla}Q\right)^2-4z\,\mathrm{Tr}\left(\hat{\varepsilon}Q\right)+Q(\hat{\Gamma}_s+\hat{\Gamma}_t+\hat{\Gamma}_c)Q\right]d^2r$$

$Q_{nm}^{\alpha\beta\ ij}$ $\quad TrQ=0 \quad Q^2=I$



The main advantage of the NLσM is that the functional itself fixes the structure of the theory, for example, for scaling description of the MIT.

parameter **z** describes renormalization of the DOS of the diffusion modes;

z can be joined with ν

How should one understand the NLσM with e-e interactions ?

$$S[Q]=\frac{\pi\nu}{4}\int \mathrm{Tr}\left[D\left(\vec{\nabla}Q\right)^2-4z\mathrm{Tr}\left(\hat{\varepsilon}Q\right)+Q(\hat{\Gamma}_s+\hat{\Gamma}_t+\hat{\Gamma}_c)Q\right]d^2r$$

$$TrQ=0 \qquad Q^2=I$$

"Quasi-classical" approximation when only slow space and time motions are kept →

A non-abelian *bosonization* (fermionic degrees are integrated out) scheme which correctly incorporates all symmetries and conservation laws:

particle number, spin conservation, and also energy conservation.
(this imposes important relationships between parameters of the NLσM)

Minimal functional which can be considered as semi-phenomenological - semi-microscopical starting point for description of disordered interacting electron systems

Why the NLσF is so robust?

Why does it preserve its form in the process of the RG, despite that in the process of the RG-procedure all channels are mixed ?

Because it is a minimal functional, rather than model!

$$S[Q]=\frac{\pi\nu}{4}\int \mathrm{T}r\left[D\left(\vec{\nabla}Q\right)^2-4z\mathrm{Tr}\left(\hat{\varepsilon}Q\right)+Q(\hat{\Gamma}_s+\hat{\Gamma}_t+\hat{\Gamma}_c)Q\right]d^2r$$

$$TrQ=0 \qquad Q^2=I$$

Minimal functional which incorporates all conservations laws and all symmetries of a disordered interacting electron system

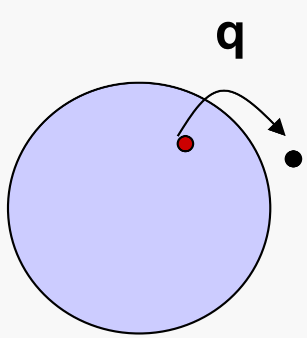
On the level of the Gaussian fluctuations *S*[Q] reproduces (trivially !) the disordered Fermi liquid

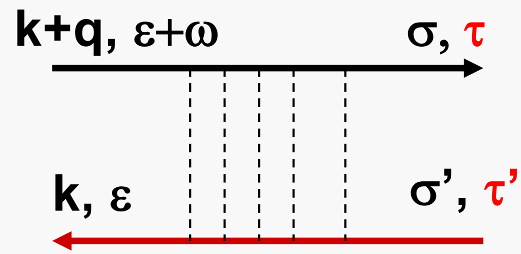
low energy physics is described by two-particle propagators
(rather than single-particle Green's functions);
they describe fluctuations of charge, spin and valley **densities**

Disorder-averaged two-particle propagators

semiconductors: spin + valleys

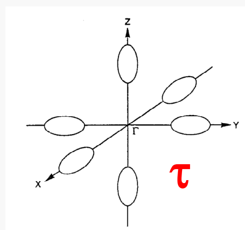
$$\left(\frac{\partial}{\partial t} - \nabla^2\right)$$



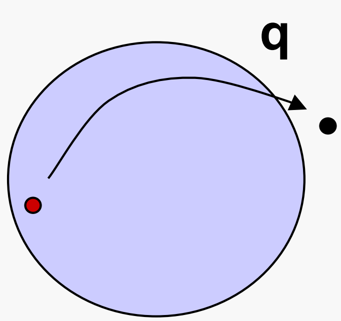


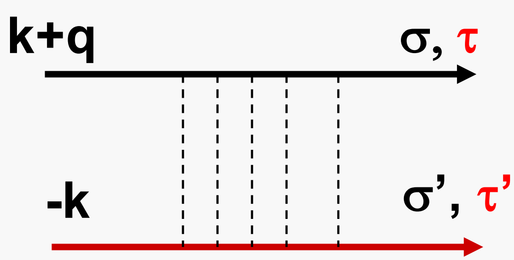
$$\approx \frac{1}{Dq^2 - i\omega}$$

particle-hole propagator (diffuson)



diffusive pole





$$\approx \frac{1}{Dq^2 - i\omega}$$

particle-particle propagator (cooperon)

quadratic expansion of the non-linear σ–functional (Gaussian fluctuations) corresponds to the Fermi-liquid theory in the presence of disorder

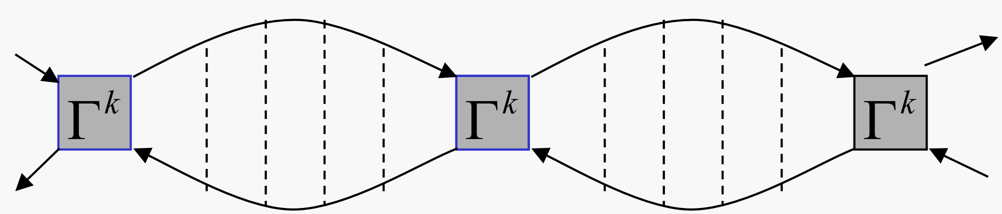
$$\tau_{tr}^{-1} >> \varepsilon, T \qquad \frac{1}{\varepsilon_F \tau_{tr}} << 1$$

multiple re-scattering of pairs of quasiparticles (**each time two into two**) leads to **diffusion** (plane waves are not eigenstates anymore :

$$\left(\frac{\omega}{\omega - \vec{v}\,\vec{k}} \right)_{clean} \Rightarrow \left(\frac{\omega}{\omega + iDk^2} \right)_{disorder}$$

Static amplitudes remain unchanged in the leading order $1/\varepsilon_F \tau_{tr} << 1$

dynamic amplitude of scattering in the particle-hole channel; only $l = 0$ harmonic survives disorder $\Rightarrow$



z can be joined with ν

Besides resistance-interaction system of the RG-equations there is

one more RG-equation: $$\frac{d \ln z}{d\xi} = \varsigma(\rho, \Gamma_\sigma / z)$$

ρ **- dimensionless resistance of a d-dimensional cubic sample, which stands constant during the RG at the fixed point corresponding to M-I transition**

For free electrons z=1 and not renormalized.
In the presence of e-e interactions, z flows even at the fixed point! Equation for z stands separately from the rest; temperature and external frequency enter as cutoffs.

one more RG-equation:

$$\frac{d \ln z}{d\xi} = \varsigma(\rho, \Gamma_\sigma / z)$$

$$\rho = \frac{r_d(\kappa)}{2\pi^2\hbar/e^2} \propto \frac{e^2}{\hbar\sigma}\kappa^{d-2}. \qquad (3.1)$$

Here, ρ is equal to the resistance r_d of a d-dimensional cube of side length $\sim 2\pi/\kappa$ measured in units of $2\pi^2\hbar/e^2$ [N.23]; κ is the momentum cutoff which decreases during the renormalization [N.24].

At the M-I transition, resistance ρ doesn't depend on scale: fixed

Then, as it follows from Eq. (3.1), in the vicinity of the transition,

$$\sigma(\kappa)/e^2 \propto \kappa^{d-2}. \qquad (3.8)$$

In the 3d case, for example, on the metallic side of the transition the critical behavior develops when $\kappa \gg \sigma(T=0)/e^2$. At non-zero temperature, in the critical regime of the MIT the process of renormalization ceases at a scale when

$$D(\kappa)\kappa^2/z(\kappa) \sim T \quad \underset{\text{using Eq. (2.11)}}{\Longrightarrow} \quad \kappa^d/\nu \sim zT. \qquad (3.9)$$

For the electric conductivity measured at external frequency $\omega \gg T$, the renormalization is cut off by ω rather than T. The above relations are a

Thus, in order to find the temperature or frequency behavior of σ at the MIT, one has to connect the momentum and energy scales in the critical region, $\kappa \sim (z \max[\omega, T])^{1/d}$. However, z itself is a scaling parameter, see Eq. (3.6). Therefore, one needs to know the critical behavior of the parameter z at the transition, which is determined by the value of $\rho\beta_z$ at the critical point:

$$\sigma(\omega, T) \sim (z \max[\omega, T])^{\frac{d-2}{d}} \sim (\max[\omega, T])^{\frac{d-2}{d}(1+\tilde{\zeta})}, \tag{3.10}$$

$$\tilde{\zeta} = -(\rho\beta_z)_{\text{critical point}} \, . \tag{3.11}$$

For free electrons z is not renormalized, and at zero temperature $\sigma(\omega) \sim \omega^{1/3}$ for $d = 3$.[26] The e–e interaction modifies this critical behavior of the conductivity through the critical exponent $\tilde{\zeta}$. If $\omega \lesssim T$, the renormalization procedure is cut off by the temperature

$$\sigma(T) \sim (zT)^{\frac{d-2}{d}} \sim T^{\frac{d-2}{d}(1+\tilde{\zeta})}. \tag{3.12}$$

The scaling behavior described above suggests that the interplay between frequency and temperature can be described by a single function

$$\sigma(T, \omega)_{\text{critical}} = T^a f(\hbar\omega/k_b T), \tag{3.13}$$

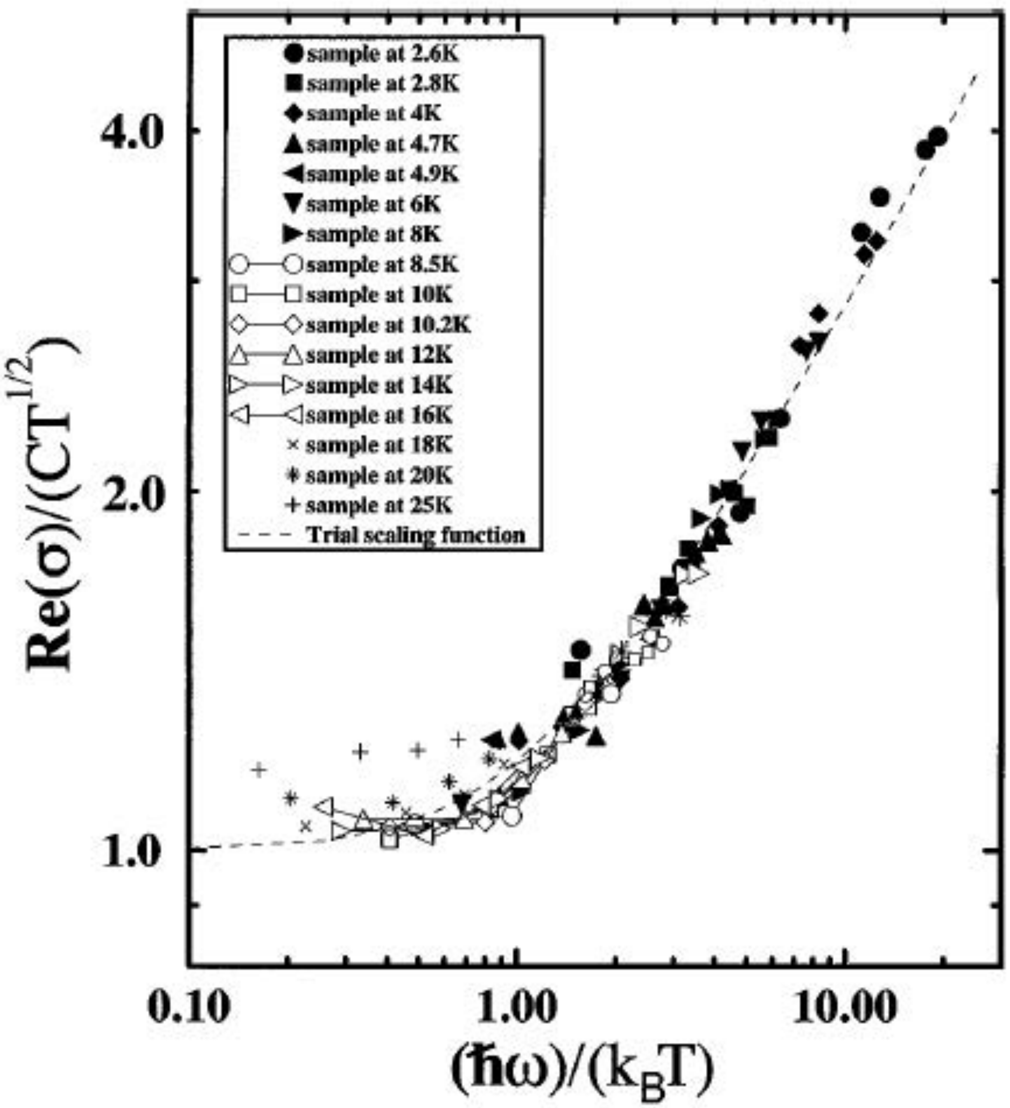


FIG. 3. Log-log plot of scaled conductivity data versus scaled frequency with the factor $C = 475\ (\Omega\,\mathrm{m}\,\mathrm{K}^{1/2})^{-1}$. The uncertainty varies inversely with $T^{1/2}$, ranging from 20% at 2.6 K to 6% at 25 K. When temperature is 16 K and below, the data for the entire frequency range collapse into one curve within the experimental noise. Higher temperature data

H.-L. Lee, … Gruner
1998, 2000, **NbSi**

Newson & Pepper (1986) $T^{1/3}$

In the lowest order of the ε-expansion, in the presence of magnetic field, z-index is zero, i.e. 1/3 …

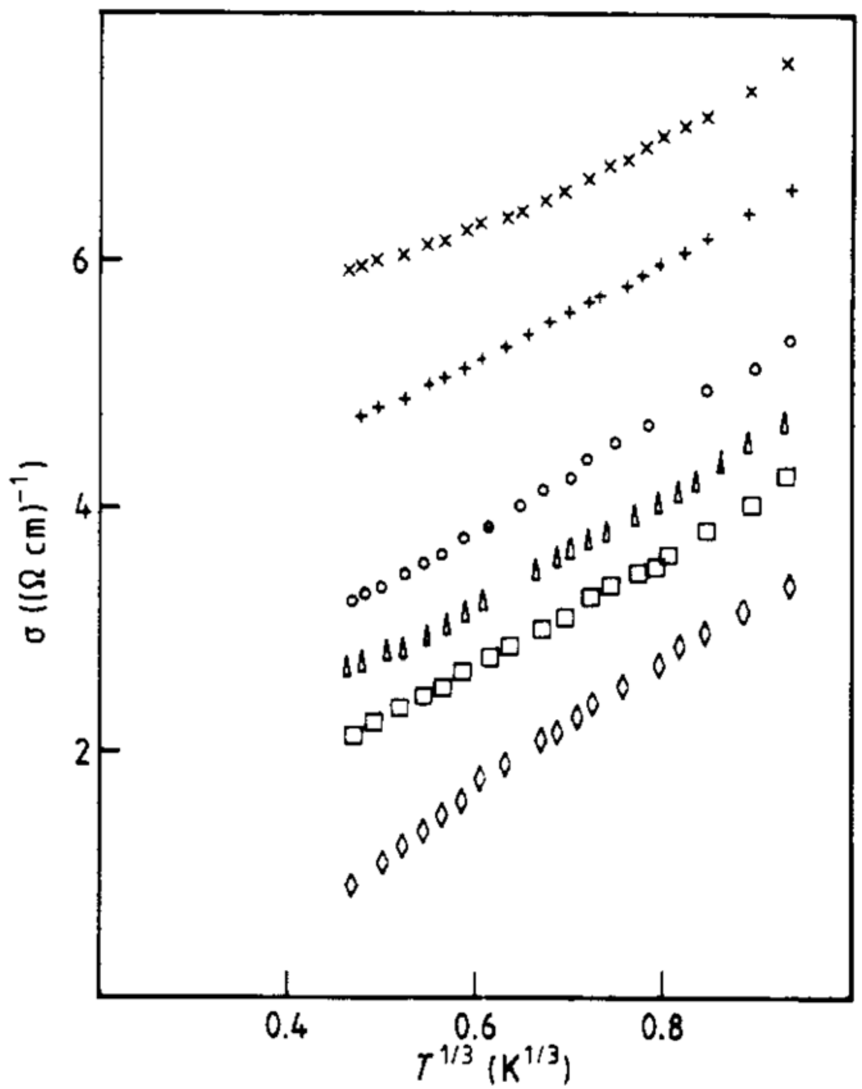


Figure 7. σ against $T^{1/3}$ for the GaAs sample for transverse magnetic fields in the proximity of the critical field ($\simeq$7.8 T). (The symbols are as for figure 3.)

There is no other way to get conductivity's exponent different from 1/3 other than via z.

The result was obtained immediately after the derivation the NLσM, AF (1983)

z z z Z Z

Why paradigmatic example:

Relation between momentum-frequency dimensions was determined not by heuristic arguments but by an equation which was calculated, and which gave different critical exponents for different universality classes of symmetry.

For a QPT, a frequency integration must be performed with participation of the soft modes, i.e., propagation functions of the dynamic critical fluctuations

$$\chi(\boldsymbol{q}, \Omega_m) \; = \; \chi_0 \xi^2 / (1 + (\boldsymbol{q} - \boldsymbol{Q})^2 \xi^2 + |\Omega_m| / \omega_{\mathrm{sf}})$$

Ornstein-Zernike (dynamic)

J. A. Hertz, “Quantum critical phenomena,” *Phys. Rev. B*, vol. 14, p. 1164, 1976.

T. Moriya, *Spin Fluctuations in Itinerant Electron Magnetism*. Berlin: Springer, 1985.

A. J. Millis, *Effect of a nonzero temperature on quantum critical points in itinerant fermion systems*, 1993.

can this frequency-term be renormalized at a QPT?

Conclusion: The metal–insulator transition in a system of diffusing electrons is an example of a quantum phase transition[32] with a temperature-frequency scaling controlled by the parameter z. Precisely the same parameter describes the scaling behavior of both the conductivity and the thermodynamics in the critical region of the transition. The structure of the theory is very general and not related to the ϵ-expansion which can be used for the calculation of $\widetilde{\zeta}$.

z had met a strong opposition in Soviet Union in eighties. The argument was that since z can be excluded by redefinition of all quantities, it is unphysical.

however,

Castellani Di Castro 1986: $c = zc_{FL}$

and later two-loop calculation by Baranov, Pruisken, Skoric' 1999

How to observe *Z* directly?

Measurements of the heat capacitance (specific heat) near the MIT are very limited due to the small number of electrons

What about heat transport?

Heat transport in disordered electron systems

How to generalize the RG-approach to thermal transport?

What are the consequences of replacing the electric field by a temperature gradient ?

Heat transport in disordered electron systems

The scaling theory used to determine transport coefficients included the calculations of correlation functions found by introducing source fields into the NLσM, and using the Einstein relations.

How to account for a temperature gradient? How to introduce the source

$$\mathrm{Nl}\sigma\mathrm{M} \quad \overset{source\ \varphi}{\longrightarrow} \quad \langle nn \rangle \quad \overset{Einstein}{\longrightarrow} \quad \sigma$$

Our approach: extend the NlσM with source fields;
study renormalizations of the fields;
calculate correlation functions; extract transport coefficients
(Luttinger's "gravitational potential").

$$\mathrm{Nl}\sigma\mathrm{M} \quad \overset{source\ ??}{\longrightarrow} \quad \langle kk \rangle \quad \overset{Einstein}{\longrightarrow} \quad \kappa$$

G. Schwiete and AF 2014-2016

Heat transport in disordered electron systems; Source fields for the heat density-density correlation function

Action:

$$S[\psi^*,\psi] = \int_{\mathbf{r},t} \left(\psi^* i\partial_t \psi - k[\psi^*,\psi]\right)$$

$$\mathcal{Z} = \int D(\psi,\psi^*)\, \mathrm{e}^{iS} \qquad k = h_0 + h_{int} - \mu n$$

$$S[\psi^*,\psi] = \int_{\mathbf{r},t} \left(\psi^* i\partial_t \psi - (1+\eta) k[\psi^*,\psi]\right)$$

Luttinger (1964)

$$\chi_{kk} = \frac{i}{2} \frac{\delta^2 \mathcal{Z}}{\delta \eta_{\mathbf{r}_1 t_1} \delta \eta_{\mathbf{r}_2 t_2}}$$

Gravitational potential acts on all terms in *k* !
As a result, many potentials arise.

Heat transport in disordered electron systems; Source fields for the heat density-density correlation function

$$S[\psi^*,\psi] = \int_{\mathbf{r},t} (\psi^* i\partial_t \psi - (1+\eta) k[\psi^*,\psi])$$

"crazy" term:

$$S_{dis} = -\int_{\mathbf{r},t} (1+\eta)\psi^* u_{dis}\psi$$

Change of variables:

$$\psi \to \frac{1}{\sqrt{1+\eta}}\psi \qquad \psi^* \to \psi^* \frac{1}{\sqrt{1+\eta}}$$

After this transformation, the derivation of the NLσM is straightforward:

$$S[Q] \sim \int d\mathbf{r}\ \mathrm{tr}\left[D(\nabla Q)^2 + 2iz\{\hat{\epsilon},\lambda\}Q\right] + Q\lambda(\Gamma_1+\Gamma_2)Q$$

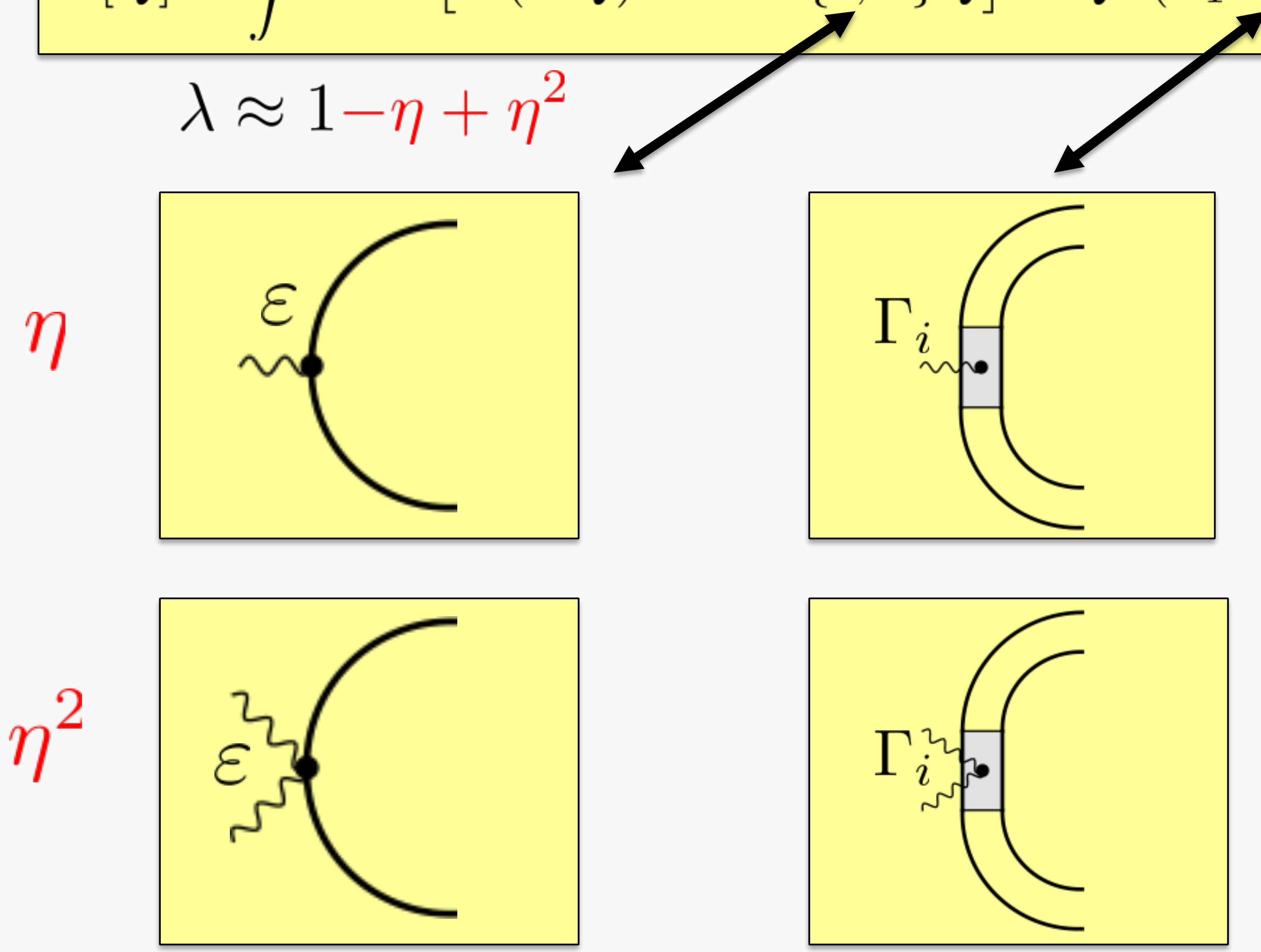
$$\lambda = \frac{1}{1+\eta} \approx 1 - \eta + \eta^2 + \ldots$$

the price:
now, nonlinear in η!

NlσM with "gravitational potentials"

$$S[Q] \sim \int d\mathbf{r}\ \mathrm{tr}\left[D(\nabla Q)^2 + 2iz\{\hat{\epsilon}, \lambda\}Q\right] + Q\lambda(\Gamma_1 + \Gamma_2)Q$$

$$\lambda \approx 1 - \eta + \eta^2$$



Technical (very unpleasant) details:

although initially there was only one gravitational potential,

$$S = \int \mathrm{tr}[D(1+\zeta_D)(\nabla Q)^2 + 2iz\{\hat{\varepsilon}, 1+\zeta_z\}Q] + \sum_{i=1,2} Q(1+\zeta_{\Gamma_i})\Gamma_i Q$$

in the course of the RG procedure, a separate coefficient was generated for each term in the action S. Fortunately, the ultimate result appears to be very simple. All the coefficients merge back to the original value:

$$\boxed{\Delta\zeta_D = \Delta\zeta_z = \Delta\zeta_{\Gamma_1} = \Delta\zeta_{\Gamma_2} = 0}$$

Fortunately, fixed point for the gravitational potentials!!!

This holds only for the correct initially conditions!
Otherwise, deep problems !

Initial conditions:

$$\zeta_D = 0 \quad \zeta_z = \zeta_{\Gamma_1} = \zeta_{\Gamma_2} = -\eta$$

Calculation of the specific heat (do we need the Fermi's formula?) I

$$\mathcal{F}_\varepsilon = \tanh\left(\frac{\varepsilon}{2T}\right)$$

$$S[Q] \sim \int d\mathbf{r}\ \mathrm{tr}\left[D(\nabla Q)^2 - 2z\{\hat{\epsilon},\lambda\}Q\right] + Q\lambda(\Gamma_1+\Gamma_2)Q$$

$$k^d_{\eta=0} = \frac{i}{2}\left.\frac{\delta\mathcal{Z}}{\delta\eta}\right|_{\eta=0} = \frac{1}{2}\int_{\mathbf{q},\omega}\underline{\mathcal{B}_\omega} D\mathbf{q}^2(z_1\mathcal{D}_1\overline{\mathcal{D}}_1 + 3z_2\mathcal{D}_2\overline{\mathcal{D}}_2 - 4z\mathcal{D}\overline{\mathcal{D}})\omega$$

Diffusion with re-scattering in the singlet and triplet channels:

$$\mathcal{D}_{1,2} = \frac{1}{D\mathbf{q}^2 - iz_{1,2}\omega} \qquad \mathcal{D} = \frac{1}{D\mathbf{q}^2 - iz\omega} \qquad \begin{aligned} z_1 &= z - 2\Gamma_1 + \Gamma_2 \\ z_2 &= z + \Gamma_2 \end{aligned}$$

Specific heat:

B_ω =cot(ω/2T)-bosonic function

$$\delta c = \partial_T k^d_{\eta=0} = \frac{1}{2}\int_{\mathbf{q},\omega}\partial_T\underline{\mathcal{B}_\omega}\ D\mathbf{q}^2(z_1\mathcal{D}_1\overline{\mathcal{D}}_1 + 3\mathcal{D}_2\overline{\mathcal{D}}_2 - 4z\mathcal{D}\overline{\mathcal{D}})\omega = zc_{FL}$$

Reproduces the result of Castellani and Di Castro (1986) within the Keldysh scheme

The correlation functions – "microscopical phenomenology”: general form of the correlation functions of conserving quantities in the diffusive limit.

Connection of the static limit of the correlation functions with thermodynamic

$$\chi_{kk} = -cT \frac{D_k \mathbf{q}^2}{D_k \mathbf{q}^2 - i\omega}$$

$$\chi_{nn} = -\frac{\partial n}{\partial \mu} \frac{D_n \mathbf{q}^2}{D_n \mathbf{q}^2 - i\omega}$$

Static limit

$$\chi_{kk}(\mathbf{q} \to 0, \omega = 0) = -cT$$

$$\chi_{nn}(\mathbf{q} \to 0, \omega = 0) = -\frac{\partial n}{\partial \mu}$$

Conservation law

$$\chi_{kk}(\mathbf{q} = 0, \omega \to 0) = 0$$

$$\chi_{nn}(\mathbf{q} = 0, \omega \to 0) = 0$$

$$\kappa = cD_k$$

$$\sigma = e^2 \frac{\partial n}{\partial \mu} D_n$$

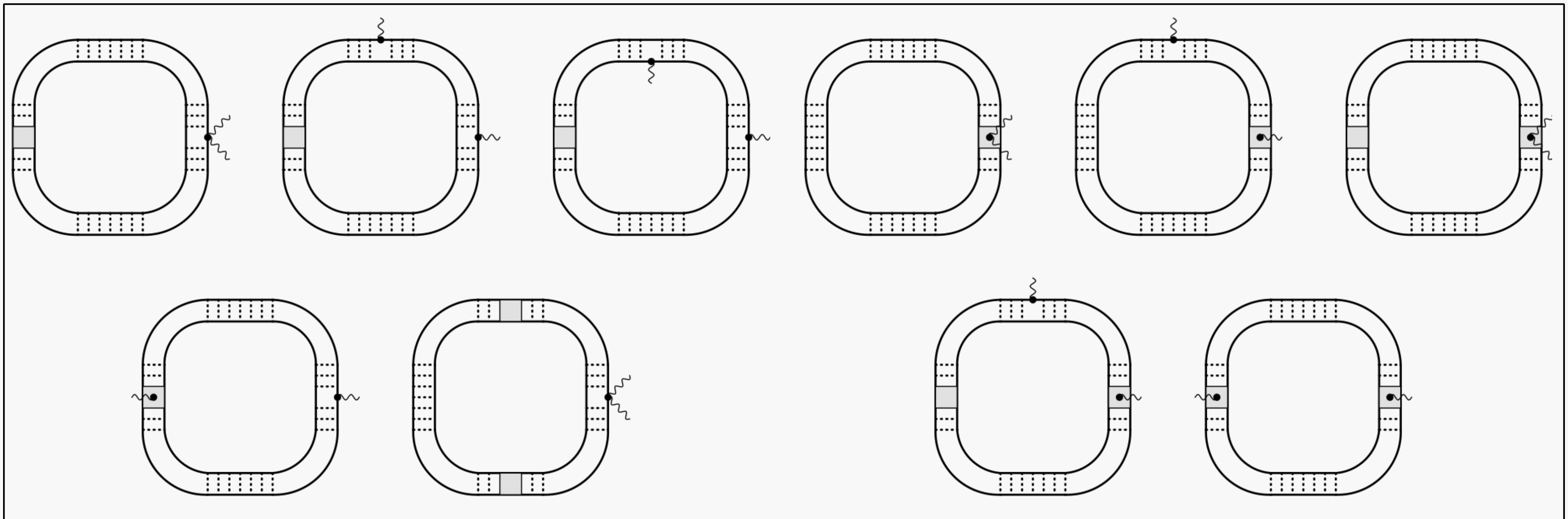
Thermal conductivity using the continuity equation

$$\kappa = -\frac{1}{T} \lim_{\omega \to 0} \lim_{k \to 0} \left[\frac{\omega}{k^2} \mathrm{Im} \chi_k(k, \omega) \right].$$

Diffusion coefficients are different for charge, spin and heat!

Calculation of the static part of the <heat density - heat density> correlation functions (another way to determine specific heat) II

$$\chi^{st}_{kk} = \frac{i}{2}\frac{\delta^2 Z}{\delta\eta^2} = -Tc$$

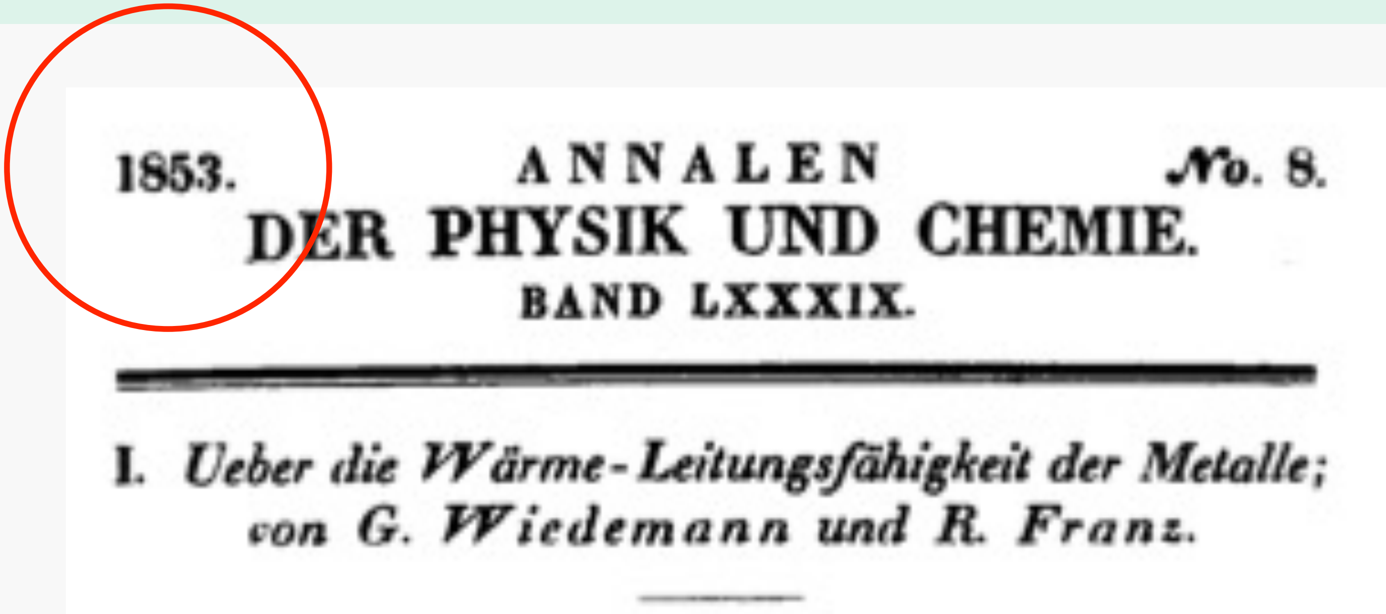


$$c = z c_{FL}$$

We have checked that the static limit reproduces thermodynamics correctly.

Now, we can extract the heat conductivity from dynamic χ → WFL?

The Wiedemann-Franz law (WFL)

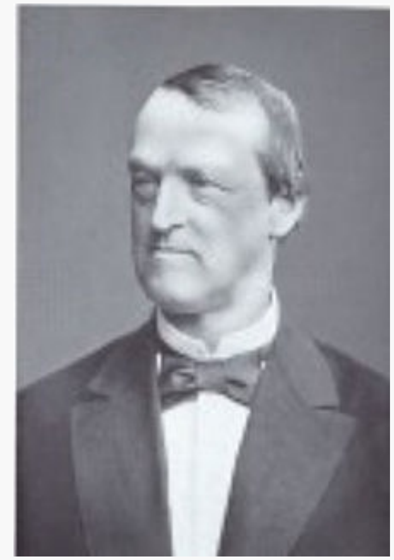


1853. ANNALEN No. 8.

DER PHYSIK UND CHEMIE.

BAND LXXXIX.

I. *Ueber die Wärme-Leitungsfähigkeit der Metalle;*
von G. Wiedemann und R. Franz.



G. Wiedemann

1853 !

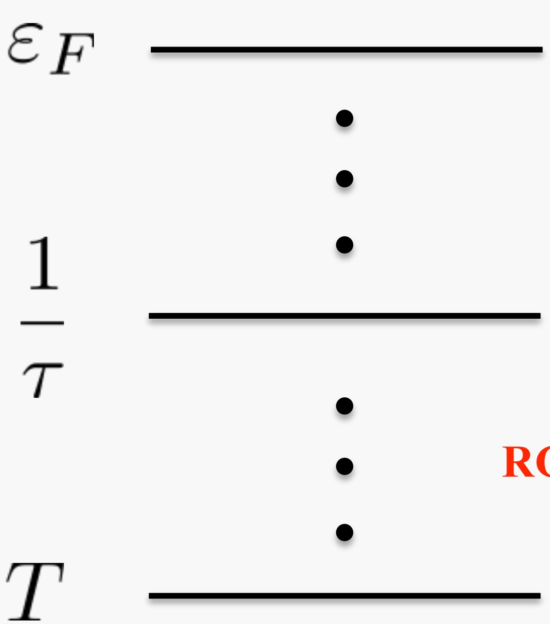
Nowadays: the WFL is used as a criterion for **non-Fermi Liquid** systems

Thermal transport and the Wiedemann Franz "Law"
Within the "crude" RG (without sub-thermal energies) the WFL holds

$$S[Q] \sim \int d\mathbf{r}\ \mathrm{tr}[D(\nabla Q)^2 + 2iz\{\hat{\varepsilon}, \lambda\}Q] + \sum_i Q\lambda\Gamma_i Q$$

$$\lambda \approx 1 - \eta + \eta^2$$

Energy scales



Georg Schwiete & AF, with Keldysh NLσM
PRB **89** (2014); PRB **90** (2014)(R); PRB **90** (2014); PRB **93** (2016);

Semi-review:
Georg Schwiete and AF in JETP 2016, vol 122, p 567

An issue in honor of L.V. Keldysh

RG regime

WFL holds

-disordered Fermi liquid

-disordered electron liquid
(long-range Coulomb interaction)

up to here WFL holds despite strong renormalizations;

we must now enter the sub-thermal interval

Full scheme versus “crude” RG: includes the sub-thermal energies interval;

Sub-thermal shell is controlled by the *u*-matrices.

At last, the Fermi’s formula is explicitly here!

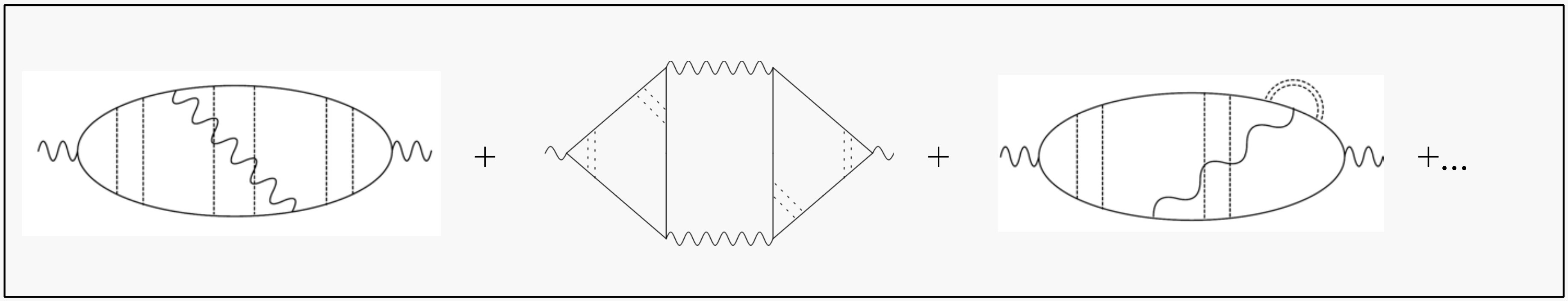
The matrix $\hat{Q}$ can be parametrized as $\hat{Q} = \hat{U}\hat{\sigma}_3\hat{\bar{U}}$, where $\hat{U}\hat{\bar{U}} = 1$; the deviations $\delta\hat{Q} = \hat{Q} - \hat{\sigma}_3$ describe diffusive degrees with energies $\lesssim 1/\tau$. For $\underline{\delta\hat{Q}}(\varepsilon\varepsilon') = u_\varepsilon \delta\hat{Q}_{\varepsilon\varepsilon'} u_{\varepsilon'}$ the temperature of electrons enters through the distribution function encoded in $\hat{u}$:

$$\hat{u}_\varepsilon = \begin{pmatrix} 1 & \mathcal{F}_\varepsilon \\ 0 & -1 \end{pmatrix}, \quad \mathcal{F}_\varepsilon = \tanh\left(\frac{\varepsilon}{2T}\right). \tag{9}$$

For **short-range interactions** no additional (log) corrections

For **long-range Coulomb interactions** **additional logarithmic corrections** arise from processes with **sub-T frequency transfers**.

Additional logarithms arise from the long-range Coulomb interaction in the sub-thermal interval. The WFL is violated



All contributions are proportional to Im(V^R):

ν **- is the frequency transferred by the dynamic Coulomb interaction**

Example:

$$\delta\chi_{kk} \propto \int_{\mathbf{k},\varepsilon,\nu} \varepsilon\nu\partial_\varepsilon\mathcal{F}_\varepsilon(\mathcal{F}_{\varepsilon+\nu}+\mathcal{F}_{\varepsilon-\nu})\mathrm{Re}\mathcal{D}^2(\mathbf{k},\nu)\mathrm{Im}V^R(\mathbf{k},\nu)$$

(screening of the Coulomb interaction contains dynamics)

For dynamically screened **long-range Coulomb interactions** **additional logarithmic corrections** arise from processes with **sub-T frequency transfers**.

the inequality $|\nu|/(D\kappa_s) < k < \sqrt{|\nu|/D}$, where $\kappa_s = 4\pi e^2 \nu_0$ is the inverse screening radius. In this interval, we can approximate the dynamically screened interaction as

$$\tilde{\Gamma}^R_{0;d}(\mathbf{k}, \nu) \approx \frac{1}{2(1+F_0^\rho)^2} \frac{i\nu}{D\mathbf{k}^2}. \tag{43}$$

(screening contains dynamics)

Eventually, the bare $1/D\mathbf{k}^2$ singularity gives rise to logarithmic corrections. It is now clear that the contributions from the sub-temperature interval are important in the case of the dynamically screened Coulomb interaction, for which $\tilde{\Gamma}^R_{0;d}(\mathbf{k}, \nu)$ is singular. For a short-range interaction, the discussed interval of frequencies does not exhibit any singularity and, therefore, is not important.

ν **- is the frequency transferred by the dynamic Coulomb interaction**

Additional logarithmic corrections to the heat conductivity arise from the dynamic Coulomb interaction only (screening contains dynamics)

$$\chi_{kk} = -cT\frac{D_k\mathbf{q}^2}{D_k\mathbf{q}^2 - i\omega}$$

$$D_k = \frac{1}{z}\left(D_n + \delta D^h\right)$$

Consistent with the conservation of energy

Additional logarithmic correction to κ:

$$\delta\kappa = \frac{T}{12}\log\frac{D\kappa_s^2}{T}$$

W-F Law is violated!

κ_s - inverse of the screening radius

From the interval:

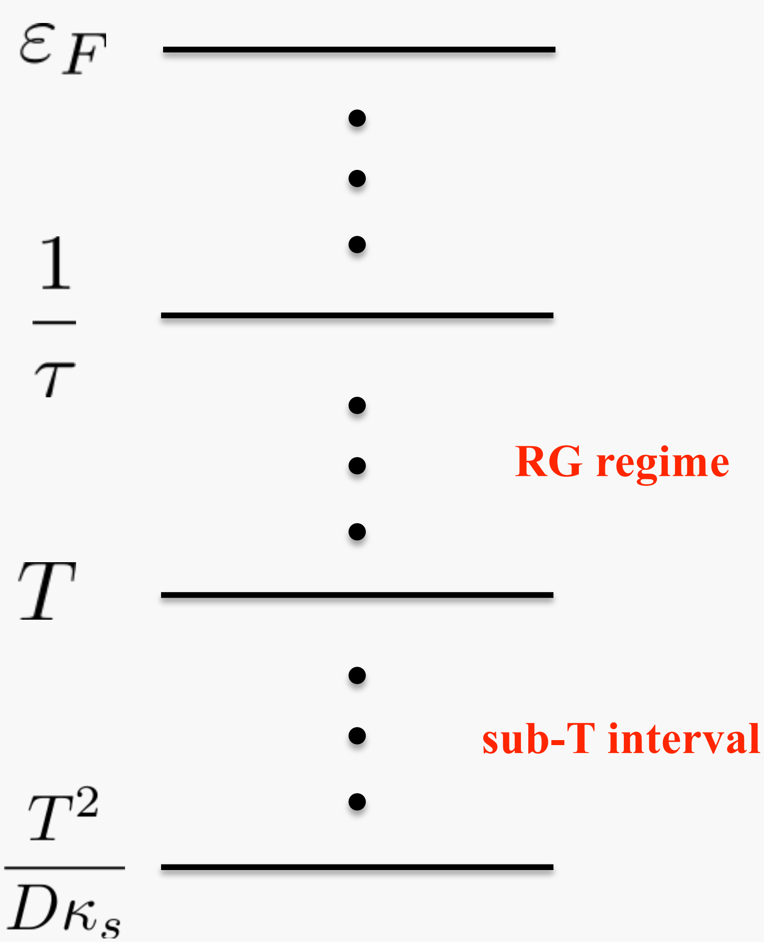
$$\frac{T^2}{D\kappa^2} < D\mathbf{k}^2 < T$$

Results: violation of the Wiedemann Franz "Law" by the Coulomb interaction

$$S[Q] \sim \int d\mathbf{r}\ \mathrm{tr}[D(\nabla Q)^2 + 2iz\{\hat{\varepsilon}, \lambda\}Q] + \sum_i Q\lambda\Gamma_i Q$$

Energy scales

$$\lambda \approx 1 - \eta + \eta^2$$



Georg Schwiete & AF, with Keldysh NLσM
PRB **89** (2014); PRB **90** (2014)(R); PRB **90** (2014); PRB **93** (2016);

Semi-review:
Georg Schwiete and AF in JETP 2016, vol 122, p 567
An issue in honor of L.V. Keldysh

WFL

-disordered Fermi liquid

-disordered electron liquid

despite strong renormalizations

WFL **-disordered Fermi liquid**

~~WFL~~ **-disordered electron liquid**

Additional logarithmic corrections!

Thus, NLσM at the RG yields the Fermi-liquid description with scale-dependent parameters. However, according to the WFL-criterion it exhibits a non-FL feature owing to long-ranged Coulomb interaction

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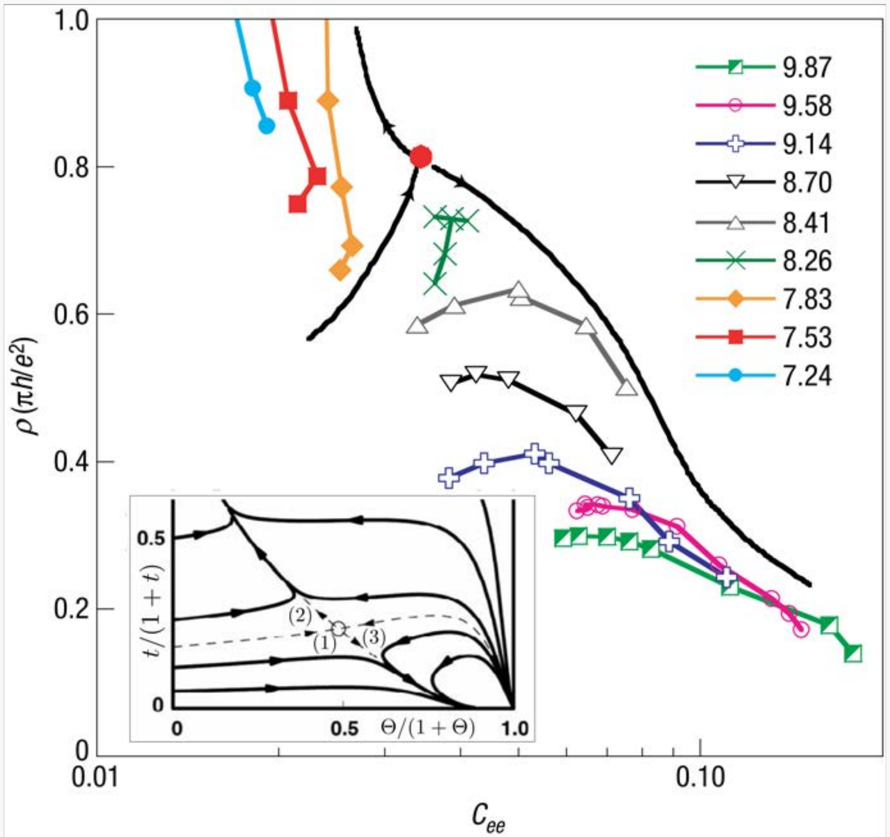
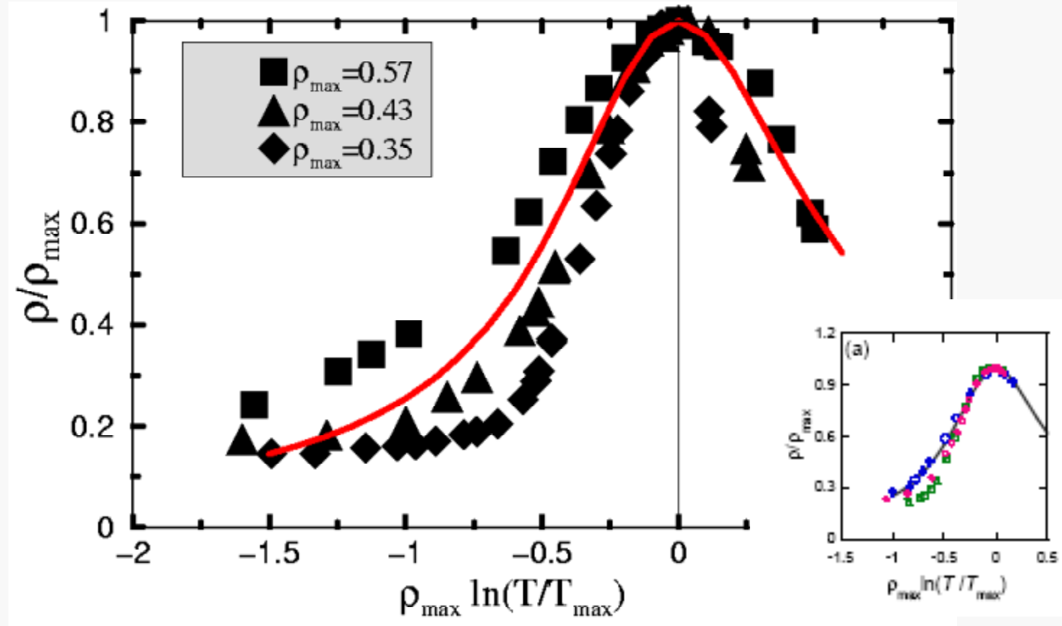
How can we be sure that all this indeed work for disordered electrons ?

Comparison with experiment: 2d electron liquid in MOSFETs



Resistance-Interaction phase diagram extracted from the analysis of the resistance and magneto-resistance (insert: two-loop calculation, A. Punnoose and AF (2005))



Data from the region C^* in a high-mobility samples (Pudalov's and Klapwijk's)

- the drop of five times in ρ(T) and its slowing down has been captured in the correct temperature interval
- **no adjustable parameters** are used

A. Punnoose and AF, PRL (2002)

S. ANISSIMOVA, S. V. KRAVCHENKO, A. PUNNOOSE, A. F AND T. M. KLAPWIJK. nature physics VOL 3, p 707, 2007

Why does my old argument based on time-dependent gauge (1987) does not hold in the case of heat transport?

The argument concerned the cancellation of the well-known logarithm-squared corrections in all quantities except the single-particle density of states, which itself is not a gauge-invariant quantity. (It can be measured only in tunneling experiments, giving rise to the zero-bias anomaly; BAA/AA, 1979.)

The argument:

1) integrate the Coulomb interaction over momenta over the sub-thermal interval:

$$\tilde{\Gamma}^{R}_{0;d}(\mathbf{k},\nu) \approx \frac{1}{2(1+F_0^{\rho})^2}\frac{i\nu}{D\mathbf{k}^2}$$

2) Obtain the effective e-e interaction that depends effectively only on the transferred frequency, but not on momentum.

3) Such an interaction can be eliminated by the time-dependent gauge transformation. Consequently, it drops out of *all quantities that are invariant with respect to the time/frequency-dependent gauge transformation*.

This is not the case for the heat conductivity, because the heat-density correlation function contains frequency in the external vertices!

Perspectives of NLσM, or this is all?

Odd-Frequency superconducting pairing (OFP), the same as spin-triplet pairing; Vadim Berezinskii (1974)

VB (1935-1980)

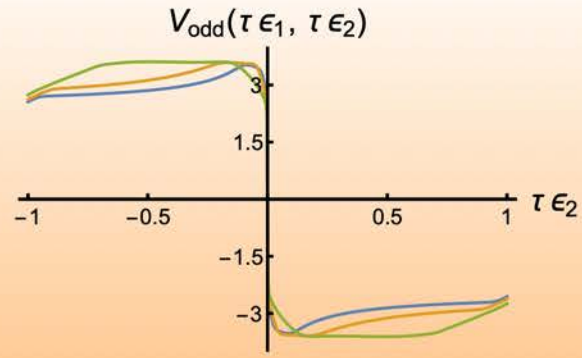
$$iS_0 = -\frac{\pi\nu}{4}\int_{\mathbf{r}} \mathrm{Tr}[D(\nabla\hat{Q})^2 + 4iZ\hat{\varepsilon}\hat{Q}]$$

$$iS_{\mathrm{triplet}} = i\frac{\pi^2\nu}{8}\mathrm{Tr}_{\mathrm{F}}\, V_{\mathrm{triplet}}(\epsilon_1,\epsilon_2)\int_{\mathbf{r}} \mathrm{Tr}_{\mathrm{KSN}}\left[\hat{\gamma}^{1/2}\tau^{\pm}\boldsymbol{\sigma}_{\alpha\beta}\underline{\hat{Q}_{\epsilon_1\epsilon_1';\beta\alpha}(\mathbf{r})}\right] \times \mathrm{Tr}_{\mathrm{KSN}}[\hat{\gamma}^{2/1}\tau^{\mp}\boldsymbol{\sigma}_{\mu\eta}\underline{\hat{Q}_{\epsilon_2\epsilon_2';\eta\mu}(\mathbf{r})}]\delta_{\epsilon_1-\epsilon_1',\epsilon_2'-\epsilon_2}$$

In the **singlet** Cooper channel stand matrices $\boldsymbol{\sigma}_0$. A **hybrid** consisting of (i) singlet Cooper channel (it contains Gor'kov-Nambu $\boldsymbol{\tau}$-matrices, but only $\boldsymbol{\sigma}_0$) and (ii) spin-density channel (it contains $\boldsymbol{\sigma}$-matrices, but no $\boldsymbol{\tau}$-matrices) has a chance to generate $S_{\mathrm{triplet}} \Rightarrow$ **FLUCTUATIONS !**

Vladimir Zyuzin and AF, 2022 and still in progress

Perspectives or this is all?
Search of Odd-Frequency superconducting pairing (OFP)



"BCS-like" Interaction in the odd-frequency spin-triplet Cooper channel (*attractive*). V. Zyuzin&AF PRB 105, 214523 (2022)

Vladimir Zyuzin and AF, still in progress

Content of the talk

MIT in disordered electron liquids

Non-linear sigma model, NLσM; in fact, not a model but a minimal functional

Connection with the disordered Fermi-liquid theory

Role of the parameter Z in the description of the dynamical properties;
relation between ω-T scaling and Z

Heat capacitance and heat conductivity

Generalization of NLσM for studying heat transport: gravitational potentials

Heat density-density correlation function

Sub-thermal corrections in the case of the Coulomb interaction

Why does my old argument based on the gauge invariance doesn't work in the case of the heat transport?

Perspectives of the theory of the NLσM

Georg Schwiete the University of Alabama

The master of “martial arts”:
Keldysh, heat transport and all that