The resilience of the Fermi Liquid in strange metals: The example of cuprates

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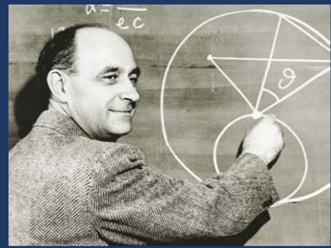
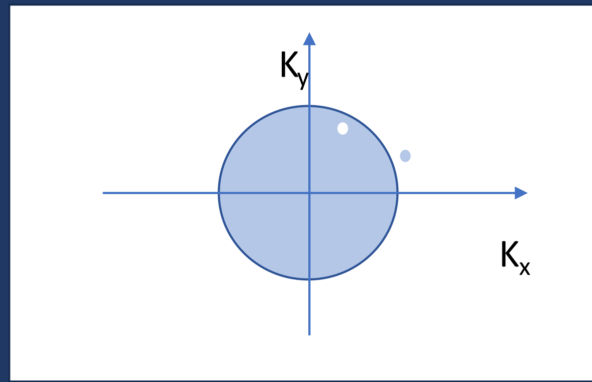
ACCADEMIA NAZIONALE DEI LINCEI CONFERENCE
FERMI LEGACY IN LOW ENERGY PHYSICS
CELEBRATING THE 100 ANNIVERSARY OF ENRICO FERMI'S PIONEERING PAPER
"SULLA QUANTIZZAZIONE DEL GAS PERFETTO MONOATOMICO" (REND. FIS. ACC. LINCEI 3, 145 (1926)
5-6 FEBRUARY 2026

The Fermi Liquid in a nutshell (1/4)





The ideal Fermi gas:

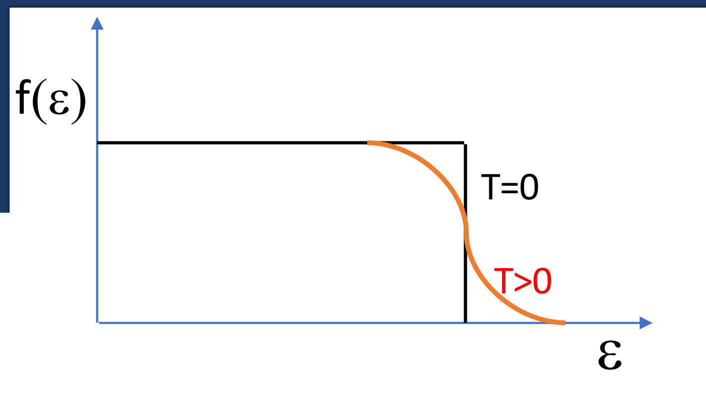
E. Fermi, **Rend. Fis. Acc. Lincei 3, 145 (1926)**

At T=0 all single particle levels are filled up to the Fermi energy

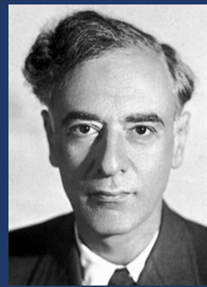
Excited states are only non interacting particle-hole pairs

Thermal average leads to the smooth change of the Fermi distribution

$$f(\varepsilon) = \frac{1}{\exp[(\varepsilon - \mu)/k_B T] + 1}$$

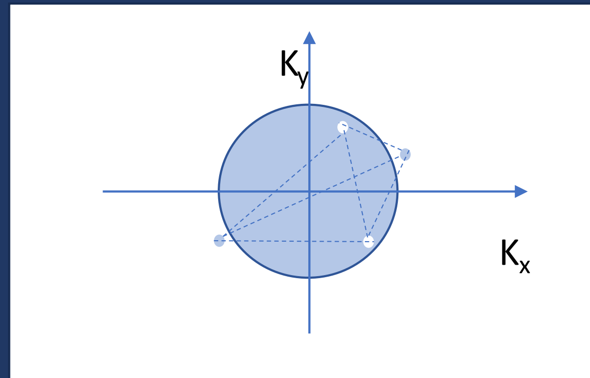


The Fermi Liquid in a nutshell (2/4)



The Fermi liquid:

L. D. Landau, *Sov. Phys. JETP.* ***3*** *(6): 920–925 (1957).*



The interaction is adiabatically switched on: N particles map into N QuasiParticles with effective mass m*, QP residuum z,…

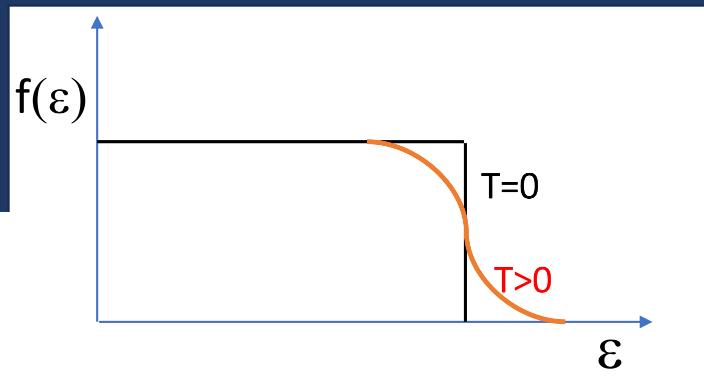
The behavior of quasiparticles is very similar to the free fermion gas

At T=0 all single QP levels are filled up to the Fermi energy

Excited states are:

- interacting particle-hole pairs
- Zero-sound collective mode for neutral He^3

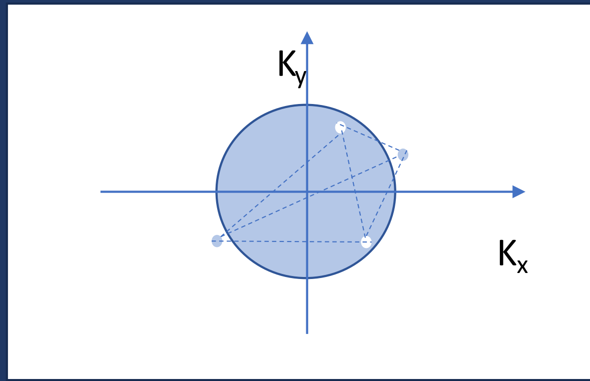
or plasmon for charged electrons



At T>0 thermal average leads to the smooth change of the Fermi distribution for the QP

$$f(\varepsilon) = \frac{1}{\exp[(\varepsilon - \mu)/k_B T] + 1}$$

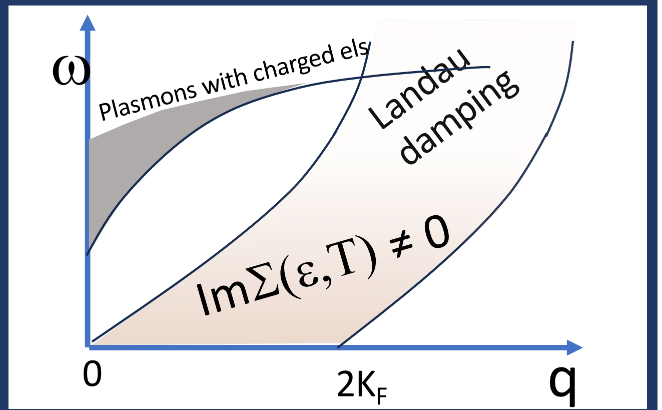
The Fermi Liquid in a nutshell (3/4)



The Fermi liquid for electrons:

The interaction between QP entails a finite lifetime, but this is long due to Pauli principle: exclusion limits the phase space for QP decay

$$1/\tau \sim \mathrm{Im}\Sigma(\varepsilon,T) \sim \varepsilon^2+T^2$$



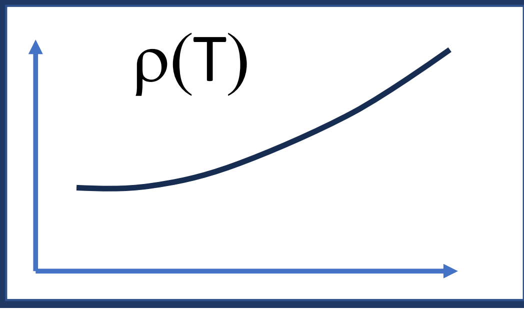
Low energy excitations providing low-T specific heat:

- QP contribution: $C_{Vfermi} \sim m^* T$ like in Sommerfeld theory

QP behave nearly like free fermions: this is why metals are metals

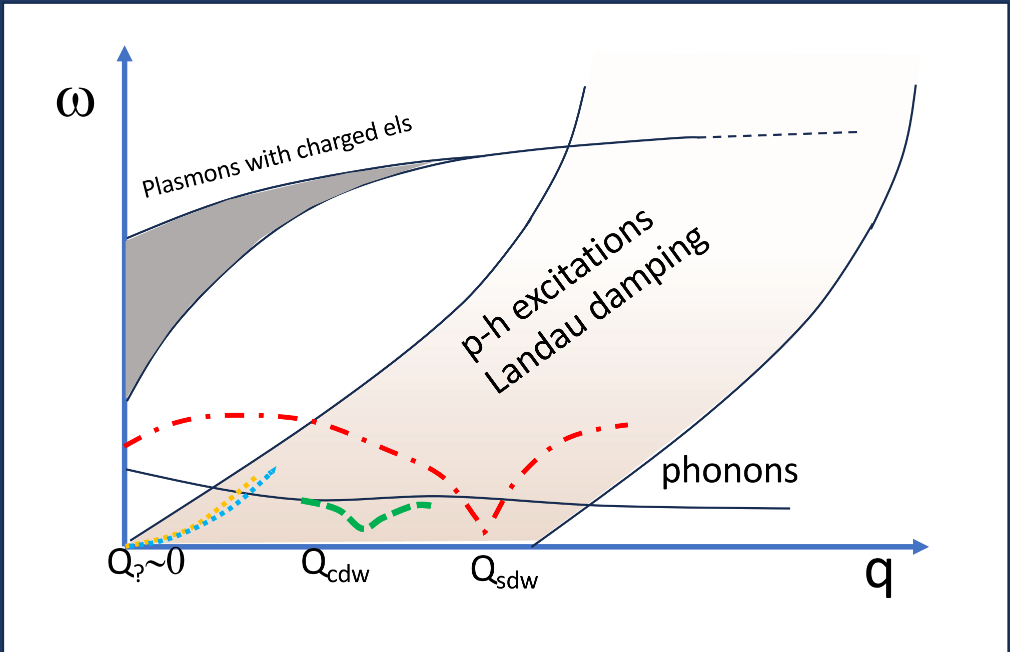
Putting aside various issues (momentum dissipation, momentum dependence of scattering mech.,…)
T^2 resistivity is expected

$$\rho(T) \sim 1/\tau_{tr} \sim 1/\tau \sim T^2$$



The Fermi Liquid in a nutshell (4/4)

The Fermi liquid can also become unstable and form ordered states

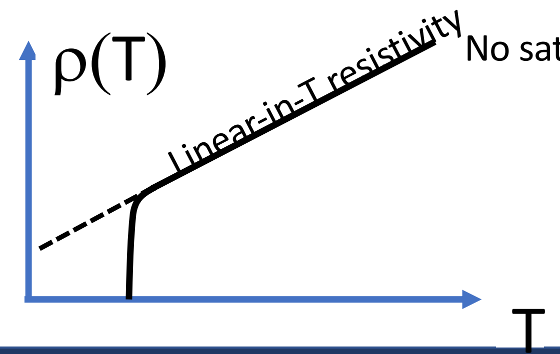


Near the instabilities many different collective excitations can populate the ω-q plane:

paramagnons, Charge Density Waves,
Pomeranchuk flucts,
Circulating Currents

The FL is very robust and general paradigm for metallic behavior

In 1987 the anomalous metallic behavior of cuprates came as a big surprise

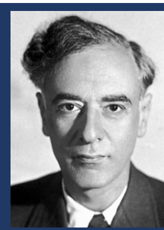


No saturation at high T

linear resistivity $\rho \sim T$

Linearity continues down to very low T when SC is destroyed by H

And a lot more anomalies… All nicely summarized by Varma's phenomenological Marginal Fermi Liquid Theory. HOW and WHY?



The Landau theory of Fermi liquids is a robust and general paradigm for metallic behavior…

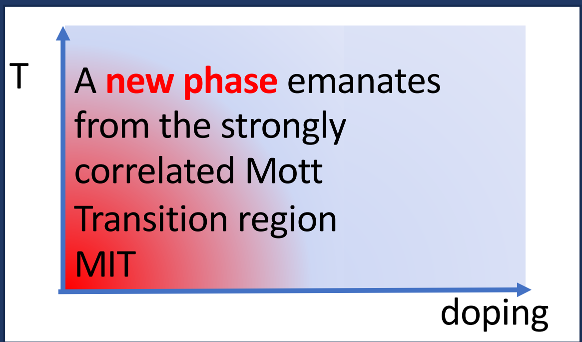
To account for disruption of FL in 2D/3D one needs **singular effective interactions**

(cf. RG calculations, Gallavotti Benfatto, Shankar…, Metzner, Castellani, Di Castro): HOW and WHY?

A very important and debated issue in cuprates

The Mottness paradigm:

MIT, magnetism, low D,..

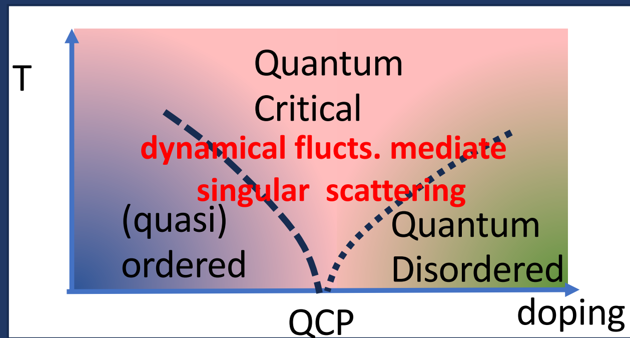


Anderson, Laughlin, Lee, Wen, Nagaosa, Sachdev, …
RVB, Luttinger Liquid, anyons, gauge fields, FL*, …
Many exotic beasts…

AF-QCP near the Mott M-I Transition
Pines, Sachdev, Chubukov,…(~1990)
Stripes
Emery, Kivelson (1993)
Ancient Romans (1993)

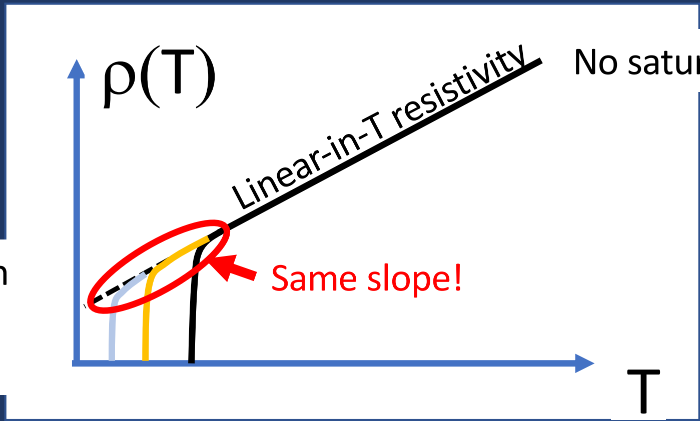
The Criticality paradigm:

Strong correlations weaken the metal and other phases may form (AF, CDW, CC, nematic,…).



C.M. Varma (1994),
Ancient Romans (1995)

WHAT IS A `STRANGE METAL'? A metal that violates the FL paradigm

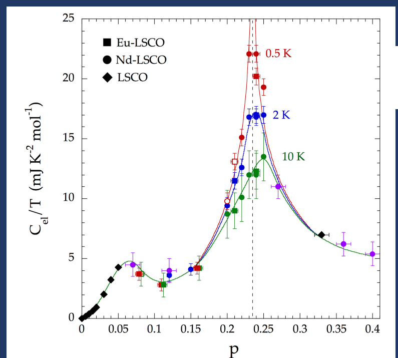


No saturation at high T

linear resistivity $\rho \sim T$

Linearity continues down to very low T when SC is destroyed by H

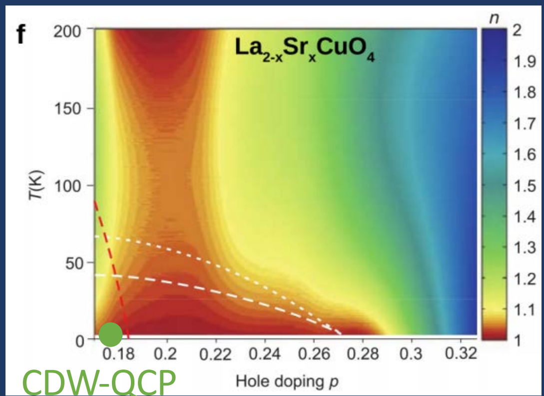
In some `2D cases' linear resistivity is accompanied by $C_V/T \sim \log(1/T)$

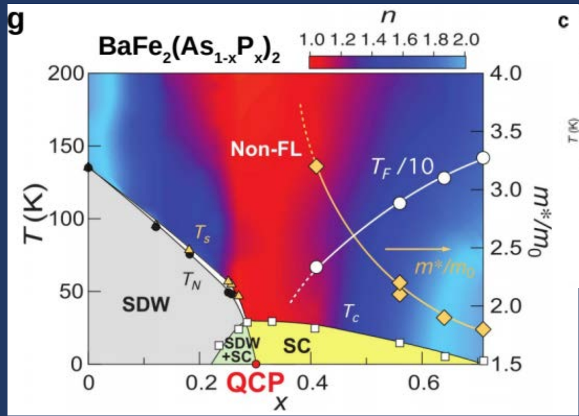


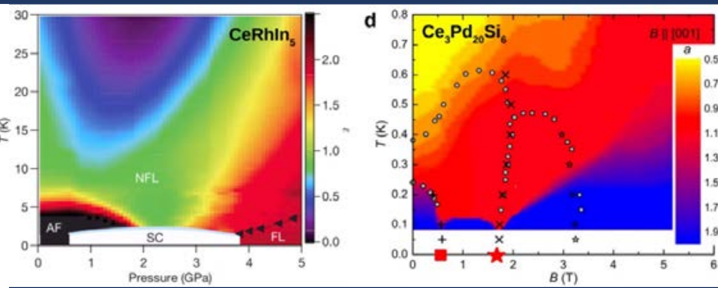
Michon et al, Nature 2019

Where are strange metals found? Where the typical ρ~T is found?

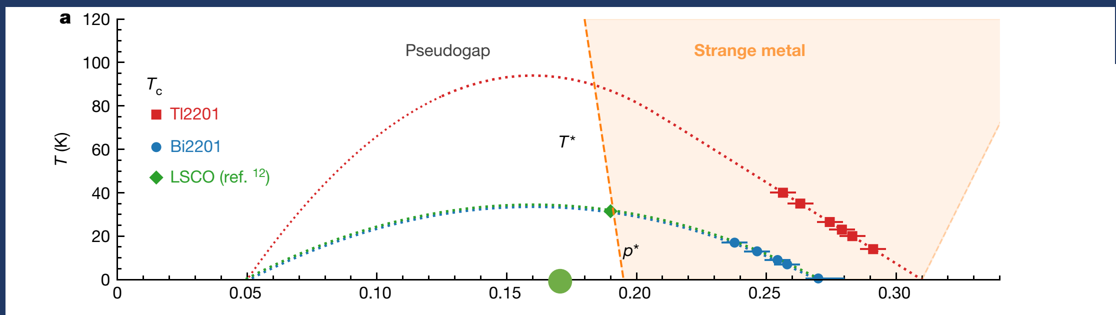
Taupin & Paschen, Crystals 2022







And the list is longer…



CDW-QCP

Ayres et al, Nature 2021

SM always around QCP's but not necessarily exactly on top: SM can occur over more or less extended parameter regions

See also Hartnoll & Mackenzie, RMP 2022

Quantum Critical Points are the ideal source of singular interactions:
Quantum fluctuations are abundant and dynamical: ideal for low-energy singular scattering.

Huge variety of systems, and involved mechanisms: 😳

AFM, FM, CDW, nematic, Pomeranchuk, circulating currents…
Q_{AF}, Q_{CDW}, $Q \sim 0, \ldots$ variety of momenta…
Narrow or broad NFL regions at low T …

Still $\rho \sim T$ essentially everywhere…

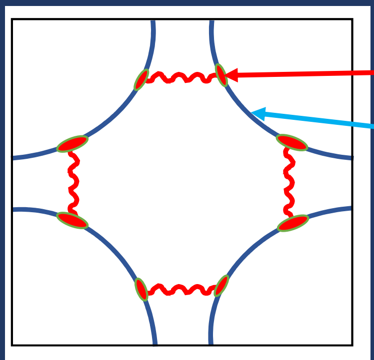
HOW CAN WE EXTRACT A GENERAL MECHANISM DESPITE THIS VARIETY? 🤔

WHAT DO WE NEED TO GET A STRANGE METAL?

Sufficient MINIMAL set of conditions to obtain SM ($\rho \sim T$):

1) If $\rho \sim T$ down to low T, **low characteristic energy ($\omega_0 < T$) scattering fluctuations**
$b(\omega) = 1/(e^{\omega/T} - 1) \sim T/\omega$: the scatterers are nearly classical fluctuations even at low T
(no H/T scaling, scattering not necessarily Planckian, pls ask…)

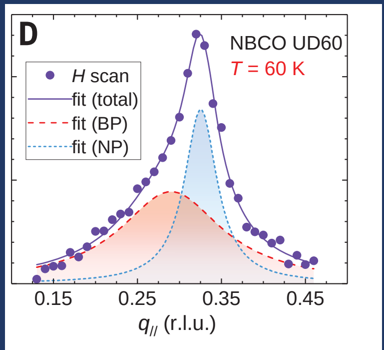
2) Nearly isotropic scattering: strong scattering **at all momenta** (Umklapp included)
if scattering is strong **at some q_c only** it doesn't work
[Hlubina, Rice, PRB 51, 9253 (1995), see also A. Rosch PRL 1999]



In Hot Spots $1/\tau \sim T$ large scattering
In cold regions $1/\tau \sim T^2$ small (Fermi-liquid) scattering
Cold regions short-circuit the hot ones and
$\rho(T) \sim T^2$ Fermi liquid behavior

DO WE HAVE SUCH STRONG ISOTROPIC SCATTERERS IN CUPRATES?

Strong hint from RIXS experiments.....



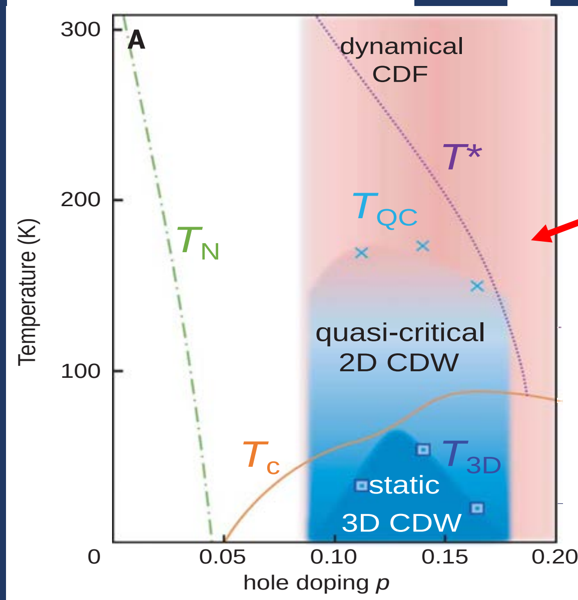
In UD cuprates at low $T<T_{QC}$ narrow-in-q CDW coexist with broad-in-q CDF

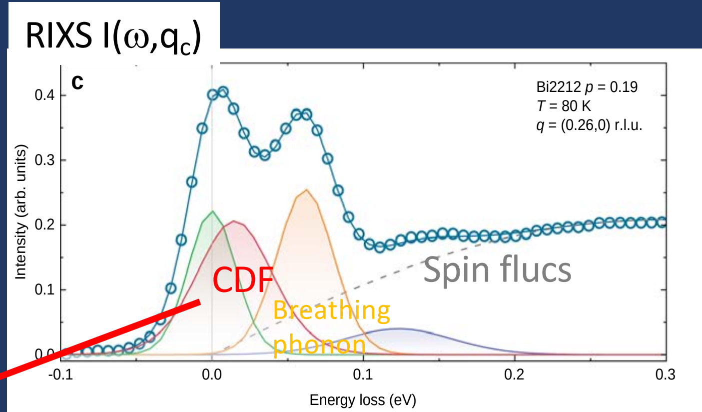
In OD or/and high T only CDF are present

Dynamical charge density fluctuations pervading the phase diagram of a Cu-based high-T_c superconductor

R. Arpaia[1,2*], S. Caprara[3,4], R. Fumagalli[1], G. De Vecchi[1], Y. Y. Peng[1†], E. Andersson[2], D. Betto[5], G. M. De Luca[6,7], N. B. Brookes[5], F. Lombardi[2], M. Salluzzo[7], L. Braicovich[1,5], C. Di Castro[3,4], M. Grilli[3,4], G. Ghiringhelli[1,8*]

Science **365**, 906–910 (2019)





Arpaia, R, et al., *Nat. Commun.* **2023**, *14*, 7198

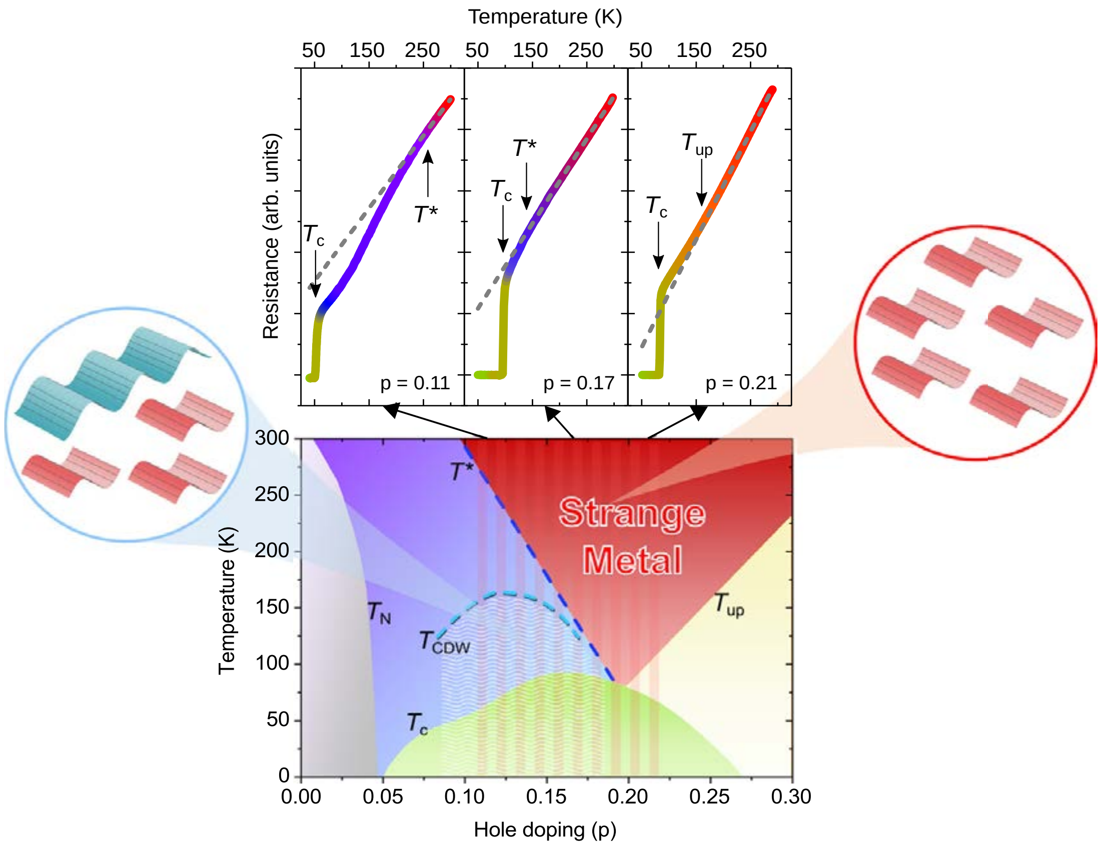
Similar things (but different interpretation) In EELS experiments by Abbamonte's group arXiv:2411.11164

CDF are a ubiquitous scattering mechanism in cuprates

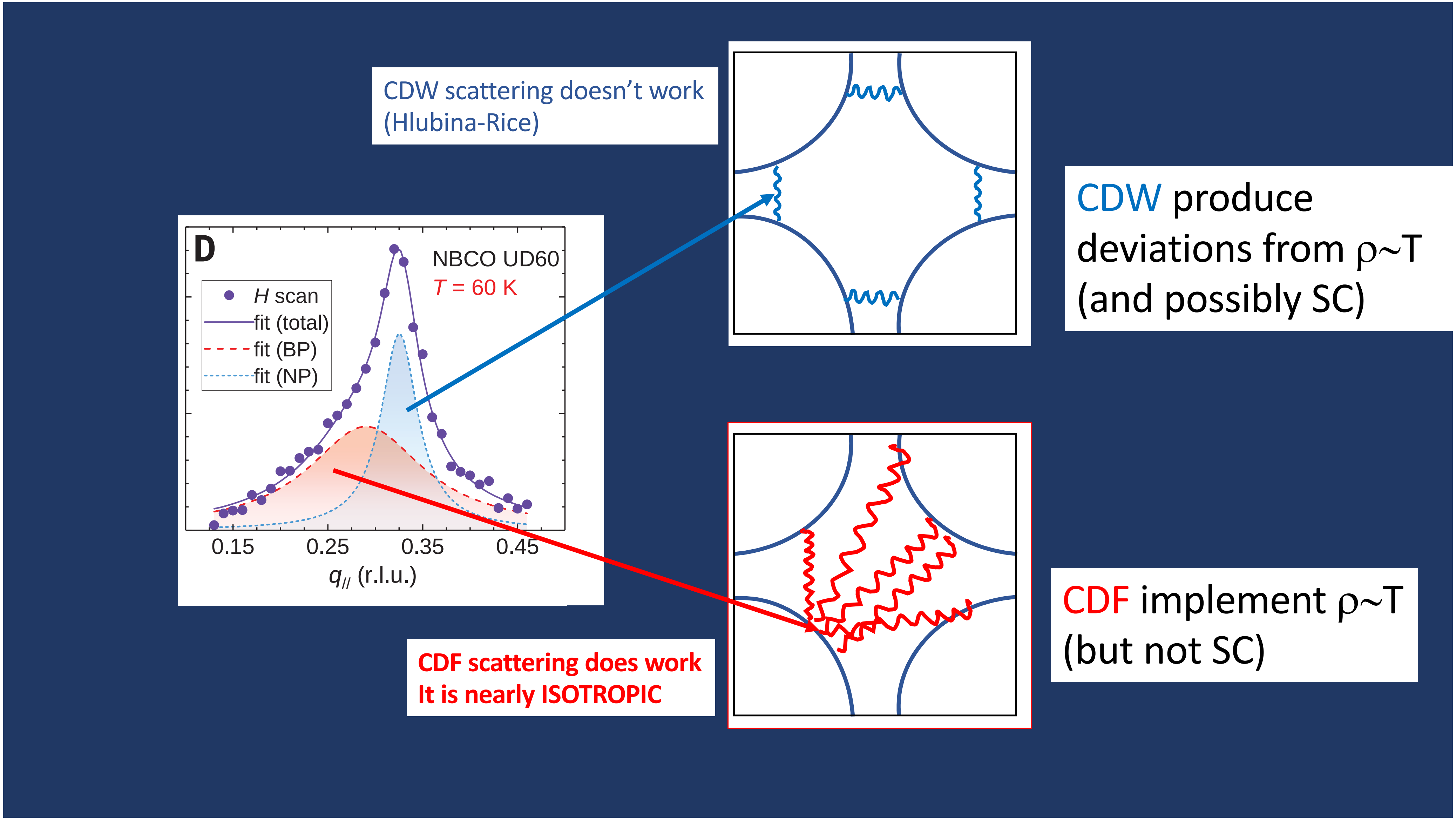
Strange metal behaviour from charge density fluctuations in cuprates

COMMUNICATIONS PHYSICS | (2021)4:7

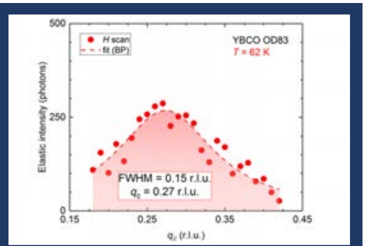
Götz Seibold[1]✉, Riccardo Arpaia[2,3], Ying Ying Peng[2,8], Roberto Fumagalli[2], Lucio Braicovich[2,4], Carlo Di Castro[5], Marco Grilli[5,6,9]✉, Giacomo Claudio Ghiringhelli[2,7] & Sergio Caprara[5,6,9]✉



CDF can account for the SM behavior


CDW scattering doesn't work
(Hlubina-Rice)
D
NBCO UD60
T = 60 K
H scan
fit (total)
fit (BP)
fit (NP)
0.15
0.25
0.35
0.45
q// (r.l.u.)
CDF scattering does work
It is nearly ISOTROPIC
CDW produce deviations from ρ~T (and possibly SC)
CDF implement ρ~T (but not SC)

IN THE SM REGION ONLY CDF (PINK) FLUCTUATIONS ARE PRESENT

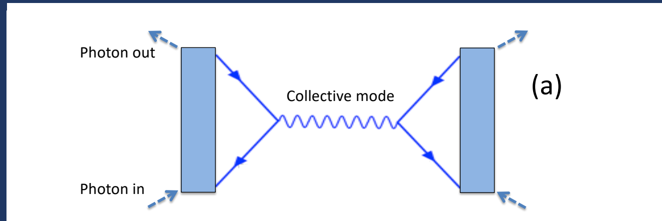


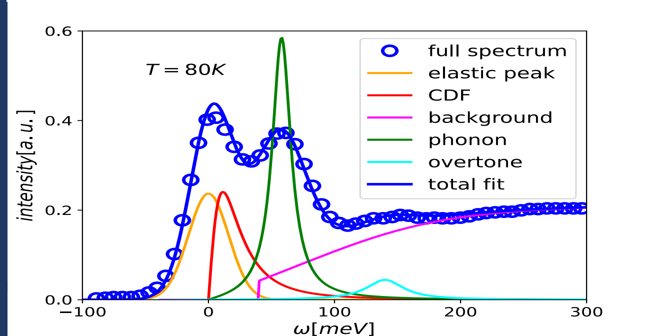
CDF dynamical corr. fcn. can have a simple gaussian textbook functional form

$$D(\mathbf{q},\omega) = \left(\mathrm{M} + \bar{\nu}|\mathbf{q}-\mathbf{q}_c|^2 - \omega^2/\overline{\Omega} - \mathrm{i}\gamma\omega \right)^{-1}$$

$$\mathrm{M} = \bar{\nu}\xi^{-2}$$

ω_0~ M is the characteristic energy of the CDF (8-25 meV, small because CDW-QCP not far)
But rather small correlation length ξ_{CDF}~1-2 λ (local in space broad peak in q)



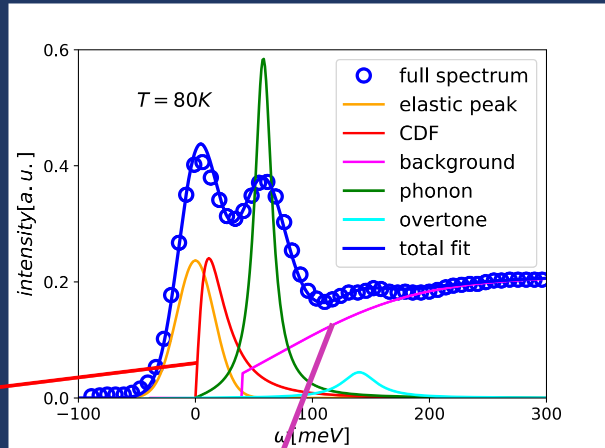


The CDF parameters (q_c, M, ν, Ω,…) can be obtained from HR and LR RIXS spectra, (and EELS…)

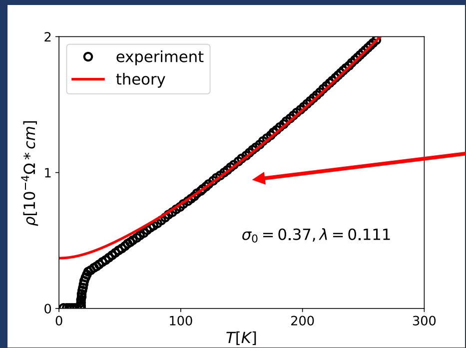
Arpaia et al, Science 2019, Arpaia & Ghiringhelli, JPSJ 2021
Seibold et al, Commun. Phys. 2021
Arpaia et al., Nat. Commun. 2023, 14, 7198.

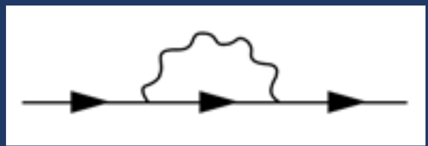
The strategy is: (same for Bi2212, YBCO, LSCO)

1) Identify from RIXS the scattering mediators

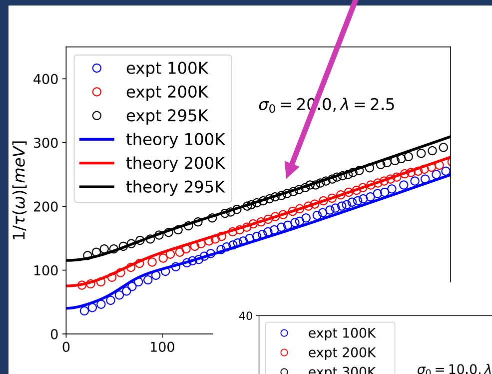


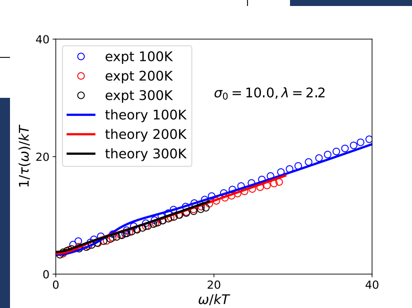
2) Calculate and fit resistivity





3) Calculate and fit the optical scattering time
(and other quantities: magnetoresistance,
Raman, ARPES, Seebeck, thermal conductivity, …)





4) Check ω/T scaling (hallmark of MFL phenomenology)

1st take-home message:

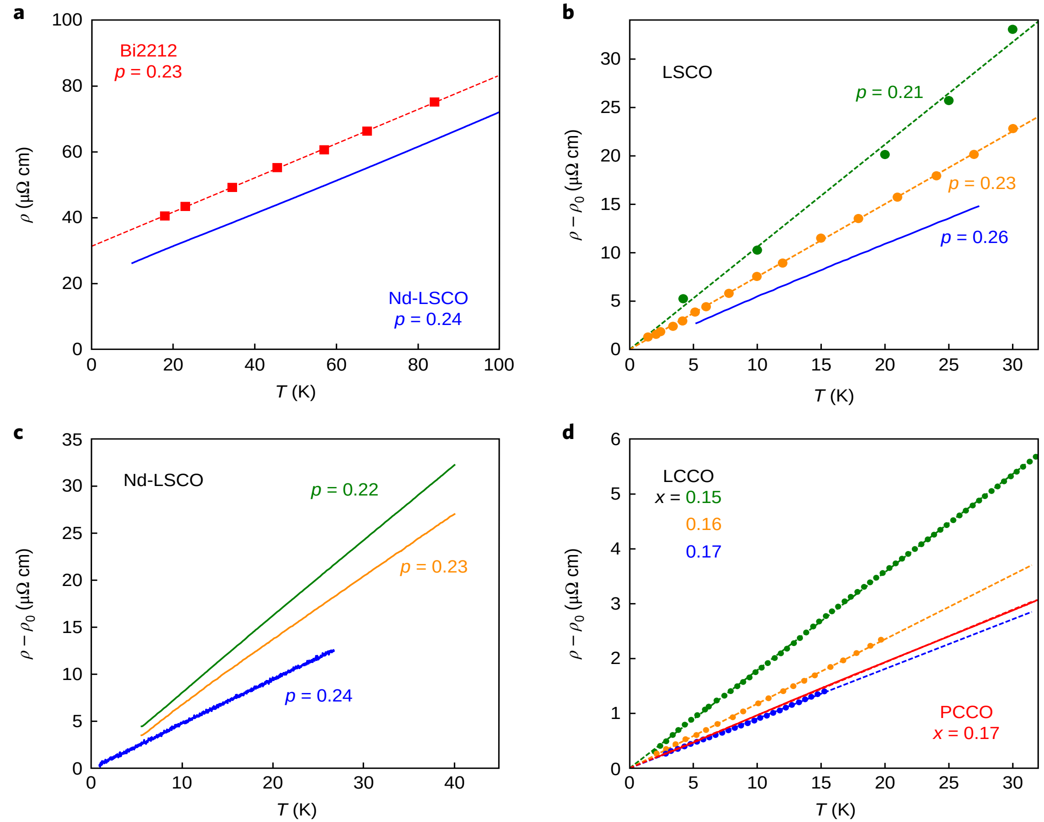
At $T>T_c$ the **strange metal is not so strange**: It may just be a FL with QP scattering with thermally excited nearly local low-energy excitations.

In cuprates these can well be fully characterized CDF fluctuations.

So, what's the problem?

But M_{CDF}~10 meV ~100 K: how can we have linear ρ~T with ω_0~M<T down to a few Kelvin?

With strong magnetic fields one can explore low temperatures $T<T_c$



Legros et al, Nat. Phys. 2019

Damping is the answer

$$D(\mathbf{q},\omega) = \left(M + \bar{\nu}|\mathbf{q}-\mathbf{q}_c|^2 - \omega^2/\overline{\Omega} - i\gamma\omega \right)^{-1}$$

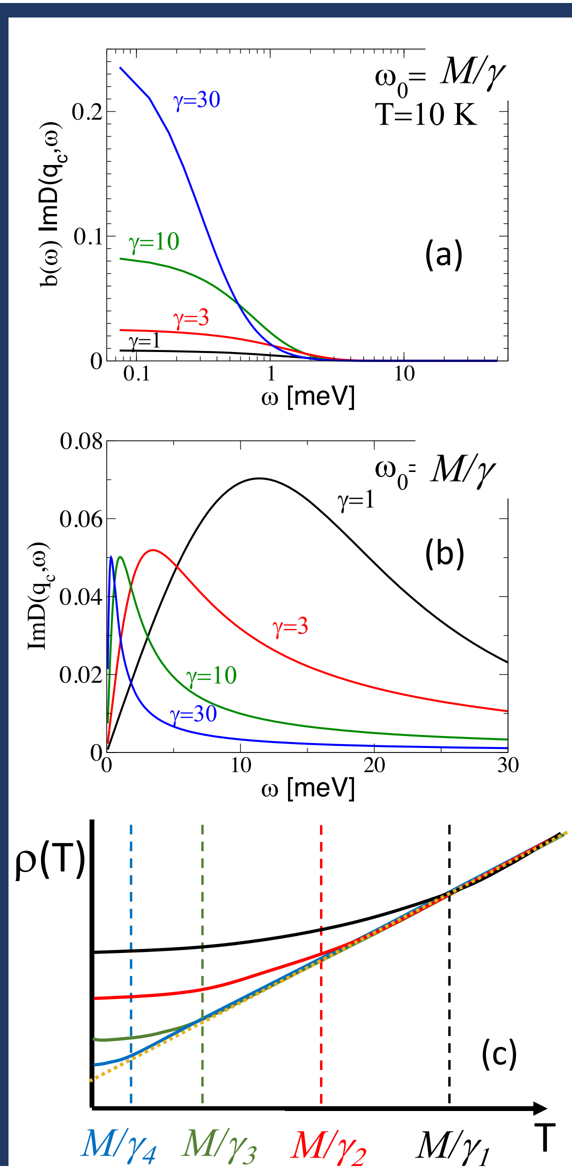
$$\mathrm{Im}\ D(\mathbf{q},\omega) = \frac{\gamma\omega}{\left(M + \bar{\nu}|\mathbf{q}-\mathbf{q}_c|^2 - \omega^2/\overline{\Omega} \right)^2 + \gamma^2\omega^2},$$

γ describes the Landau damping: the mode decays in p-h pairs in time $\tau=\gamma\tau_0$

When γ grows, the characteristic energy $\omega_0= M/\gamma$ of the CDF decreases

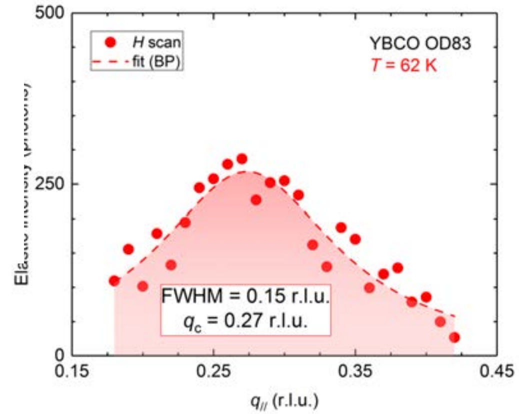
$T_{FL}\sim M/\gamma\sim 100K/\gamma$ **shift the focus from M to γ!**

Larger γ = longer lifetime τ of CDF $\Rightarrow$ linear $\rho\sim T$ down to lower temperature



2^{nd} take-home message:
The dissipation parameter γ can rule the decrease of ω_0=M/γ **for finite ξ**
M=$\nu\xi^{-2}$ stays finite: **no critical slowing down due to $\xi \rightarrow \infty$**

Since ξ **stays small** the momentum distribution stays broad $\Rightarrow$ **isotropic scattering coexists with small energy ω_0=M/γ**



Dissipation-driven strange metal behavior
Sergio Caprara, Carlo Di Castro, Giovanni Mirarchi, Götz Seibold & MG
Commun. Phys. 2022

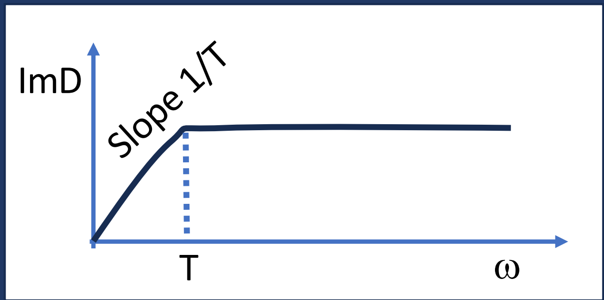
Let's assume that γ increases by decreasing T, e.g. $\gamma\sim\log(1/T)$, then the FL scale M/γ shrinks

GENERAL CONSEQUENCE: At T=0 the system is still FL, but the (T,ω) range of FL shrinks by decreasing T (increasing γ) $\Rightarrow$ SHRINKING FERMI LIQUID (SFL)

Let's benchmark SFL with MFL

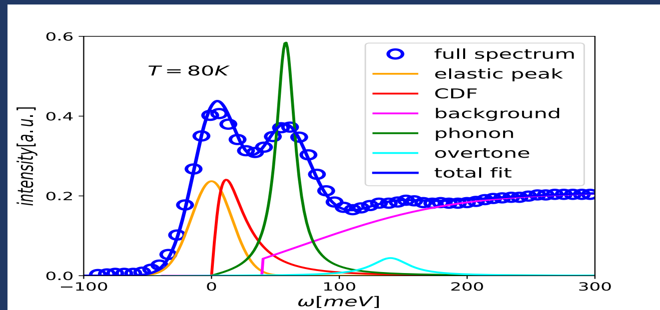
MFL

Interaction due to momentum indep. (local) mediator



SFL

Interaction due to momentum indep. (local) CDF, phonons, paramagnons, p-h pairs…



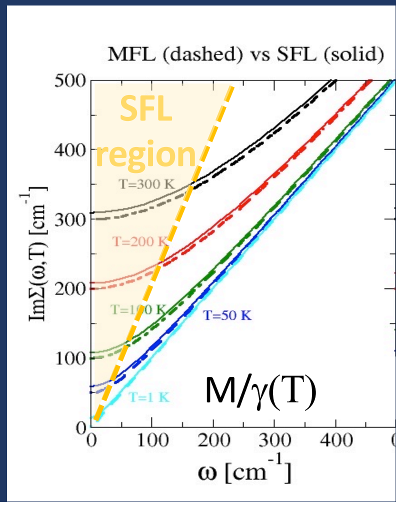
Self-energy

$$|Im\Sigma(\omega,T)|\sim g\sqrt{\omega^2+T^2}$$

Standard MFL ω/T **scaling** form

MFL has divergent QP mass

$m_{QP}^*\sim\log(1/T)$



Self-energy

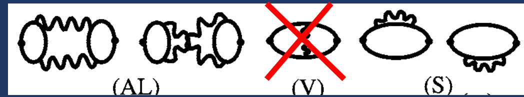
$$|Im\Sigma(\omega,T)|\sim g\left(\sqrt{(M/\gamma)^2+\omega^2+T^2}-M/\gamma\right)$$

SFL has an **almost scaling** form when $M/\gamma(T)\rightarrow 0$

FL for $\omega<M/\gamma$

$m_{QP}^*=m/Z$ is finite

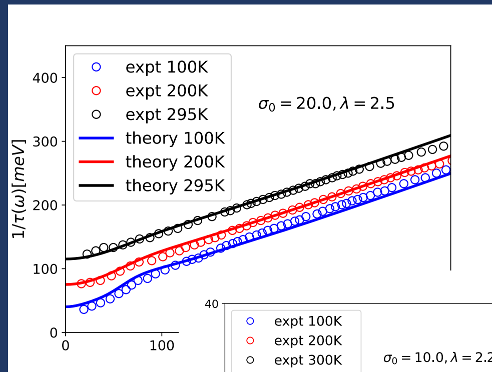
SOME EXPERIMENTAL CONSEQUENCES:
OPTICAL CONDUCTIVITY

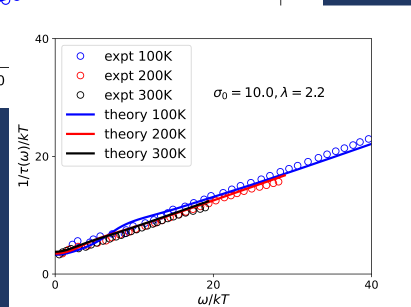


The interaction is (almost) momentum independent ⇒ vertex corrections negligible in current-current response

⇒$\Sigma(\omega,T)$ (almost) fully determines the optical conductivity $\sigma(\omega,T)$

- At $\omega > M/\gamma$, $\sigma(\omega,T)$ quite similar to the MFL case (see Michon et al, Nat Commun. 2023)





SFL has **quasi-scaling form if M/γ is small…**

$$|Im\Sigma(\omega,T)| \sim g\left(\sqrt{(M/\gamma)^2+\omega^2+T^2}-M/\gamma\right)$$

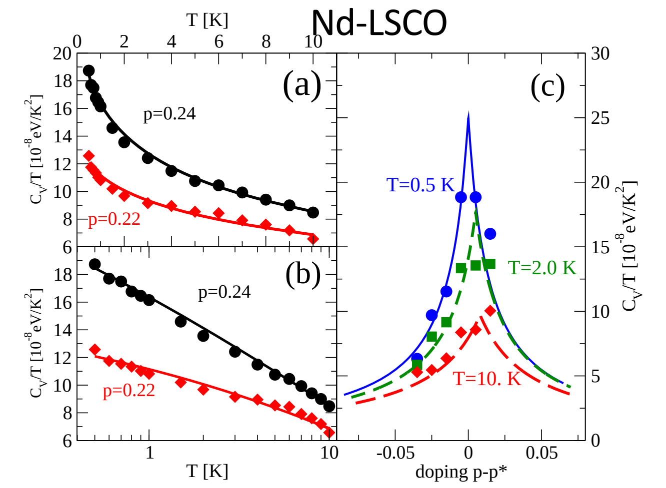
Mirarchi et al. *Condens.Matter* **2024**,*9*,14.

Optical conductivity expts seem to agree well with MFL and with SFL as well…

SO FAR WE LOOKED AT POSSIBLE CONSEQUENCES OF

ASSUMING $\gamma(T) \sim \log(T_0/T)$

ARE THERE INDICATIONS THAT γ INDEED DOES GROW LARGE?



Log(1/T) divergence of C_V/T down to low temperature

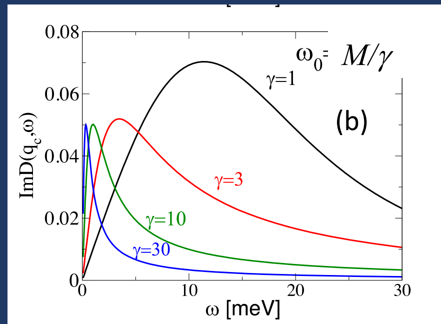
Data from Nd-LSCO, Eu-LSCO
Michon et al, Nature 2019

specific heat **both from electrons and collective CDF**

$C_V^{el} \sim m_{QP}^* \sim 3\text{–}5\ m_e$ **finite electron contribution**

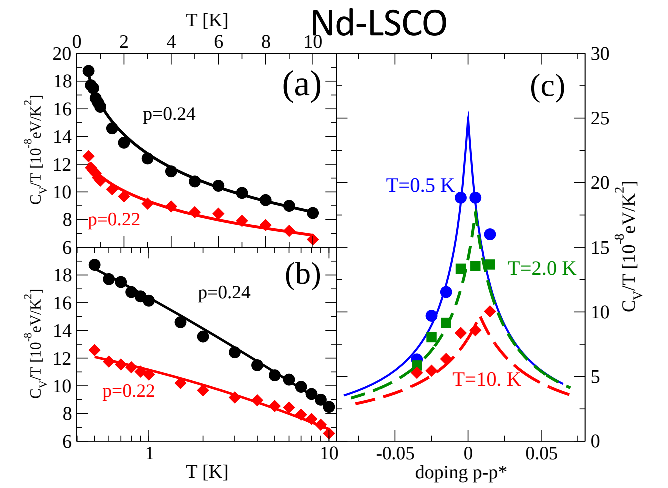
$C_V^{bos}/T \sim \gamma \sim \log(1/T)$ **singular bosonic contribution**

See also, Shang-Shun Zhang , Erez Berg, and
Andrey V. Chubukov, PRB 2023



$$\frac{C_V^B}{T} \approx k_B^2 \frac{\gamma}{3\bar{\nu}} \log\left(1 + \frac{\pi\bar{\nu}}{M}\right)$$

The Hertz-Millis
Log term stays finite



Log(1/T) divergence of C_V/T down to low temperature

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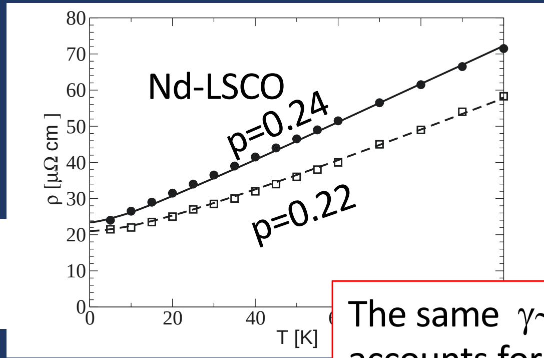
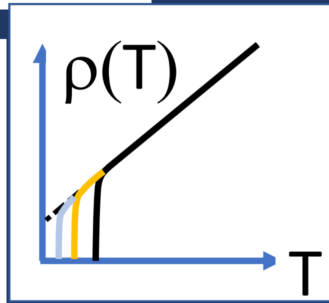
Michon et al, Nature 2019





The same γ~log(1/T) accounts for T-linear resistivity down to low T with the same slope
Caprara et al. Commun. Phys. 2022

specific heat **both from electrons and collective CDF**

$C_V^{el} \sim m_{QP}^* \sim 3-5\ m_e$ **finite electron contribution**

$C_V^{bos}/T \sim \gamma \sim \log(1/T)$ **singular bosonic contribution**

See also, Shang-Shun Zhang , Erez Berg, and Andrey V. Chubukov, PRB 2023

$$\frac{C_V^B}{T} \approx k_B^2 \frac{\gamma}{3\bar{\nu}} \log\left(1 + \frac{\pi\bar{\nu}}{\mathrm{M}}\right)$$

The Hertz-Millis Log term stays finite

WHY SHOULD γ GROW? AND WHY LOGARITHMICALLY ~log(1/T)?

INTRINSIC MECHANIMS:

CDF interact and tend to form a **self-generated glass:**

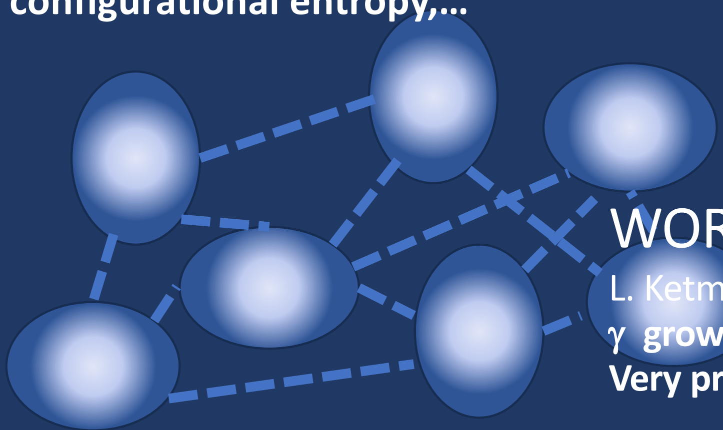
Old story:

J. Schmalian, P. Wolynes et al…. Stripe glasses 2000…

V. Dobrosavljevic, E. Miranda 2005,…Cluster Glass

Overcooled liquid of CDF

Many open issues: interplay between dynamical slowing down and (quantum) glass formation, stability of configurational entropy,…



WORK IN PROGRESS

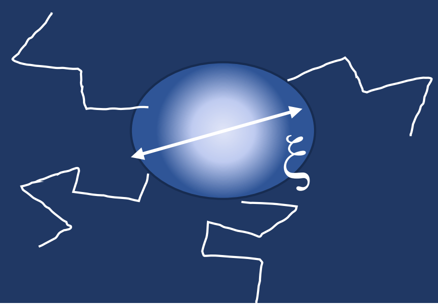
L. Ketmaier, MG, J Kurchan

γ **grows ~(1/T)**

Very preliminary

EXTRINSIC MECHANISM:

CDF decay in diffusive p-h modes



In 2D the damping γ has a log renormalization

$$\delta\gamma = \gamma - \gamma_0 = A \log \max\left[(\tau T)^{-1}, 1\right],$$

The γ grows ~log (1/T)

[MG, C. Di Castro, G. Mirarchi, G. Seibold, S. Caprara Symmetry 15, 569 (2023)]

A quick personal survey and comparison

Ancient Romans

CDF local non-critical flucts. **near but away from** QCP (ξ small, finite M)
Low energy with an increasing damping (slow relaxation)
$\gamma \to \infty$, $M/\gamma \to 0$). Disorder: not important (just a bit?)
Scaling? NO, but almost (matter of fact)

Glassy phase (AR) and Stripe
Glass (Schmalian, Wolynes,...)
Cluster Glass (Dobrosaljevic,...)
Griffith's phase (MMS, Vojta,)
Boson localiz. (Sachdev)

Local low energy bosons mediate scattering

Sachdev & Co.

Yukawa-SYK model(s)
Critical flucts **at QCP** become local due to disordered e-bos coupling.
Low energy because $M \to 0$, ξ diverges
Near QCP also because of boson localization

Varma

Circulating currents $\to$ Dissipative XY-model
Topo excitations **with z=∞ QCP**
effectively **local** flucts. due to factorized
$D(\omega,q)=\chi(q)D_{MFL}(\omega)\ldots$
Low energy because $M_{q,\omega} \to 0$, $\xi_{r,\tau}$ diverge
MFL like $D_{MFL}(\omega)$

Collaborators: <u>THEORY</u>

The Ancient Romans

(Sapienza):

<u>S. Caprara,</u>

C. Castellani framework

<u>C. Di Castro,</u>

<u>G. Mirarchi (->Wuerzburg)</u>

<u>S. Bhattacharyya</u>

Cottbus (BTU):

<u>G. Seibold</u>

<u>RIXS EXPERIMENTS: Politecnico di Milano</u>

G. Ghiringhelli

L. Braicovich

Y. Y. Peng

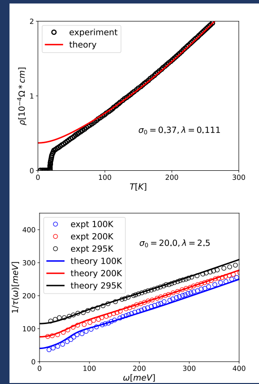
R. Arpaia

F. Lombardi (Chalmers)

and many others

N. B. Brooks, B. Keimer, M. Le Tacon, M. Salluzzo, ...

CONCLUSIONS



1) In cuprates CDF work well as strong low-energy scatterers at $T>T_c$ ⇒ **SM from CDF, observed measured modes**

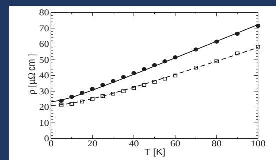
No exotic stuff: FL+CDF

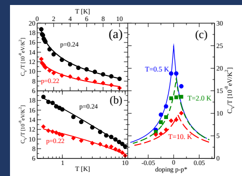
2) At low T if the dissipation parameter γ grows large and ξ stays small

Allow small energy and isotropic scattering

if $\gamma \sim \log(1/T)$ the SFL mimics well the MFL and it accounts for:

$\rho \sim T$ at low T, C_V, Seebeck, MFL-like $\Sigma(\omega,T)$, $\sigma(\omega,T)$…





3) Slowing down of short range fluctuations

CDF decay in diffusive particle-hole pairs?

New T=0 glassy phase of CDF over a finite interval of QCP tuning parameter?

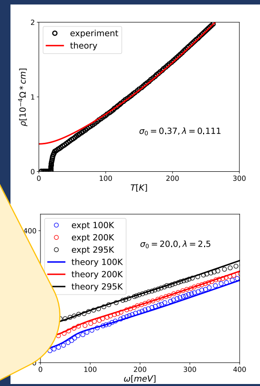
No exotic stuff: FL+ slower and slower CDF….

But maybe a new local criticality with $\gamma \sim \tau \sim \xi_\tau$?

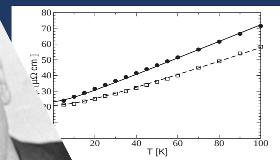
CONCLUSIONS

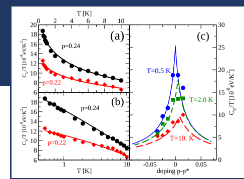
1) In cuprates CDF work well as strong low-energy sc...
at $T>T_c$ ⇒ **SM from CDF, observed measur...**

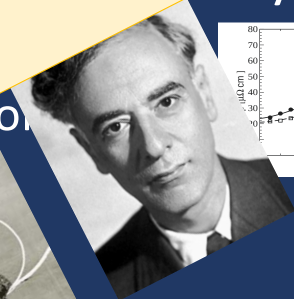
No exotic stuff: FL+CDF



2) At low T if the dissipation pa... stays small
Allow small energy and i...
if $\gamma \sim \log(1/T)$ the **SFL m...** fo...
$\rho \sim T$ at low T, C...







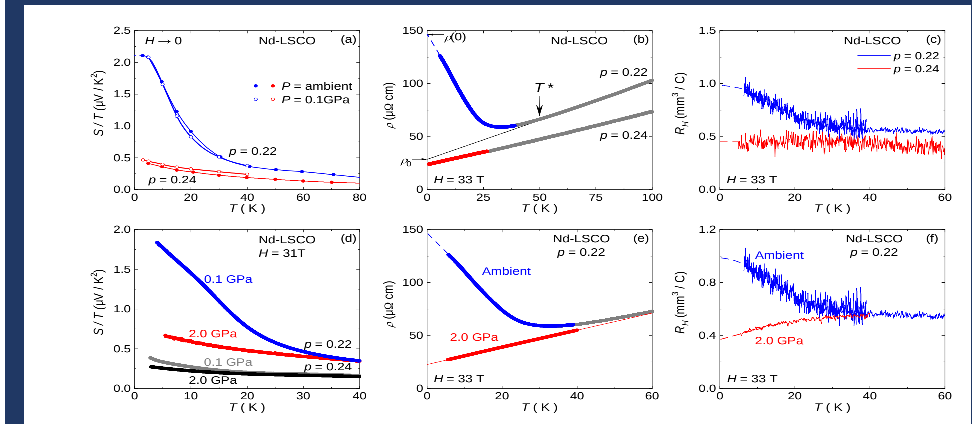
3) Slo... luctuati...
CDF dec... ole pairs?
New T=0 ... DF over a finite interval of QCP tuning parameter?

No exotic stuff: FL+ slower and slower CDF.....

If you hear hooves, think 'horse', not 'zebra'.

SLIDES DI APPOGGIO

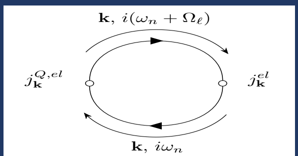
Effect of charge density fluctuations on thermopower properties



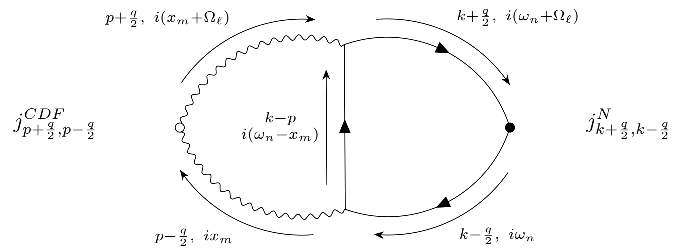
Gourgout et al., PHYSICAL REVIEW RESEARCH **3**, 023066 (2021)

When $\gamma \sim \log(1/T)$ then the same behavior occurs for Seebeck and C_V/T
G. Mirarchi et al.

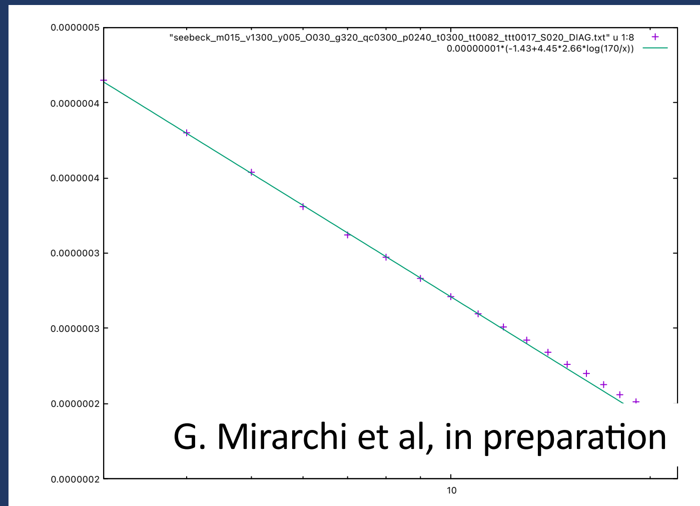
Also the CDF-drag contribution to thermopower is proportional to $\gamma \sim \log(1/T)$



CDF DRAG CONTRIBUTION TO THE SEEBECK COEFFICIENT

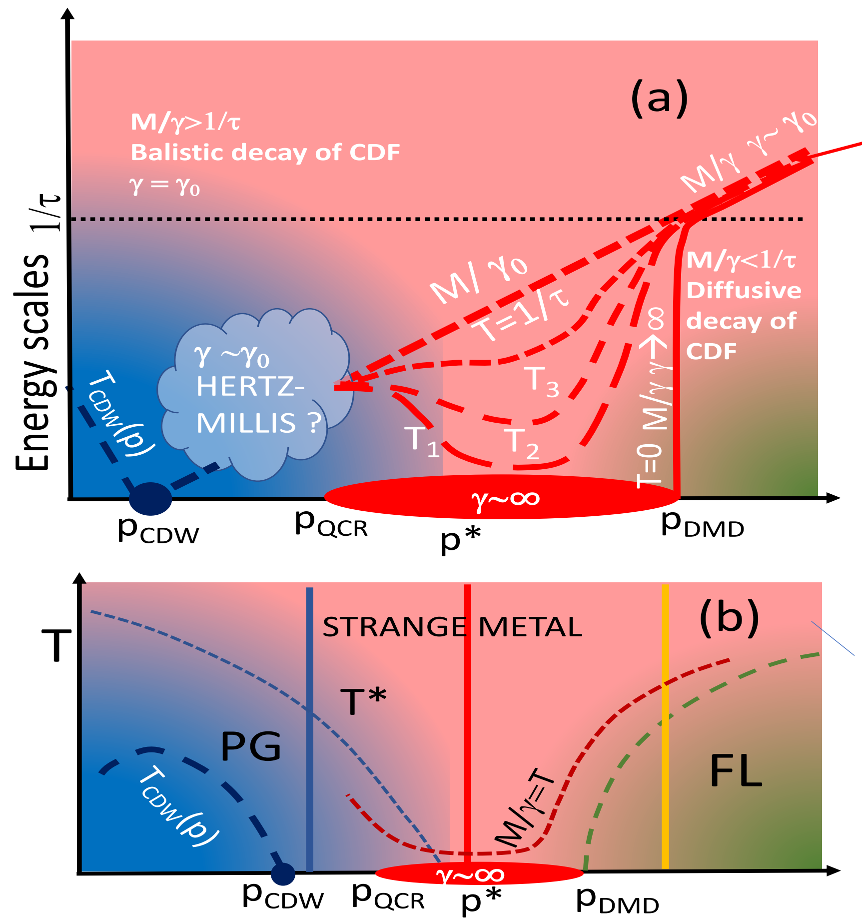


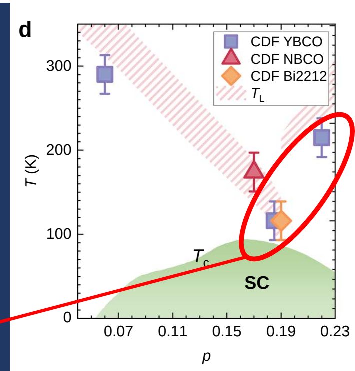
$$\Gamma^{12}_{drag} \propto g^2 T^2 \gamma$$



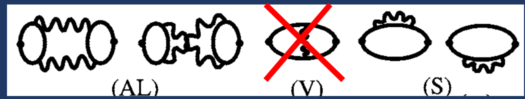
G. Mirarchi et al, in preparation

Where and how γ grows?



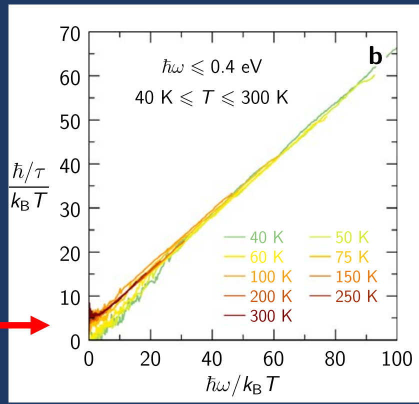


SOME EXPERIMENTAL CONSEQUENCES



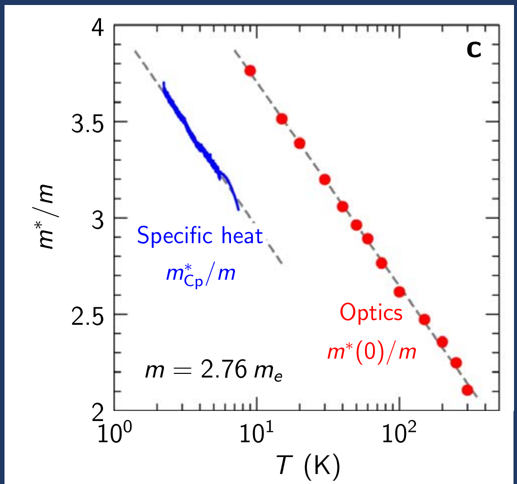
The interaction is (almost) momentum independent ⇒ vertex corrections negligible in current-current response ⇒Σ(ω,T) (almost) fully determines the optical conductivity σ(ω,T)

- At ω>T, σ(ω,T) quite similar to the MFL case
(see Michon et al, Nat Commun. 2023)



- But notice that σ(ω,T) scaling in not perfect at low ω when T<100 K
M/γ is small but finite and spoils perfect scaling

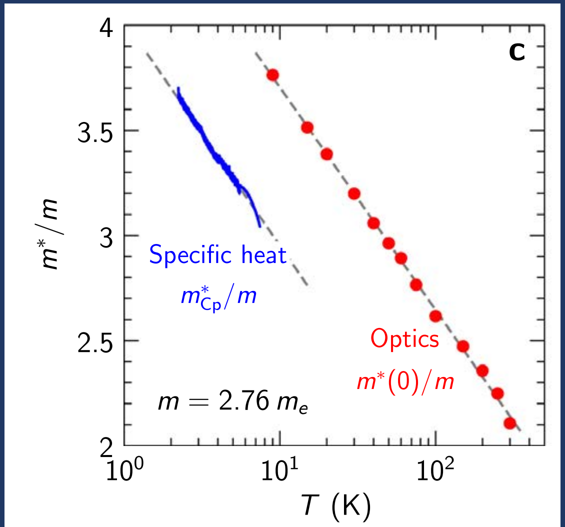
How about the effective mass m*(ω~0)?



It seems that m*/m from specific heat C_V/T and from optics m*(0)/m are similar and divergent. But in SFL m* is finite! Is there a problem for the SFL scenario?

Michon et al, Nat Commun. 2023

SOME EXPERIMENTAL CONSEQUENCES



Michon et al, Nat Commun. 2023

It seems that m*/m from specific heat C_V/T and from optics m*(0)/m are similar and divergent. But in SFL m* is finite!

Actually there is a way out for SFL:
Different origin of diverging m*s

- $C_V/T \sim \gamma \sim \log(1/T)$ (from bosonic modes, see next slide)
- m*(0)/m comes from finite frequency fermionic $\Sigma(\omega,T)$ which is quite similar to the MFL one at $\omega > M/\gamma \Rightarrow$ m*(0)/m diverges also in the SFL case….

Remember: the equal slope of m*/m from C_V/T and $\sigma(\omega,T)$ has been **imposed** in the experimental paper by choosing the total spectral weight (reasonable choice, but not mandatory…)

MAGNETORESISTANCE

First trivial/crucial remark: the T-linear resistivity and ω/T scaling is due to the Bose statistics at $T>\omega$:

$b(\omega)=1/(e^{\omega/T}-1)\sim T/\omega \quad \Rightarrow 1/\tau \sim T/\omega \, \mathrm{Im}D(\omega)$

By no means the magnetic field **H** can play the same role of T: no way to get $1/\tau \sim (T^2+H^2)$, H/T scaling and so on

What experiments say?

P. Giraldo-Gallo et al. Science 2018, (LSCO)
Ayres, J. et al. Nature 2021, (Tl2201, Bi2201)
Hayes, I. M. et al., Nat. Phys. 2016 (pnictides)

$1/\tau \sim \max[T,H] \sim (T^2+H^2)$, near a critical doping value

Ataei et al. Nat. Phys. 2022
The scattering rate is the sum of an elastic (*T*-independent) anisotropic term and an inelastic (*T*-dependent) isotr. term:

$$1/\tau\,(\varphi, T) = c[\,1/\tau_0 + 1/\,\tau_{aniso}\,|\cos(2\varphi)|^{\nu}] + \alpha k_B T/\hbar.$$

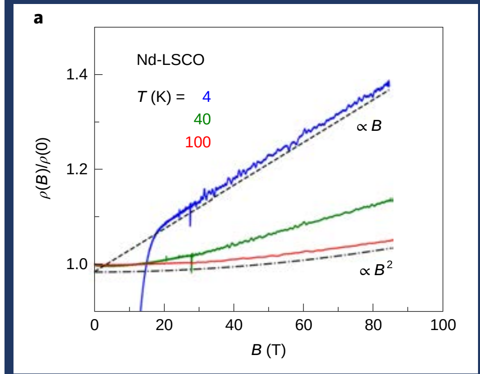
`the behaviour of electrons in a magnetic field in these strange metals is entirely the result of their orbital motion, and there is no evidence that the scattering rate has any field dependence.'

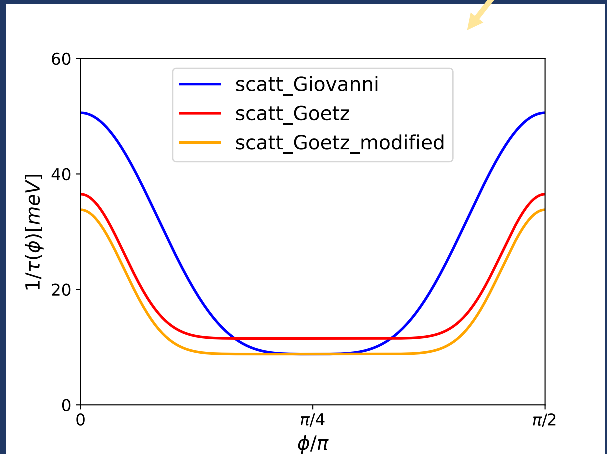
⇓

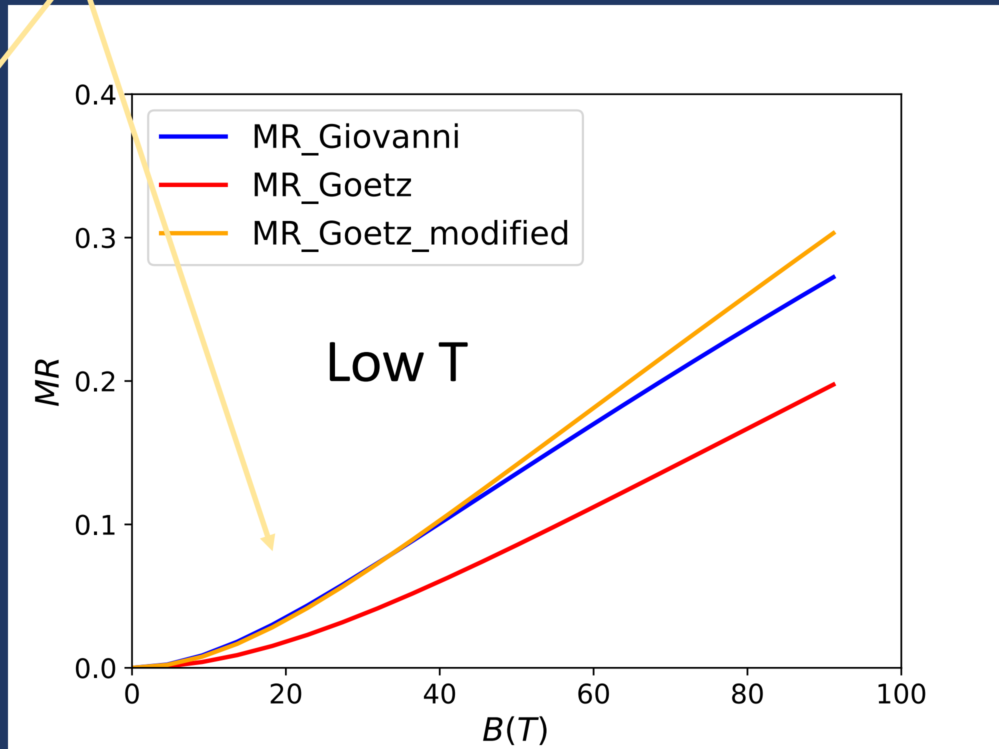
The *H*-linear dependence at low *T* is accounted for by Boltzmann theory, given the strongly anisotropic elastic scattering rate

Ataei et al. Nat. Phys. 2022

$$1/\tau(\varphi, T) = c[\,1/\tau_0 + 1/\tau_{aniso}\,|\cos(2\varphi)|^{\nu}] + \alpha k_B T/\hbar.$$





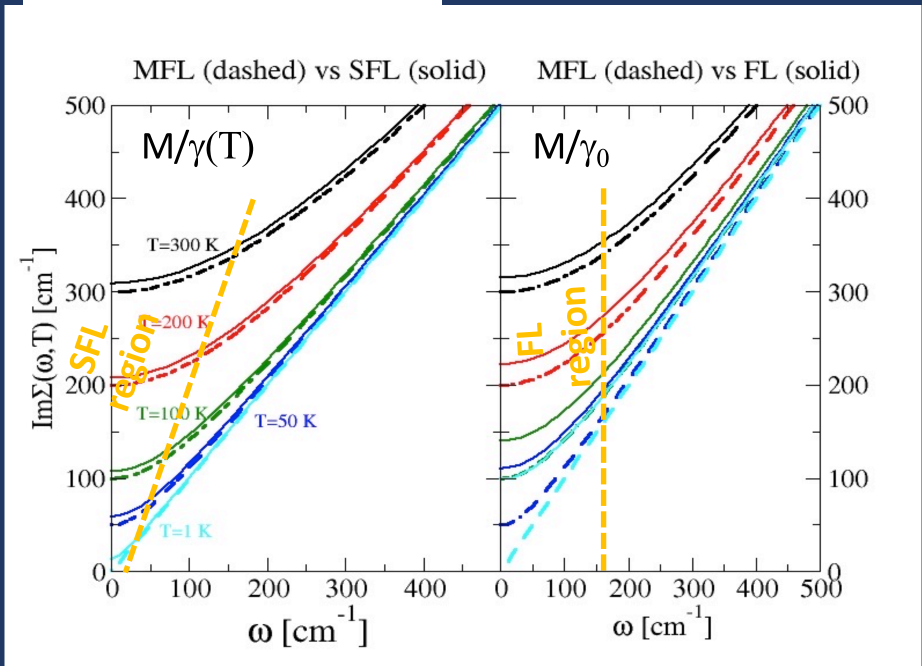


At low T the elastic scattering dominates

At higher T the inelastic part dominates and H^2 magnetoresistance is recovered

SFL vs MFL comparison

Mirarchi et al, Condens.Matter **2024**,*9*,14



MFL

$$|Im\Sigma(\omega,T)| \sim g\sqrt{\omega^2+T^2} \sim max[\omega,T]$$

Standard MFL ω/T **scaling** form

MFL has divergent QP mass $m_{QP}^* \sim \log(1/T)$

SFL

$$|Im\Sigma(\omega,T)| \sim g\left(\sqrt{(M/\gamma)^2+\omega^2+T^2}-M/\gamma\right)$$

SFL has an **almost scaling** form when $M/\gamma(T) \rightarrow 0$

FL for $\omega < M/\gamma$

$$\mathrm{Re}\Sigma(\omega,T=0) = -\frac{g^2 N_0}{\gamma}\arctan\left(\frac{\gamma\omega}{M}\right)$$

$$Z := \left(1-\frac{\partial \mathrm{Re}\Sigma(\omega,T=0)}{\partial\omega}\bigg|_{\omega=0}\right)^{-1} = \frac{M}{M+g^2 N_0}$$

Crucial difference for SFL: $m_{QP}^* = m/Z$ is finite

Question: Why should γ grow large? And why should $\gamma \sim \log(1/T)$?

EXTRINSIC MECHANISM

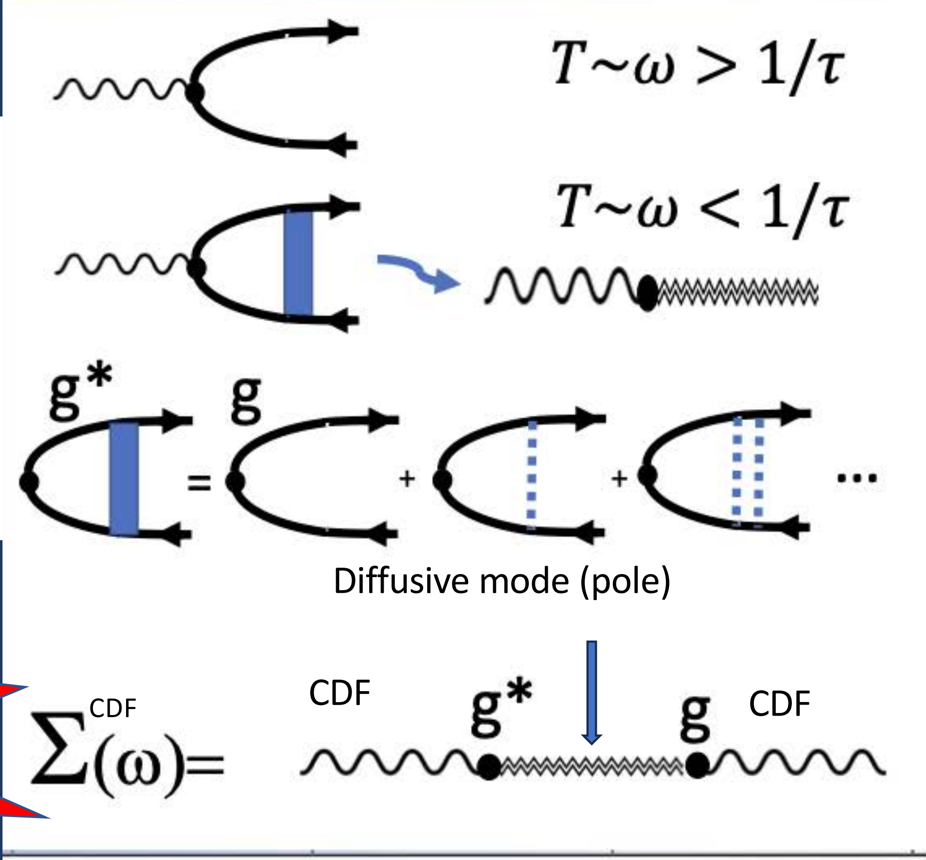
A model (for 2D only) [MG, C. Di Castro, G. Mirarchi, G. Seibold, S. Caprara Symmetry 15, 569 (2023)]

At low energy and T the CDF can decay in a diffusive p-h mode

In 2D the damping γ has a log renormalization

$$\delta\gamma = \gamma - \gamma_0 = A \log \max\left[(\tau T)^{-1}, 1\right],$$

The γ grows ~log (1/T)



Notice:
No strong disorder,
just few impurities of
a Drude metal

Why the model works only in 2D?

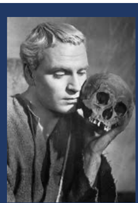
$$\Sigma^{\mathrm{CDF}}(\omega_n) = g^2 N_0 \int_{Q_{\min}}^{Q_{\max}} \frac{\mathrm{d}^2 q}{4\pi^2} \frac{Dq^2}{Dq^2 + |\omega_n|}$$

← Diffusive dens-dens corr fcn

$$= \frac{g^2 N_0}{4\pi D} \int_{T}^{1/\tau} \mathrm{d}(Dq^2) \left(1 - \frac{|\omega_n|}{Dq^2 + |\omega_n|}\right) = \delta M - |\omega_n| \delta\gamma.$$

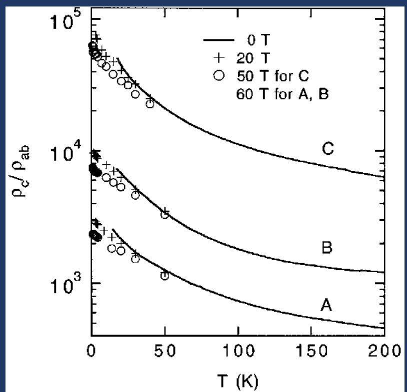
The self-energy of CDF due to diffusive modes is only singular in D=2

$$\delta\gamma = \gamma - \gamma_0 = A \log \max\left[(\tau T)^{-1}, 1\right]$$

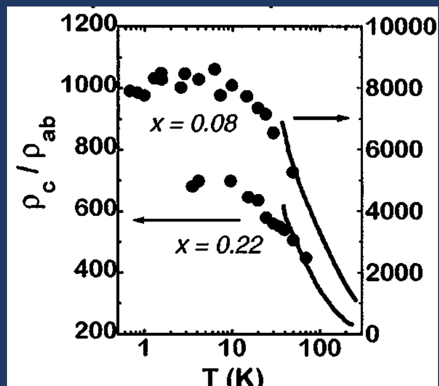


2D or not 2D, this is the question

Still….in some systems around p* the system **does become increasingly anisotropic (2D)**

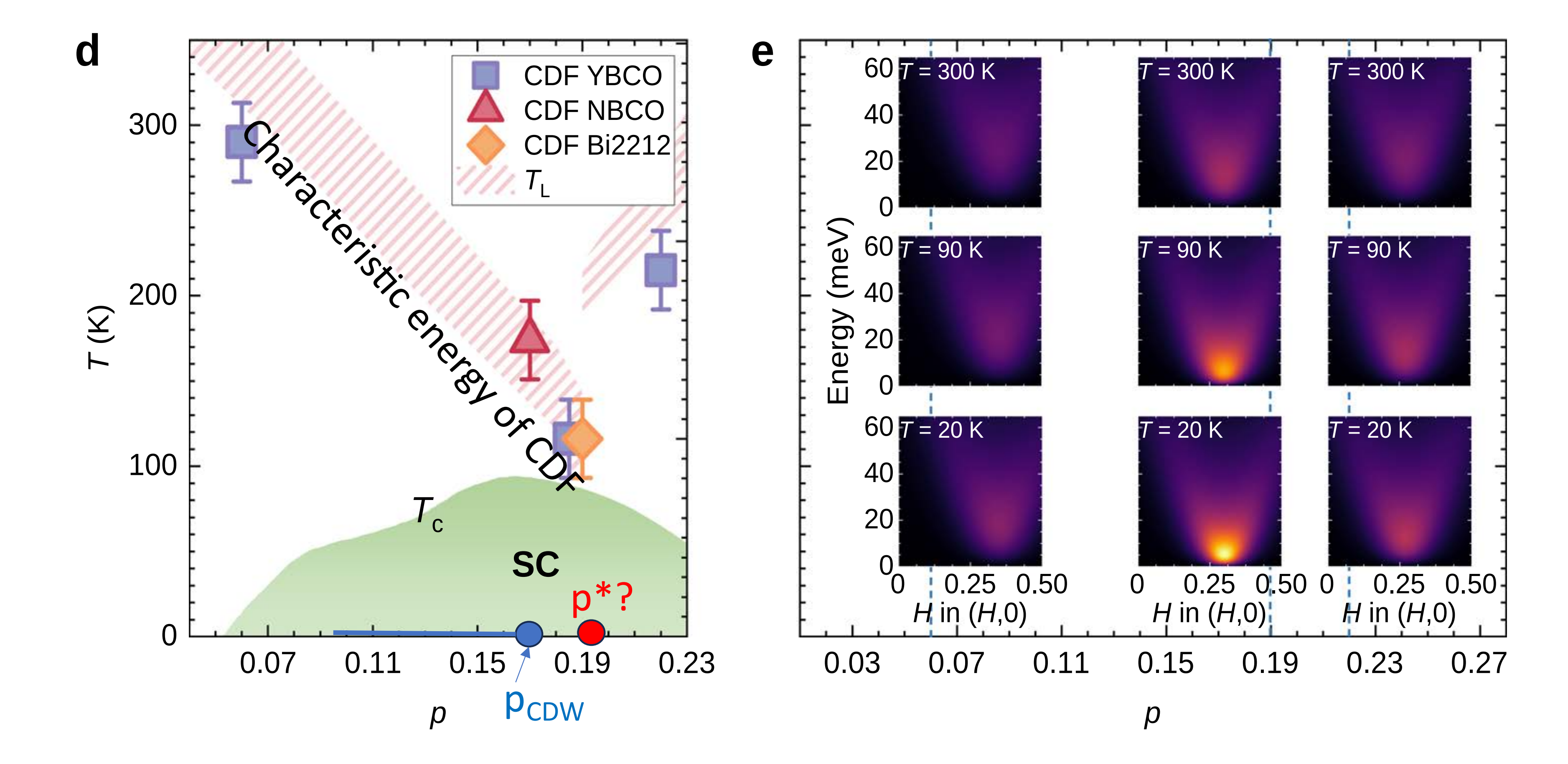


Y. Ando et al. PRL 1996
$Bi_2Sr_{2-x}La_xCuO_y$



Boebinger et al, PRL 1996
$La_{2-x}Sr_xCuO_4$

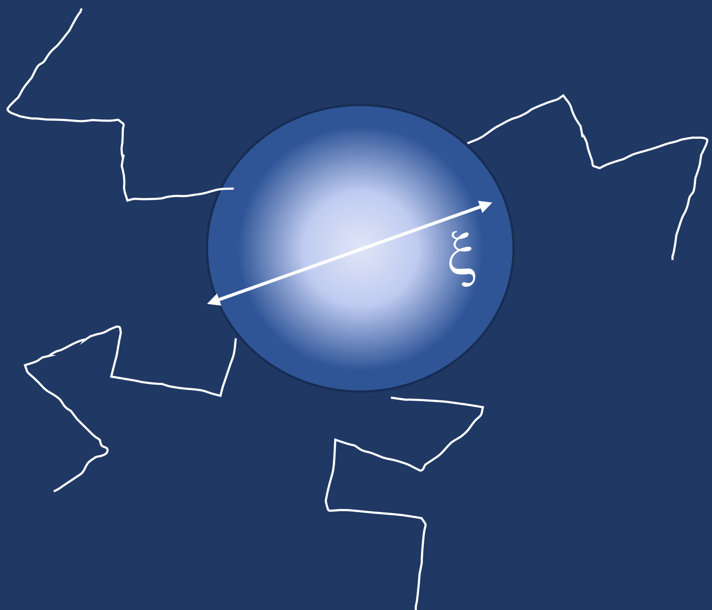
P. M. Lozano, G. D. Gu, Qiang Li, and J. M. Tranquada
arXiv: 2307.13740



Arpaia, R, et al., *Nat. Commun.* **2023**, *14*, 7198

QUICK SUMMARY AND PERSPECTIVE: WHAT IS THE MAIN IDEA

Proximity to a QCP to have abundant order parameter fluctuations. But stay away from it:



A local order parameter fluctuation embedded in a bath of fermionic quasiparticles

Similarities with

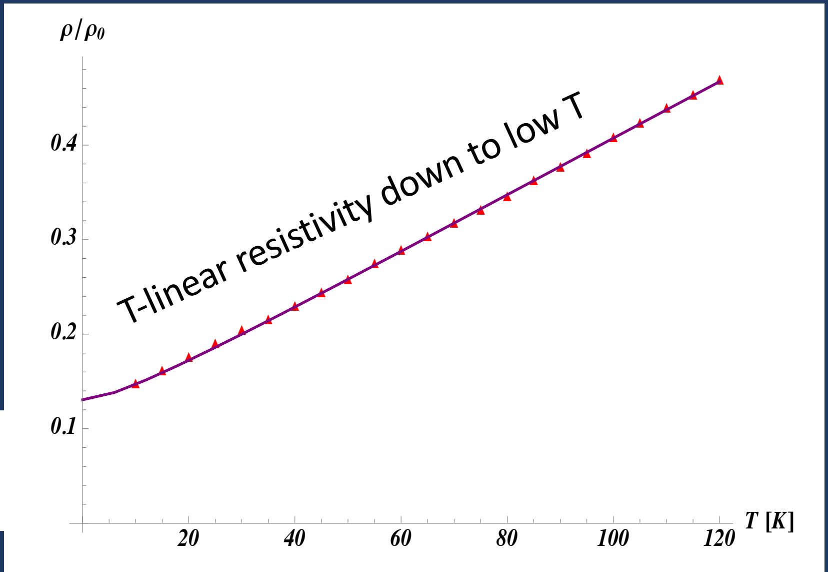
- SYK model (Sachdev, Patel, Parcollet, Schmalian, Valentinis,…)
- Spin-Boson (Schmalian, Berg,…) (also C_v=T log(1/T) from bosons)
- Kondo-destruction as if $\gamma \sim 1/\omega^{1-\alpha}$ but $\xi \rightarrow \infty$

COMMON IDEA:
let fermions interact with local d.o.f. at **low-energy**
(built-in in SYK models, due to Spin-Boson and CDF in the SFL model)

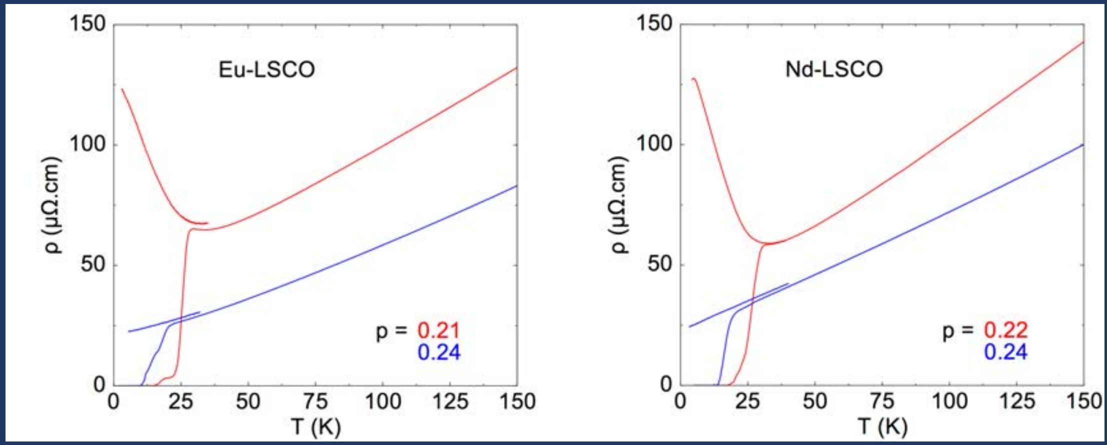
What happens when γ grows?

Linear resistivity down to lower and lower temperatures

$M/\gamma<T$



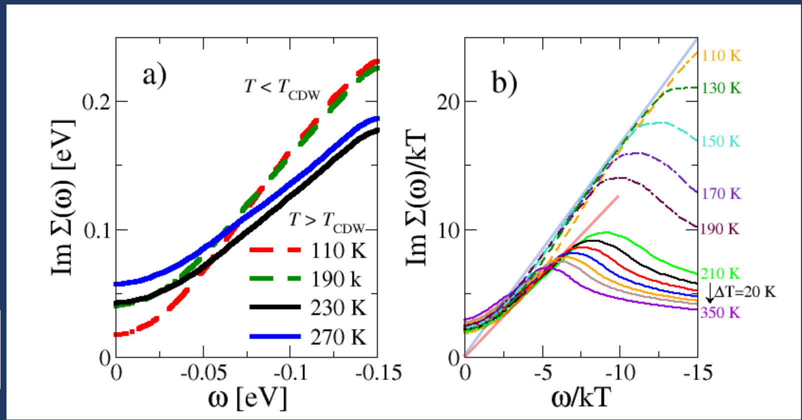
[MG, C. Di Castro, G. Mirarchi, G. Seibold, S. Caprara Symmetry 15, 569 (2023)]

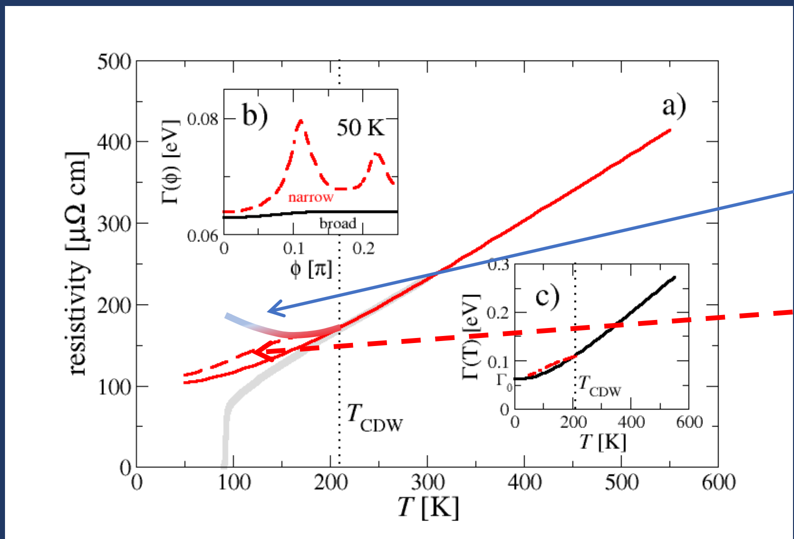


Michon et al, Nature 2019

When both CDW and CDF are present (UD samples)

Seibold et al., arXiv:1905.10232





The inhomogeneous mixture of CDW and CDF is treated:

- summing the two scattering channels (Mathiessens rule)
- with Effective medium theory EMT

$$Z = \frac{M}{M + g^2 N_0 \left[\frac{M^2}{M^2 + (2\gamma k_B T)^2} + \frac{16}{3\pi^2} \Phi\left(x = \frac{3\pi^2}{8} \frac{\gamma k_B T}{M} \right) \right]}$$

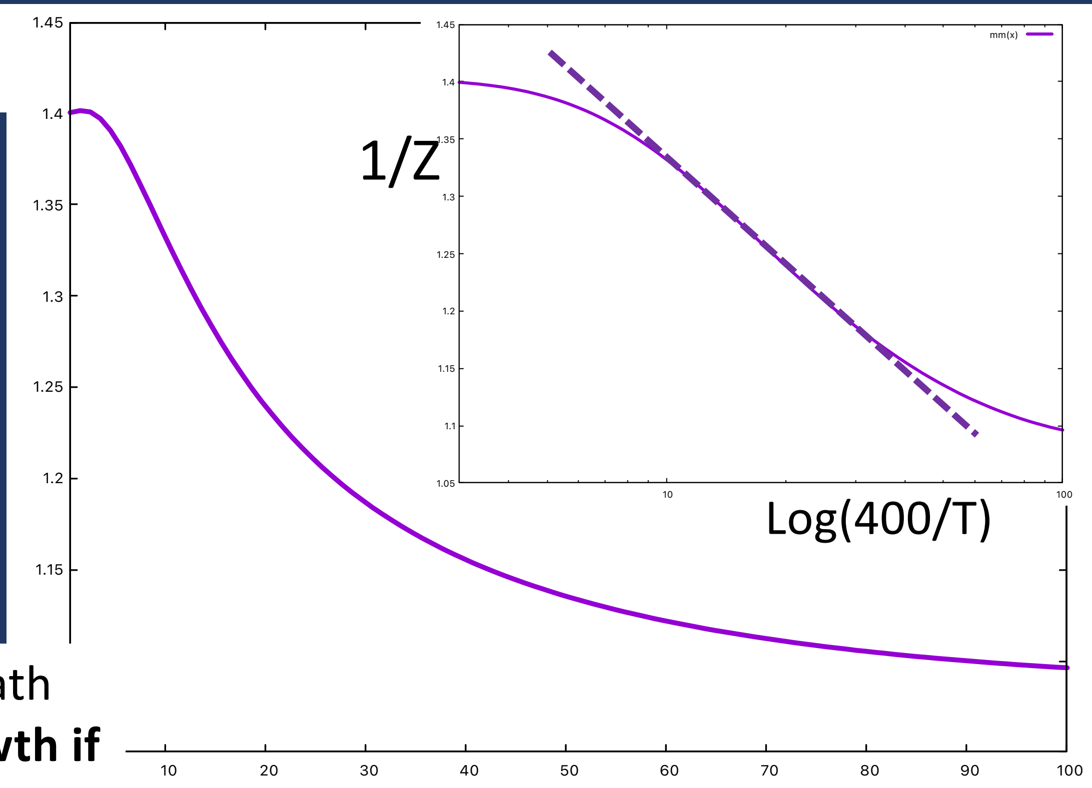
where

$$\Phi(x) := \frac{\arctan(x)}{x} - \frac{1}{1 + x^2}$$

The dimensionless coupling
$\lambda = g^2 N_0 / M \sim 0.4$
Determines m*/m at T=0

m*/m=1+λ=1.4

Seibold et al, Commun. Phys. 2021



m*/m can contribute to the specific heath C_V/T , **but it cannot explain a large growth if λ is small/moderate**

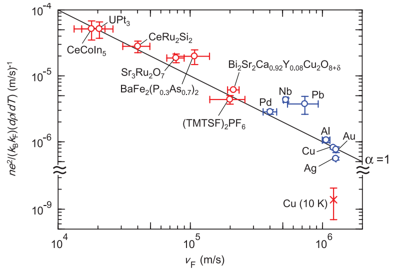
ARE STRANGE METALS ALL `PLANCKIAN'?

$1/\tau = \alpha K_B T$ with $\alpha \sim 1$

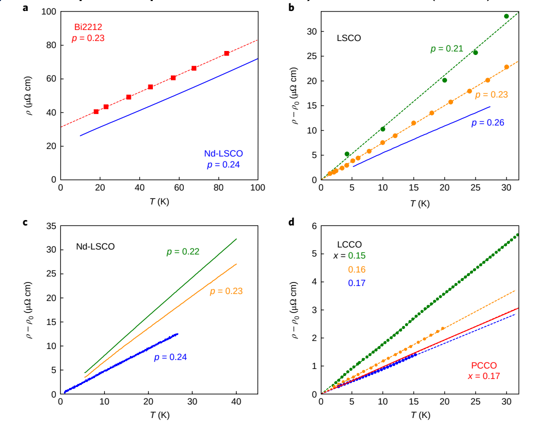
Similarity of Scattering Rates in Metals Showing *T*-Linear Resistivity

J. A. N. Bruin,[1] H. Sakai,[1] R. S. Perry,[2] A. P. Mackenzie[1]

Science 2013



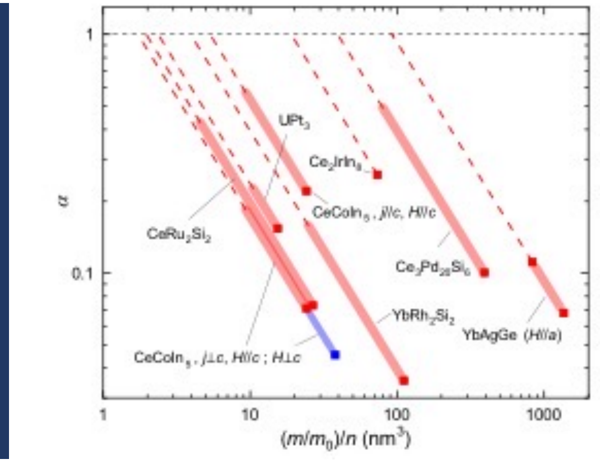
Legros, A. et al. Universal T-linear resistivity and Planckian dissipation in overdoped cuprates. *Nat. Phys.* **15**, 142 (2019).



Are Heavy Fermion Strange Metals Planckian?

Mathieu Taupin and Silke Paschen *

Crystals **2022**, *12*, 251. https://doi.org/10.3390/cryst12020251



Planckian relaxation delusion in metals
M V Sadovskii, 2021 *Phys.-Usp.* **64** 175

Not all strange metals are Planckian?

See also Hartnoll & Mackenzie, RMP 2022

SOME EXPERIMENTAL CONSEQUENCES

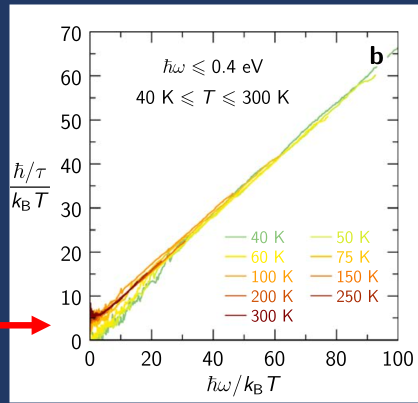
The interaction is (almost) momentum independent ⇒ vertex corrections negligible in current-current response ⇒$\Sigma(\omega,T)$ (almost) fully determines the optical conductivity $\sigma(\omega,T)$

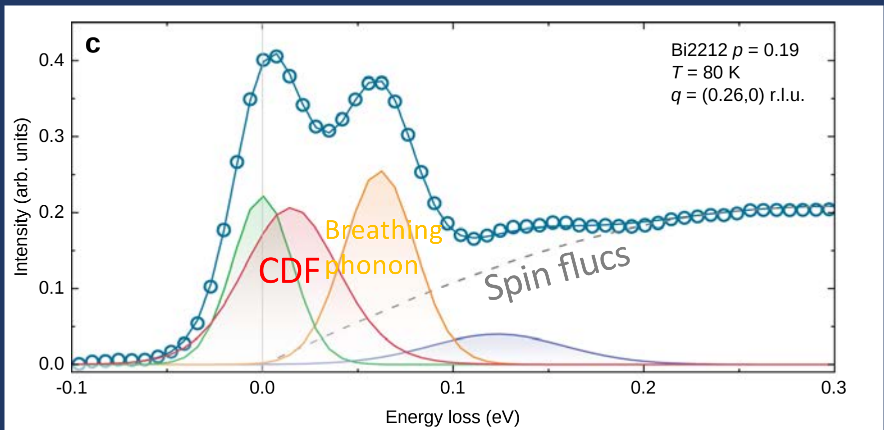
- At $\omega>T$, $\sigma(\omega,T)$ quite similar to the MFL case

(see Michon et al, Nat Commun. 2023)

- But notice that $\sigma(\omega,T)$ scaling in not perfect at low ω when T<100 K

M/γ is small but finite and spoils perfect scaling

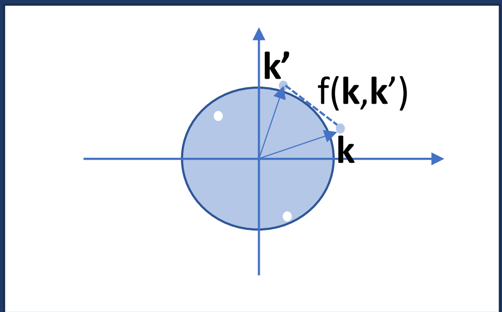




Arpaia, R, et al., *Nat. Commun.* **2023**, *14*, 7198

The Fermi Liquid in a nutshell (4/4)

The Fermi liquid:

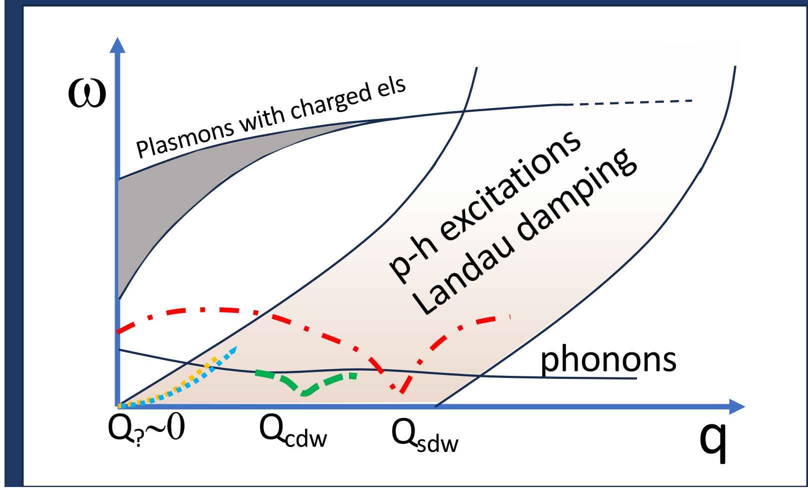


The residual interaction between QP is contained in the interaction function f(k,k')

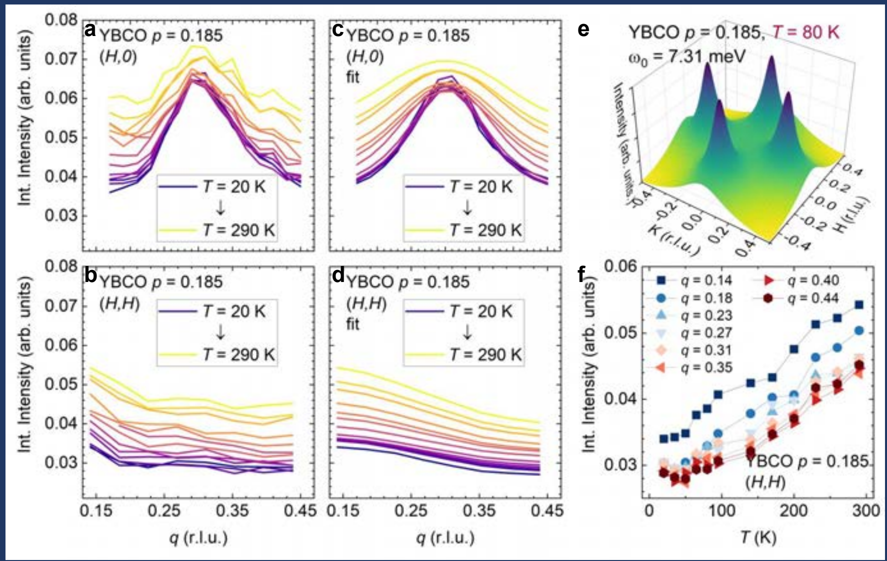
From f(k,k') the Landau parameters $F_{(s,a)l}$ are derived that describe physical quantities ($F_{s0} \rightarrow$ compressibility, $F_{a0} \rightarrow$ magn. suscept.,...)

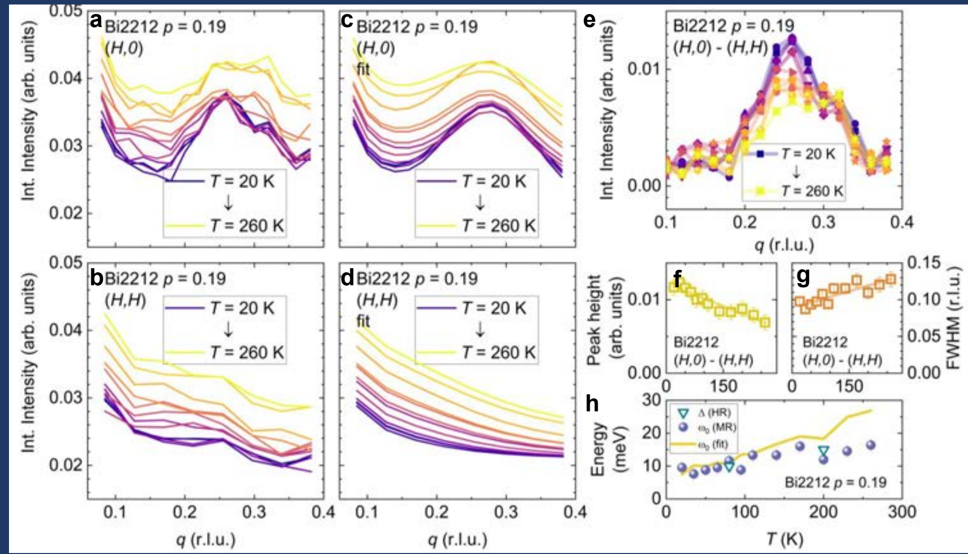
This also entails the Pomeranchuk stability conditions of the FL.
E.g. for the symmetric channel case

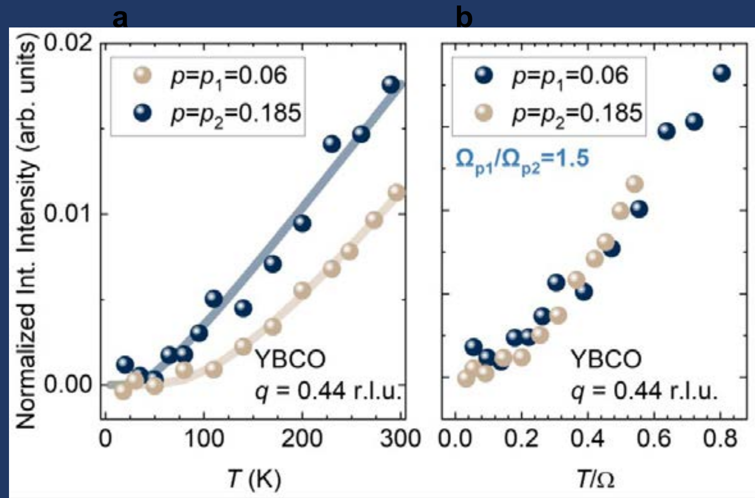
$$1 + \frac{F_{sl}}{2l+1} > 0$$



Near the instabilities many different collective excitations can populate the ω-q plane:
paramagnons, Charge Density Waves, Pomeranchuk flucts, Circulating Currents







Arpaia, R, et al., *Nat. Commun.* **2023**, *14*, 7198