

From fermions to spinons and holons, a tale of dimensionality

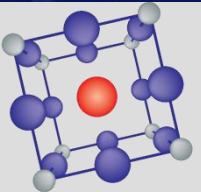
<http://giamarchi.dqmp.ch>



**UNIVERSITÉ
DE GENÈVE**



FONDS NATIONAL SUISSE
SCHWEIZERISCHER NATIONALFONDS
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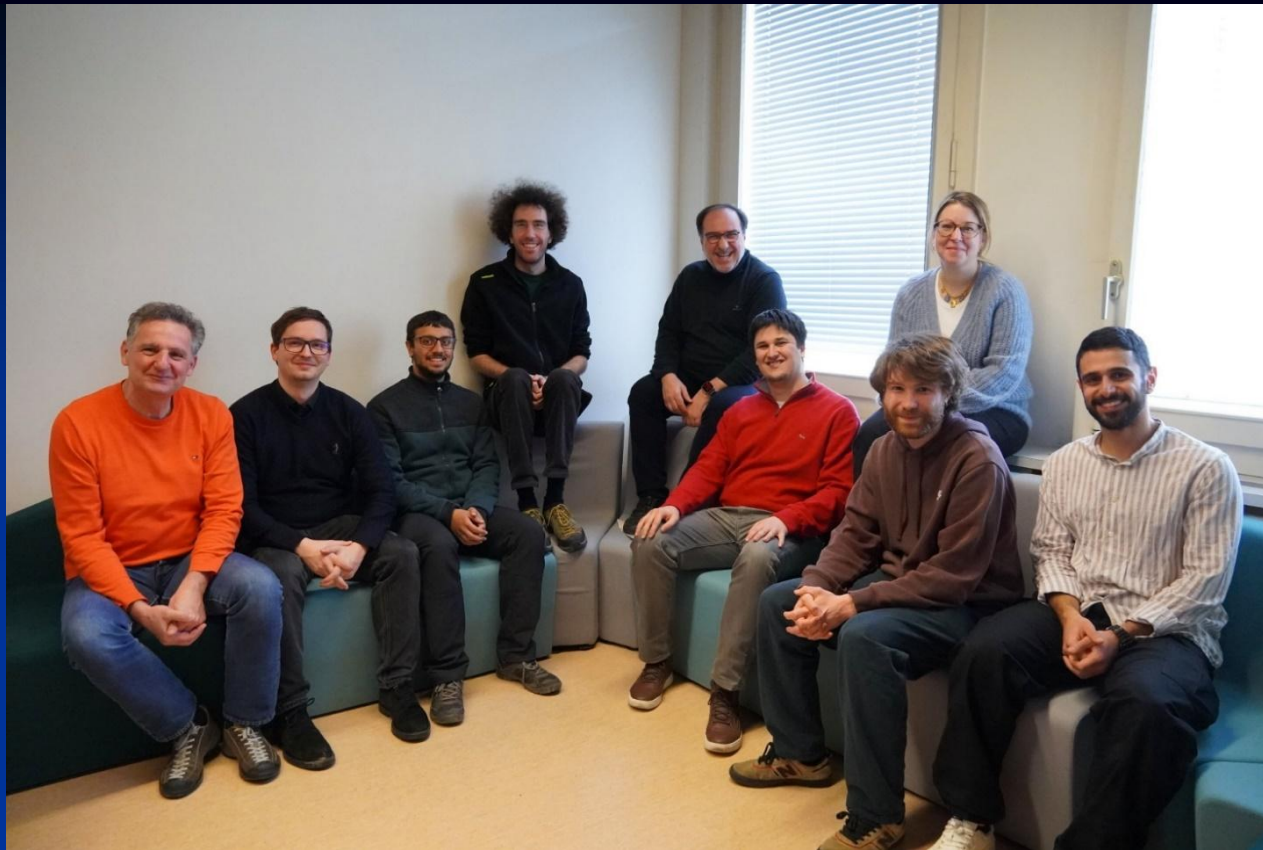




R. Citro



E. Orignac



C. Kollath



M. Cazalilla

C. Berthod, C. Halati, S. Sur, G. Morpurgo, TG, L. Gotta, F. Hartmeier, D. Bertolusso, L. Pizzino

H.J. Schulz, M. Cazalilla, R. Citro, C. Kollath, E. Orignac,

Condensed matter: P. Bouillot, R. Chitra, A. Tsvelik, C. Varma,

Exp Groups: C. Berthier, L. Degiorgi, M. Dressel, B. Grenier, D. Jerome, S. Petit, C. Ruegg, V. Simonet, A. Zheludev,...

Cold atoms: V. Cheianov, E. Demler, A. Ho, L. Sanchez-Palencia, U. Schollwoeck, P. Torma, M. Zvonarev,...

Exp Groups: I. Bloch, J.P. Brantut, T. Esslinger, L. Fallani, R. Hulet, M. Inguscio, F. Minardi, G. Modugno, H.C. Nagerl, G. Roati,

How to master materials ?



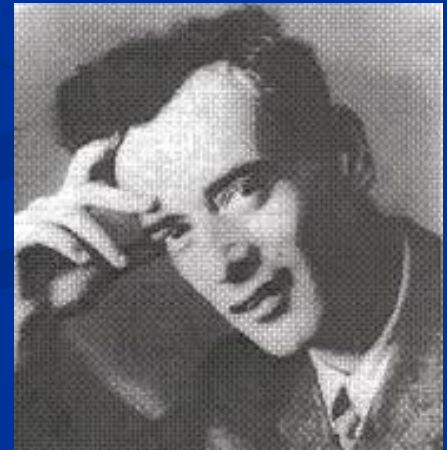
- Understood: free electrons
- Real systems : Coulomb interaction

$$E \sim 10\,000\text{ K} !$$

- Properties of realistic systems ?

- Free electron theory works quite well : Landau Fermi liquid

$$m \rightarrow m^*$$



One dimensional quantum systems



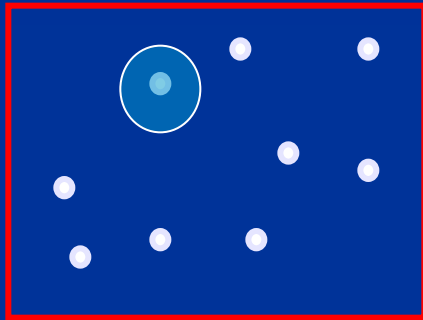
A good reason to work on 1D

However, my personal reason for working on one-dimensional problems is merely that they are **fun**. A man grows stale if he works all the time on the insoluble and a trip to the beautiful work of one dimension will refresh his imagination better than a dose of LSD.

Freeman Dyson (1967); review in Physics today of the book by E.H. Lieb and D.C. Mattis

Why is 1D special (and interesting) ?

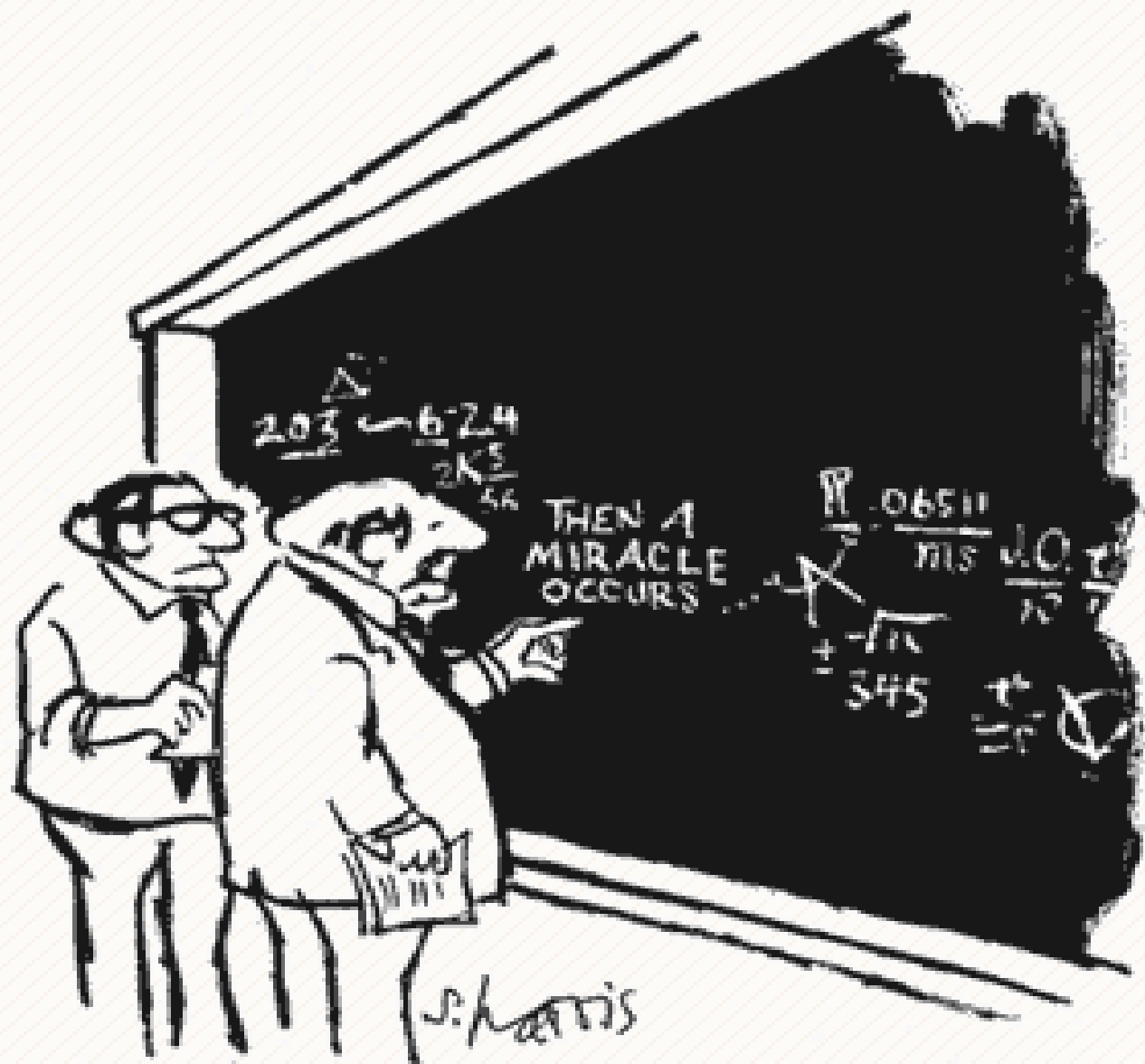
- No individual excitation can exist (only collective ones)



- Strong quantum fluctuations

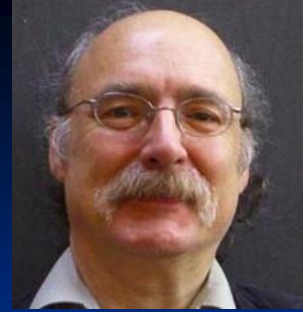
$$\psi = |\psi| e^{i\theta}$$

Difficult to order



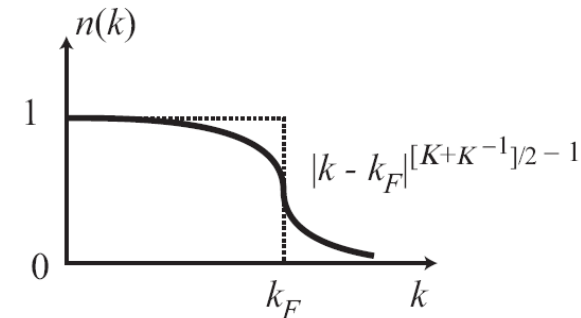
"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."

Tomonaga-Luttinger liquids



- No separation of statistics and interactions
- Criticality:

$$\begin{aligned}\langle \rho(x, \tau) \rho(0) \rangle = & \rho_0^2 + \frac{K}{2\pi^2} \frac{y_\alpha^2 - x^2}{(x^2 + y_\alpha^2)^2} + \rho_0^2 A_2 \cos(2\pi \rho_0 x) \left(\frac{\alpha}{r}\right)^{2K} \\ & + \rho_0^2 A_4 \cos(4\pi \rho_0 x) \left(\frac{\alpha}{r}\right)^{8K}\end{aligned}$$



- Few parameters control all physical properties:
 - u: velocity of excitations (sound velocity)
 - K: controls all exponents
 - A: non-universal amplitudes

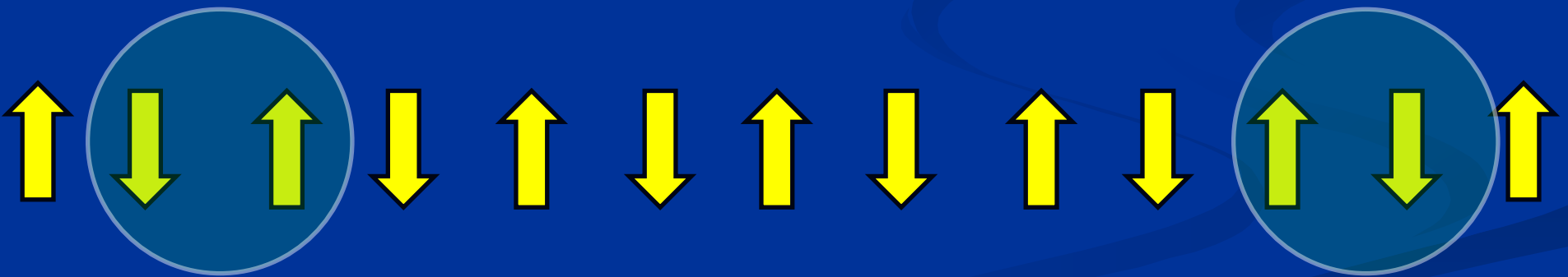
Fractionalization of excitations

$$H = J \sum_i \vec{S}_i \cdot \vec{S}_{i+1}$$



$$\Delta S = -1 \quad E = \hbar \omega(q)$$

Magnon



$$\Delta S = -1/2$$

Spinons

$$\Delta S = -1/2$$

Magnons and spinons: $1 = \frac{1}{2} + \frac{1}{2}$

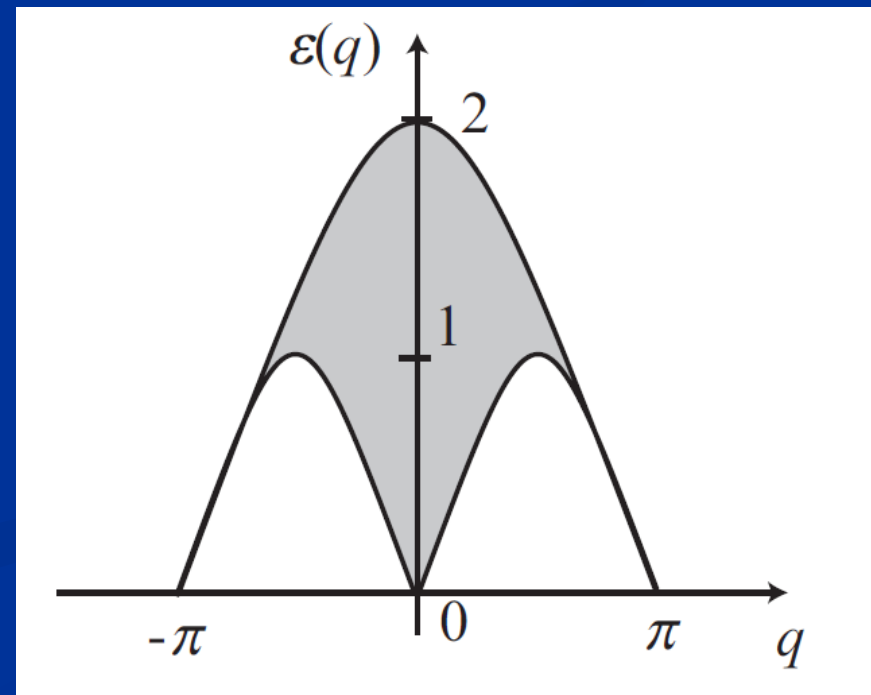


- Hidden (topological) order parameters

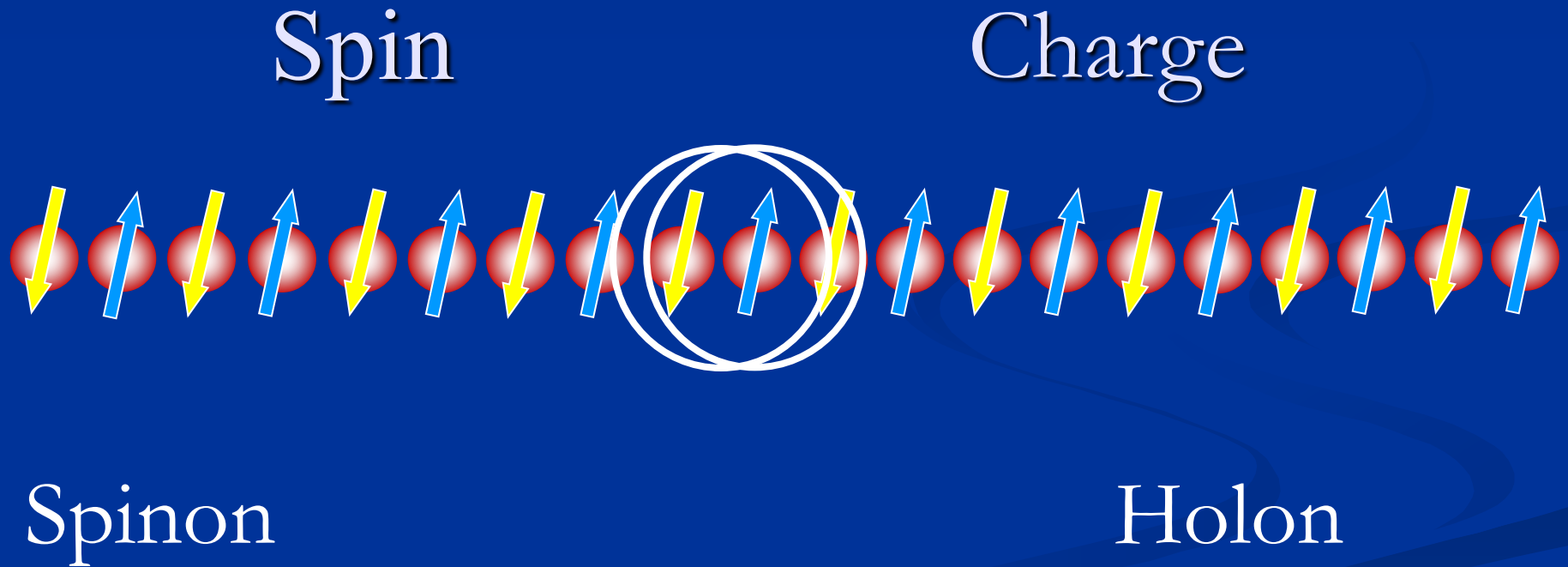
- Continuum of excitations

$$E(\mathbf{k}) = \cos(k_1) + \cos(k_2)$$

$$\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2$$



Deconstruction of the electron: spin-charge separation

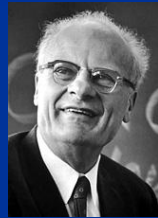


How to treat (20th century)

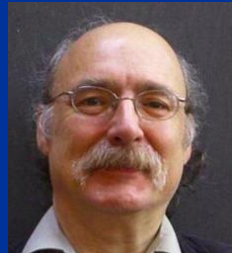
■ “Standard” many body theory



■ Exact Solutions (Bethe ansatz)



■ Field theories (bosonization, CFT)

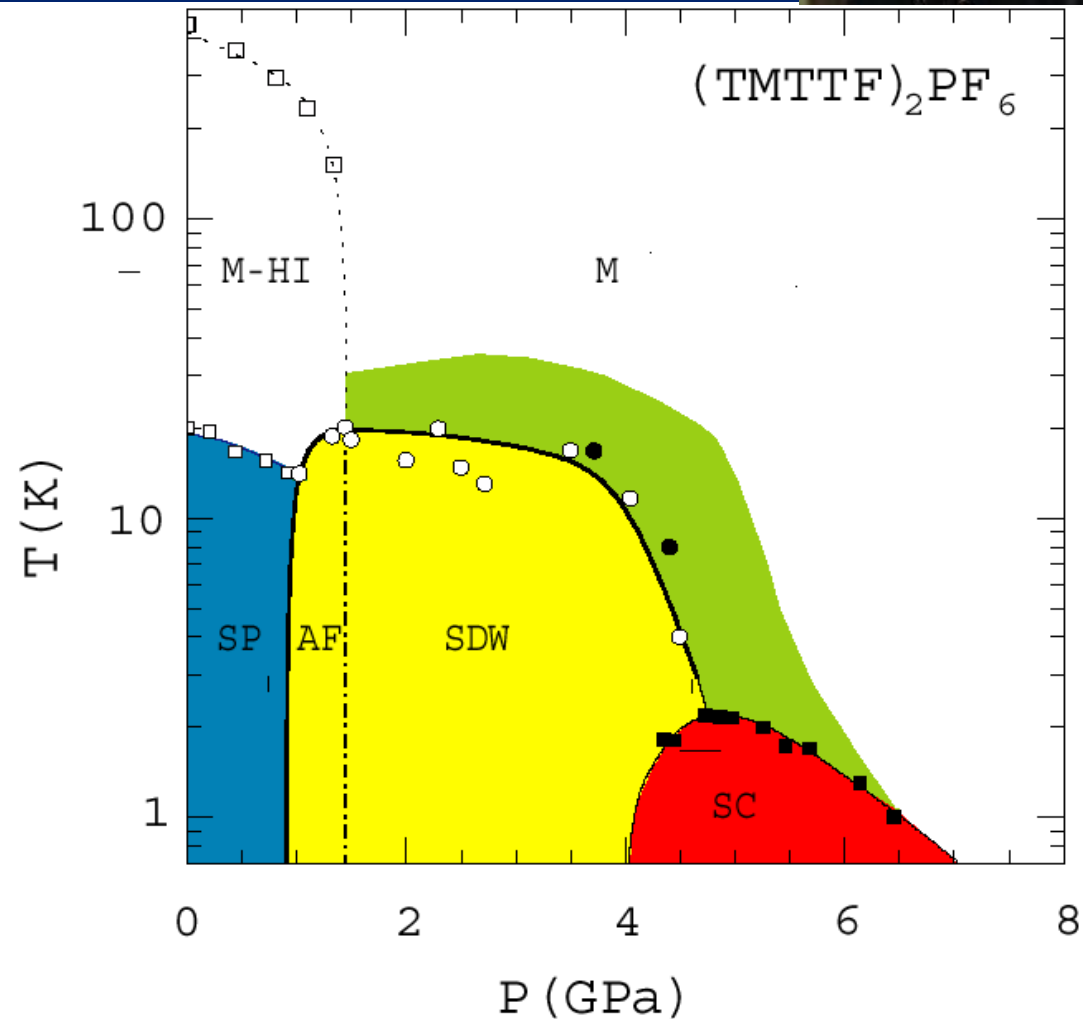
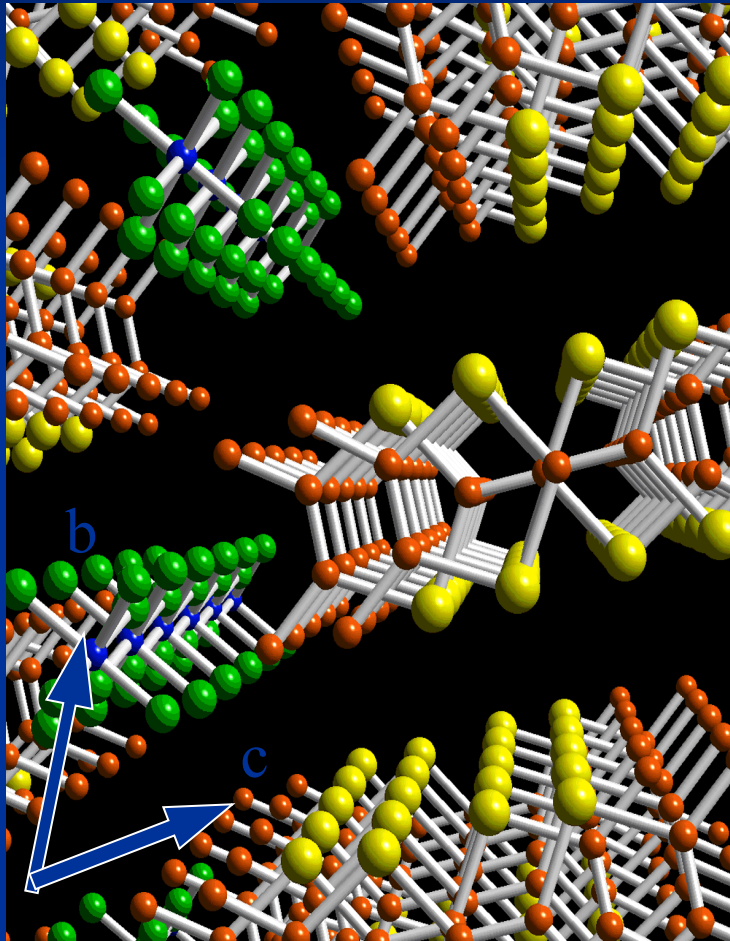


■ Numerics (DMRG, MC, etc.)





Organic conductors



D. Jaccard et al., J. Phys. C, 13 L89 (2001)

Optical conductivity

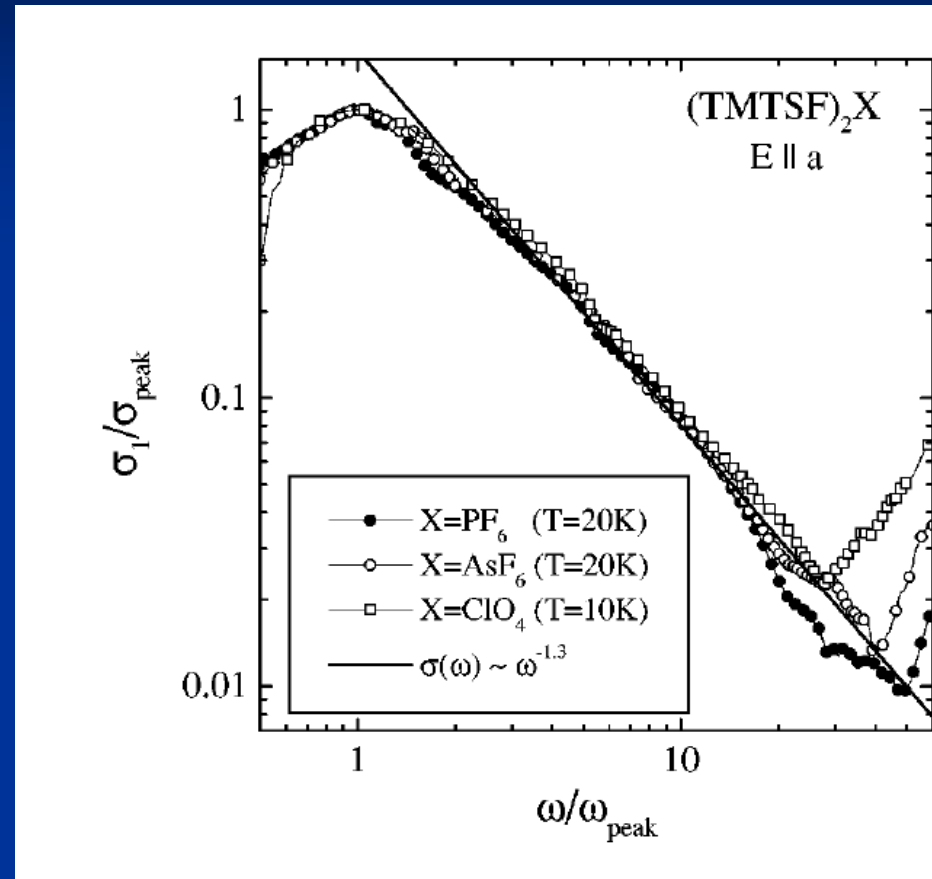


$$\sigma(\omega) \propto \omega^\nu$$

TG PRB (91) :

Physica B 230 (1996)

First observation of LL !!

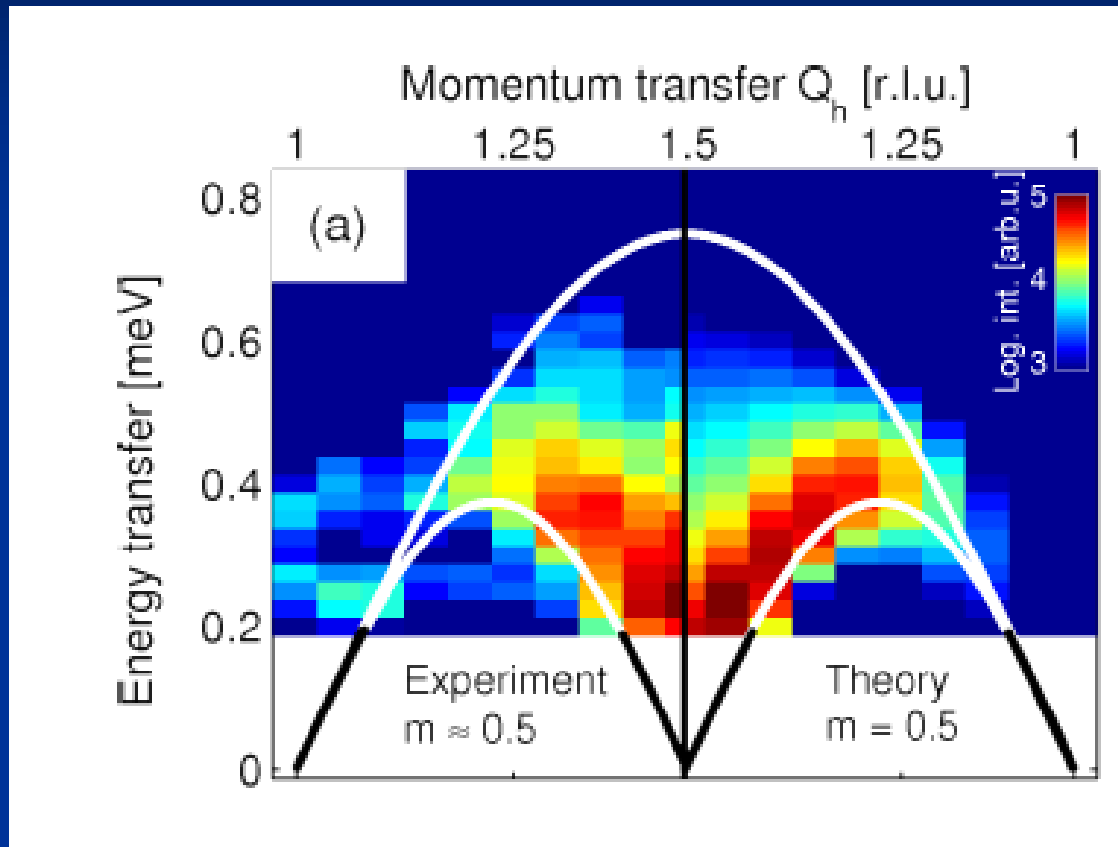


A. Schwartz et al. PRB 58
1261 (1998)

Fractionalization

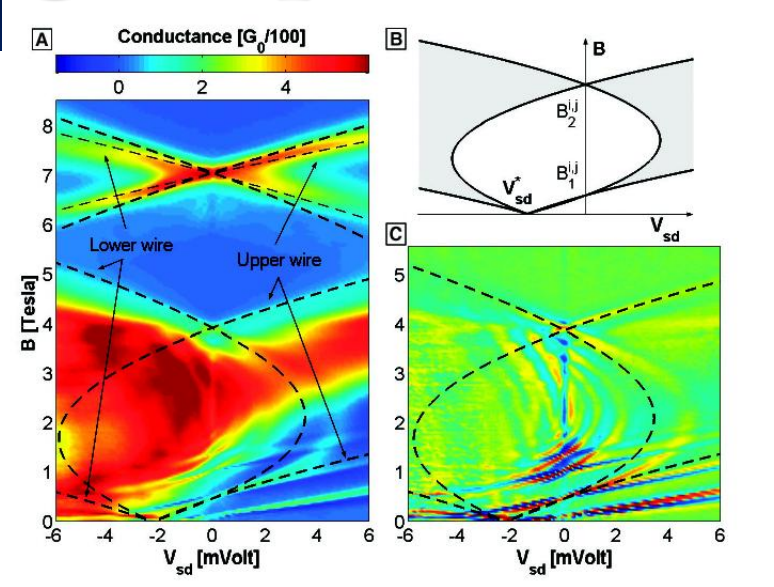
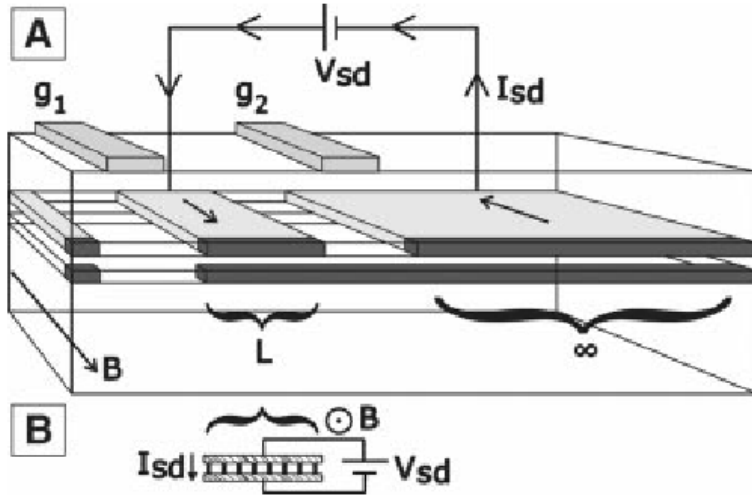


Quantum spin systems



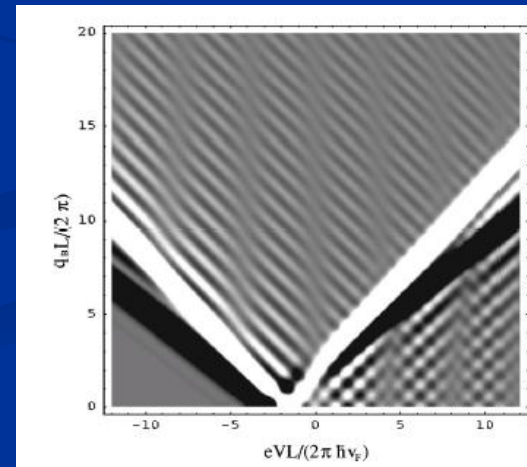
B. Thieleman et al. PRL 102, 107204 (2009)

Spin charge separation



O.M Ausslander et al., Science **298** 1354 (2001)

Y. Tserkovnyak et al., PRL **89**
136805 (2002) ; PRB **68**
125312 (2003)



1D in the 21st century



Drastic evolution of the 1d world

- New methods (DMRG, correlations from BA, etc.)
- New systems (cold atoms, magnetic insulators, etc.)
- New questions (strong SOC, out of equilibrium, etc)

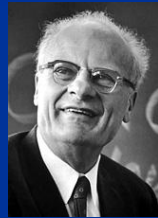
Novel theoretical description

- Field theory:
 - Asymptotically true: ($r \gg a$; $t \gg t_0$; $T \ll J$)
 - Amplitudes and TLL parameters unknown
- Numerics:
 - Efficient at short time, short distance
 - Takes into account the full microscopic model
- Best of both worlds: combine numerics (DMRG) or BA and field theory : essentially exact !

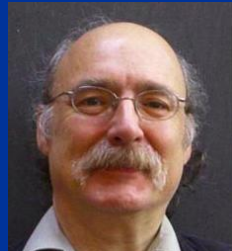
■ “Standard” many body theory



■ Exact Solutions (Bethe ansatz)



■ Field theories (bosonization, CFT)



■ Numerics (DMRG, MC, etc.)





Charge velocity

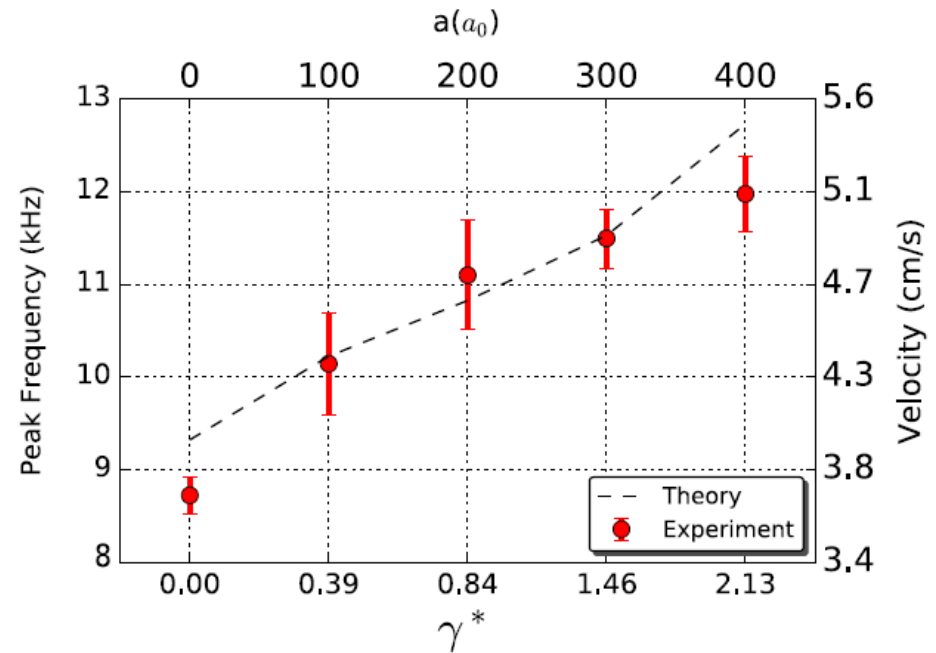
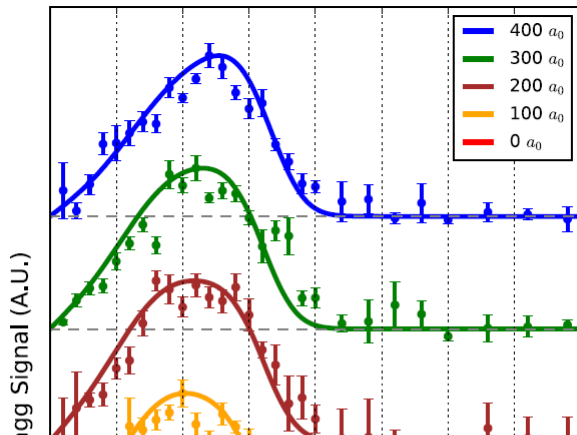


PHYSICAL REVIEW LETTERS **121**, 103001 (2018)

Editors' Suggestion

Measurement of the Dynamical Structure Factor of a 1D Interacting Fermi Gas

T. L. Yang,¹ P. Grišins,² Y. T. Chang,¹ Z. H. Zhao,¹ C. Y. Shih,¹ T. Giamarchi,² and R. G. Hulet¹



ULTRACOLD ATOMS

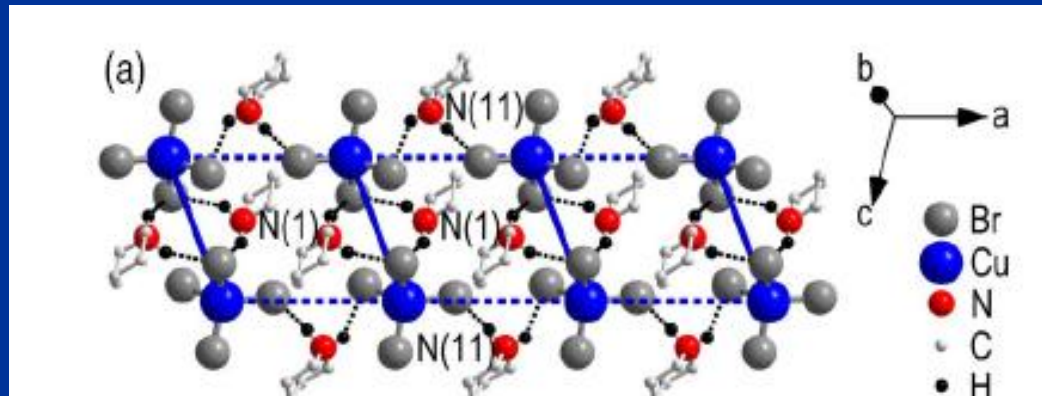
CONFIRM 55-YEAR-OLD

PHYSICS THEORY

<https://www.futurity.org/one-dimensional-electrons-physics-1858622/>

Quantum spin systems

TG, Ch. Rüegg, O. Tchernyshyov, Nat. Phys. 4 198 (08)

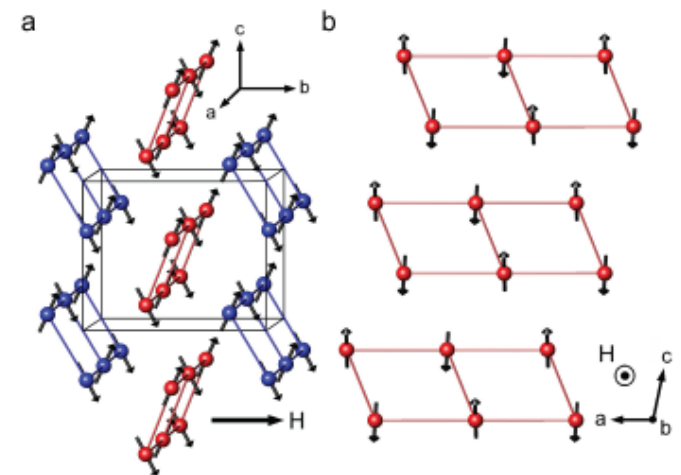


M. Klanjsek et al.,

PRL 101 137207 (2008)

B. Thielemann et al.,

PRB 79, 020408 © 2009

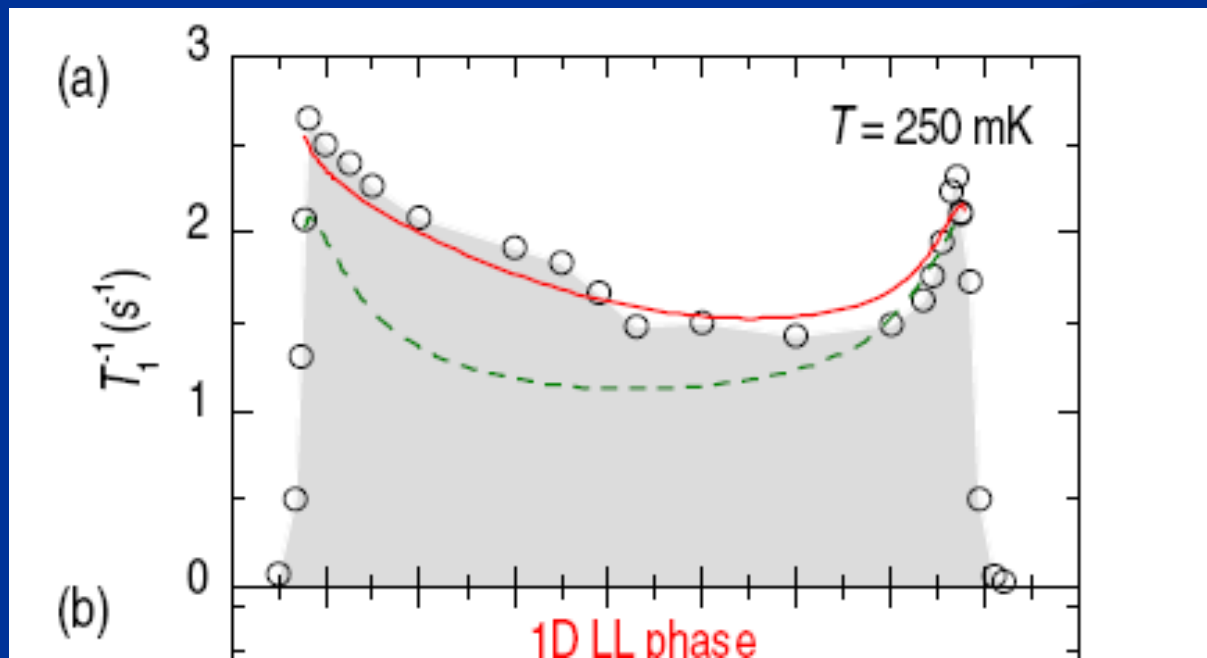




NMR relaxation time

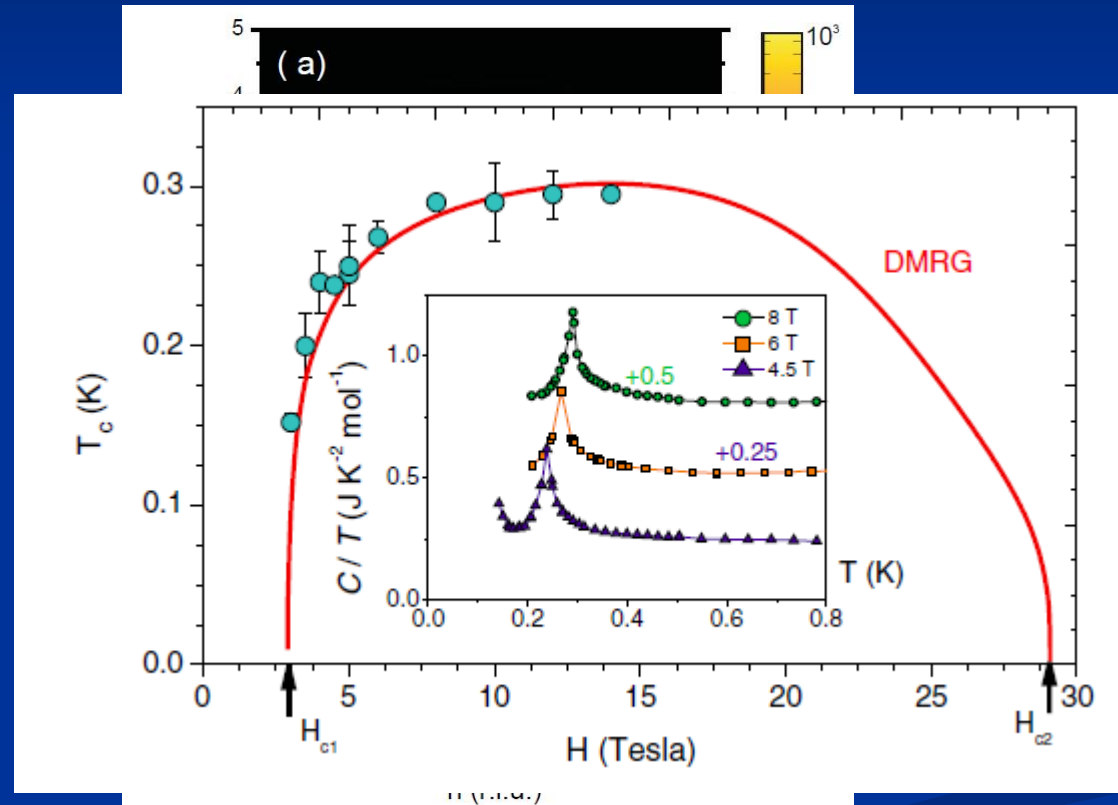
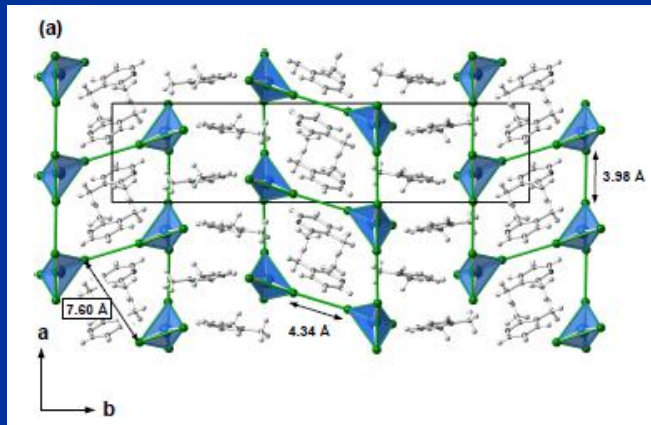
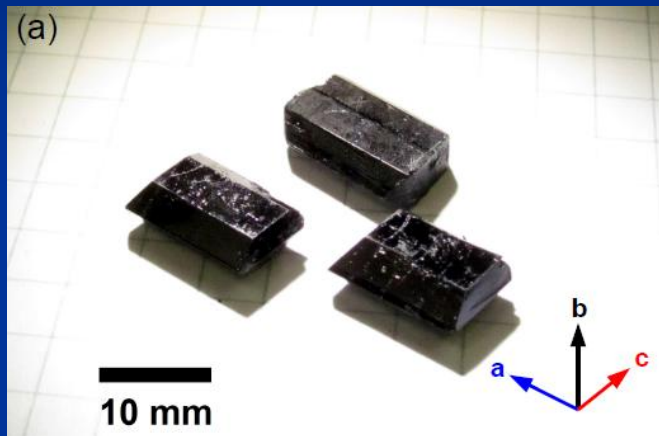
M. Klanjsek et al., PRL 101 137207 (2008)

$$T_1^{-1} = \frac{\hbar \gamma^2 A_{\perp}^2 A_0^x}{k_B u} \cos\left(\frac{\pi}{4K}\right) B\left(\frac{1}{4K}, 1 - \frac{1}{2K}\right) \left(\frac{2\pi T}{u}\right)^{(1/2K)-1},$$



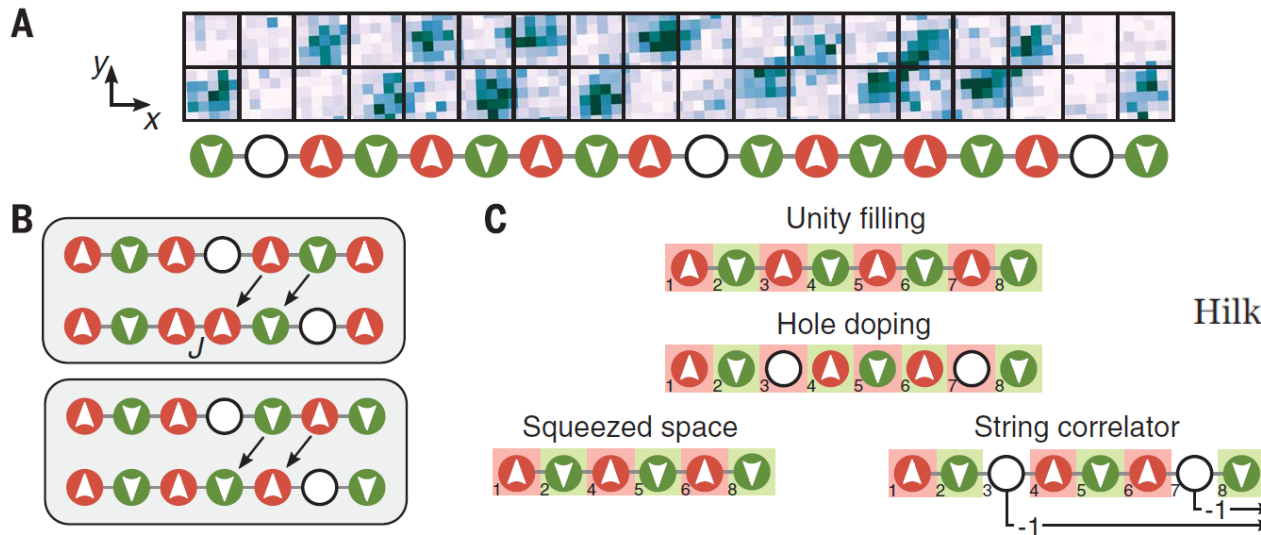
Hamiltonian reconstruction

D. Schmidiger et al. PRL 108 167201 (2012)



$$\mathcal{H} = J_{\text{leg}} \sum_{l,j} S_{l,j} \cdot S_{l+1,j} + J_{\text{rung}} \sum_l S_{l,1} \cdot S_{l,2} - g\mu_B H \sum_{l,j} S_{l,j}^z.$$

Spin-charge separation



Hilker *et al.*, *Science* **357**, 484–487 (2017)



QUANTUM GASES

Spin-charge separation in a one-dimensional Fermi gas with tunable interactions

Ruwan Senaratne^{1†}, Danyel Cavazos-Cavazos^{1†}, Sheng Wang^{2,3}, Feng He^{3,4}, Ya-Ting Chang¹, Aashish Kafle¹, Han Pu¹, Xi-Wen Guan^{2,5*}, Randall G. Hulet^{1*}

SCIENCE • 16 Jun 2022 • Vol 376, Issue 6599 • pp. 1305-1308

A. Kafle, R. Senaratne, D. Cavazos-Cavazos, H.-Y. Cui, T.G., H. Pu, X.-W. Guan, R. G. Hulet, arxiv/arXiv:2512.08866

Quantum simulations from CM



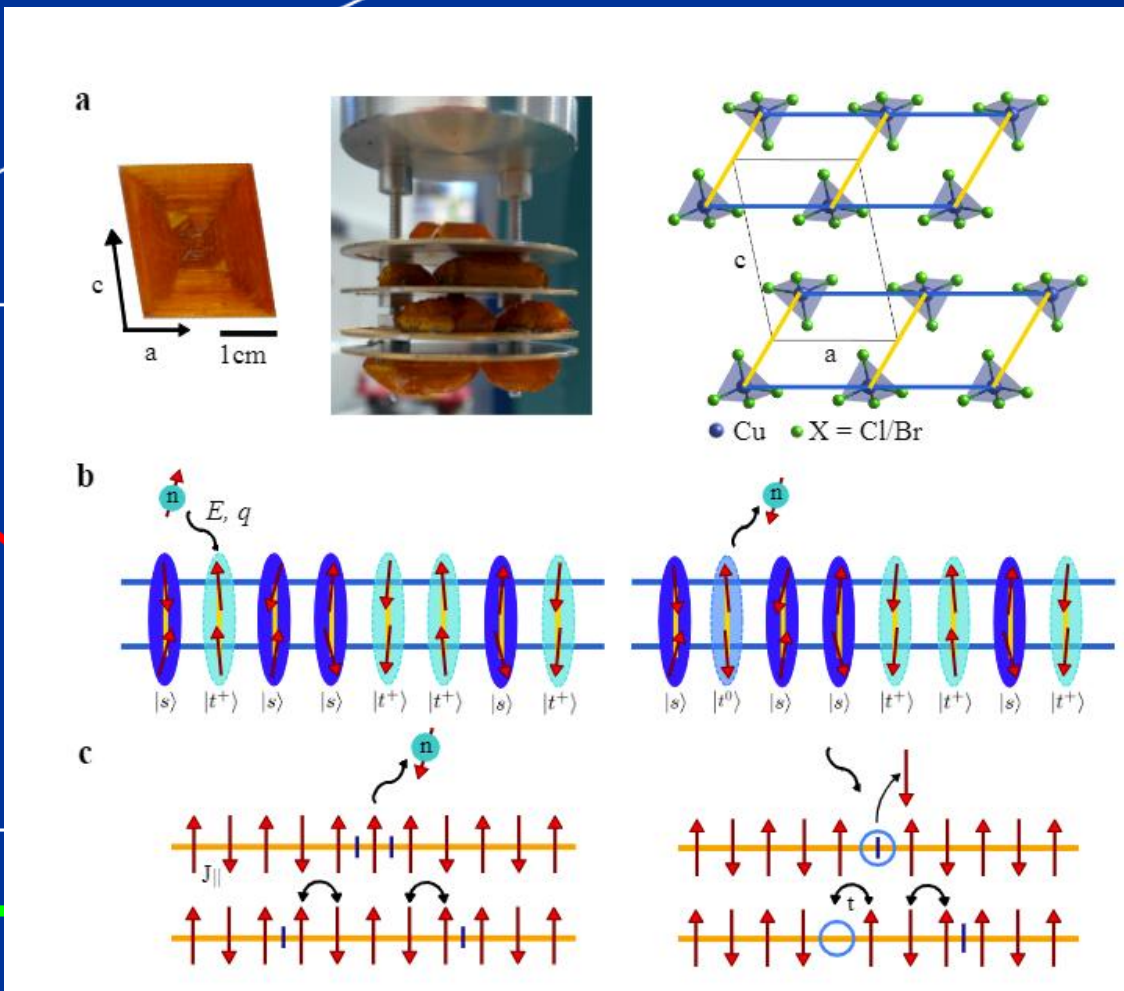
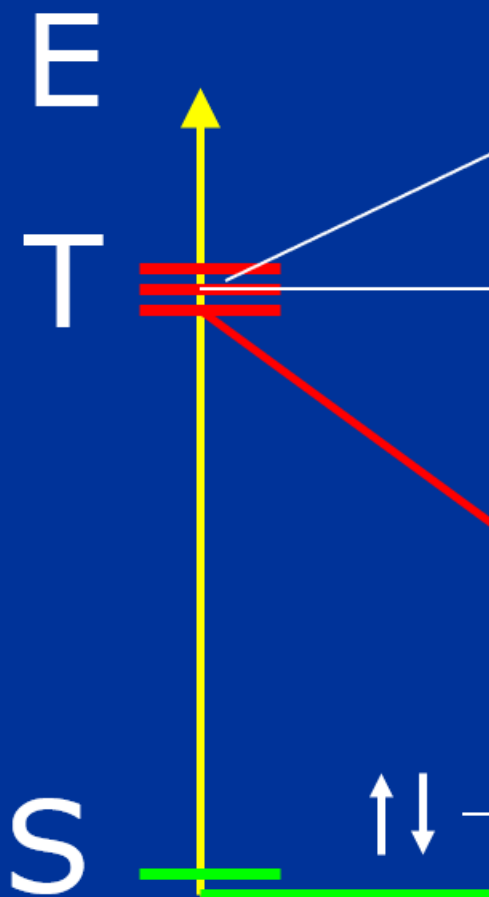


Supersymmetric spin/charge dyn.



P. Bouillot et al. PRB 83, 054407 (2011)

B. Wehinger, F. Lisandrini et al Nat Comm 16 3228 (2025)



Quantitative agreement with the predictions of the Tomonaga- Luttinger liquid

End of the story ???????

NO! TLL plays the same role
than Fermi liquid did

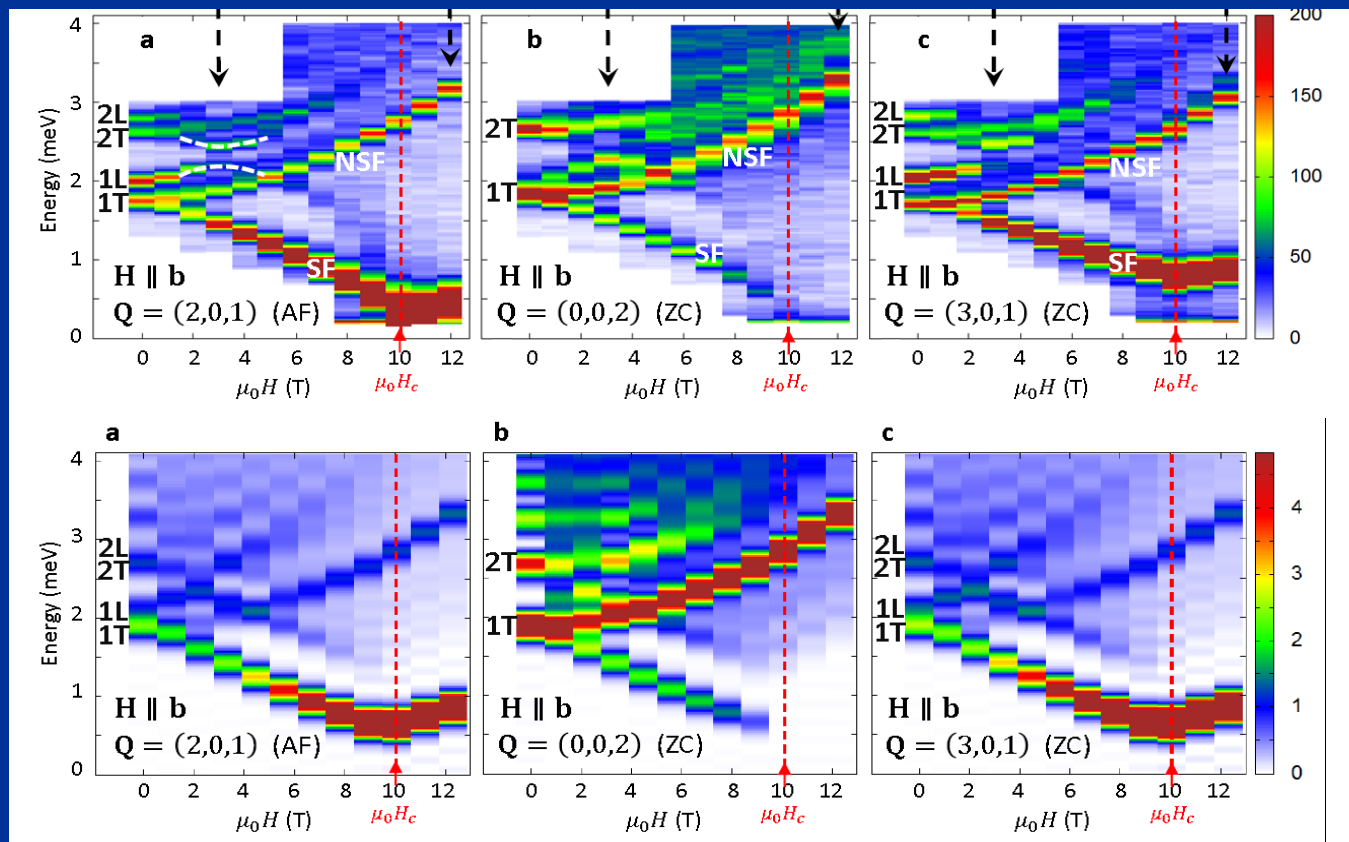
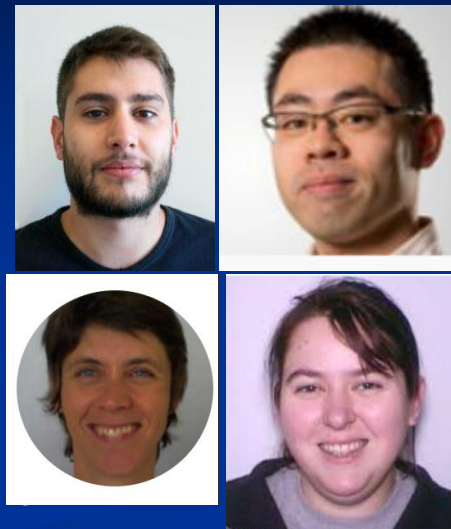
Beyond Tomonaga-Luttinger liquids

- 1D additional perturbation:
Lattice (Mott transition), disorder (Bose glass) etc.
Multicomponents, mixtures,
- New type of quantum critical points (e.g. topological)...

Double topological transition

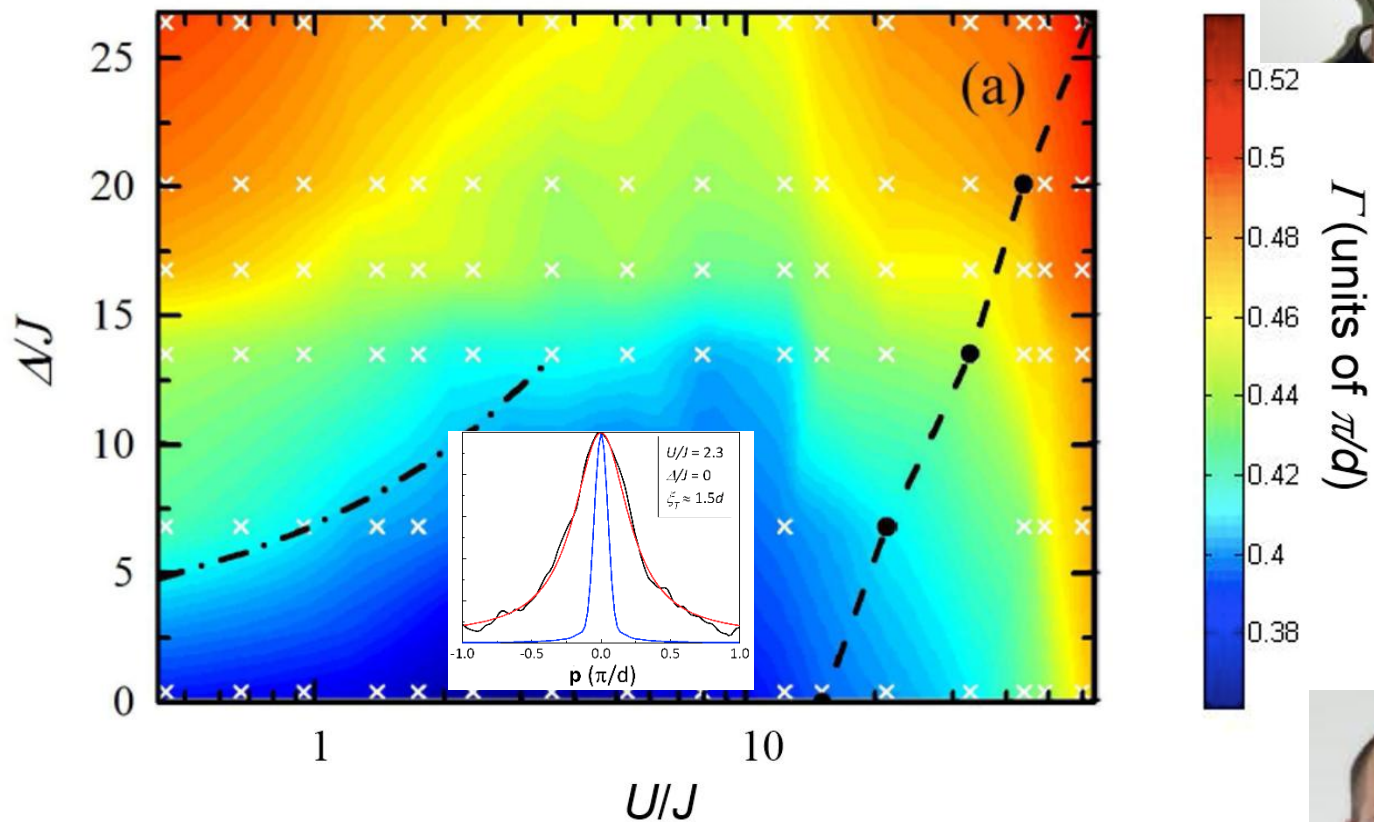
Topological quantum phase transition in the Ising-like antiferromagnetic spin chain $\text{BaCo}_2\text{V}_2\text{O}_8$

Q. Faure, S. Takayoshi, et al. Nat. Phys 14, 716 (2018)



Quasi-periodics and interactions

C. D'Errico, E. Lucioni et al. PRL (2014);
L. Gori et al PRA 93 033650 (2016)

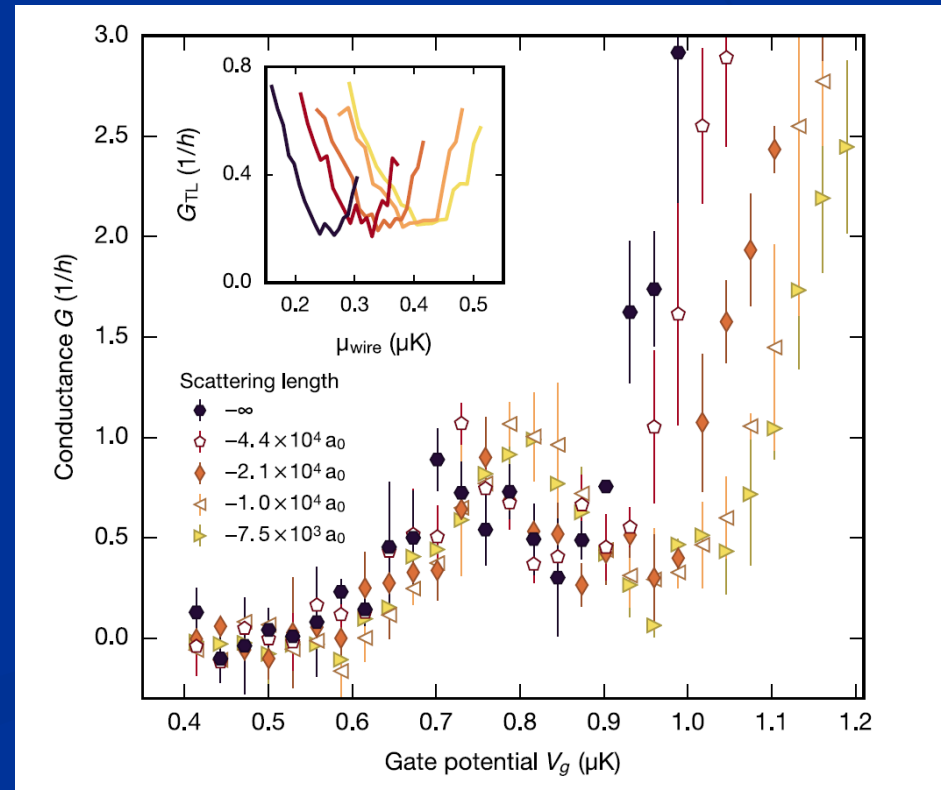
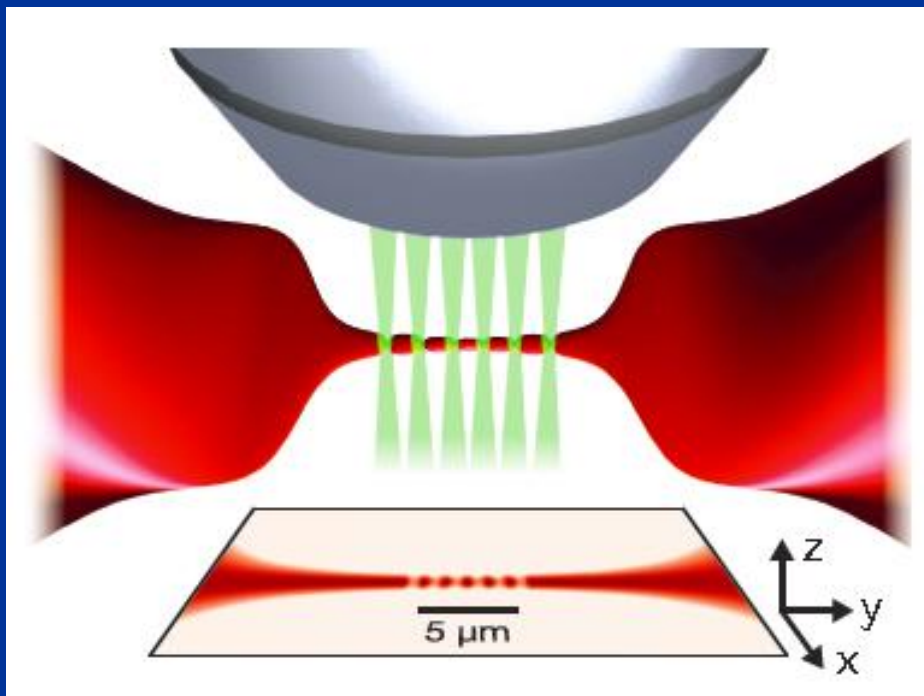


Beyond Tomonaga-Luttinger liquids

- 1D additional perturbation:
Lattice (Mott transition), disorder (Bose glass) etc.
Multicomponents, mixtures,
- New type of quantum critical points (e.g. topological)...
- Transport; Out of equilibrium situations.

Quantum transport in 1D

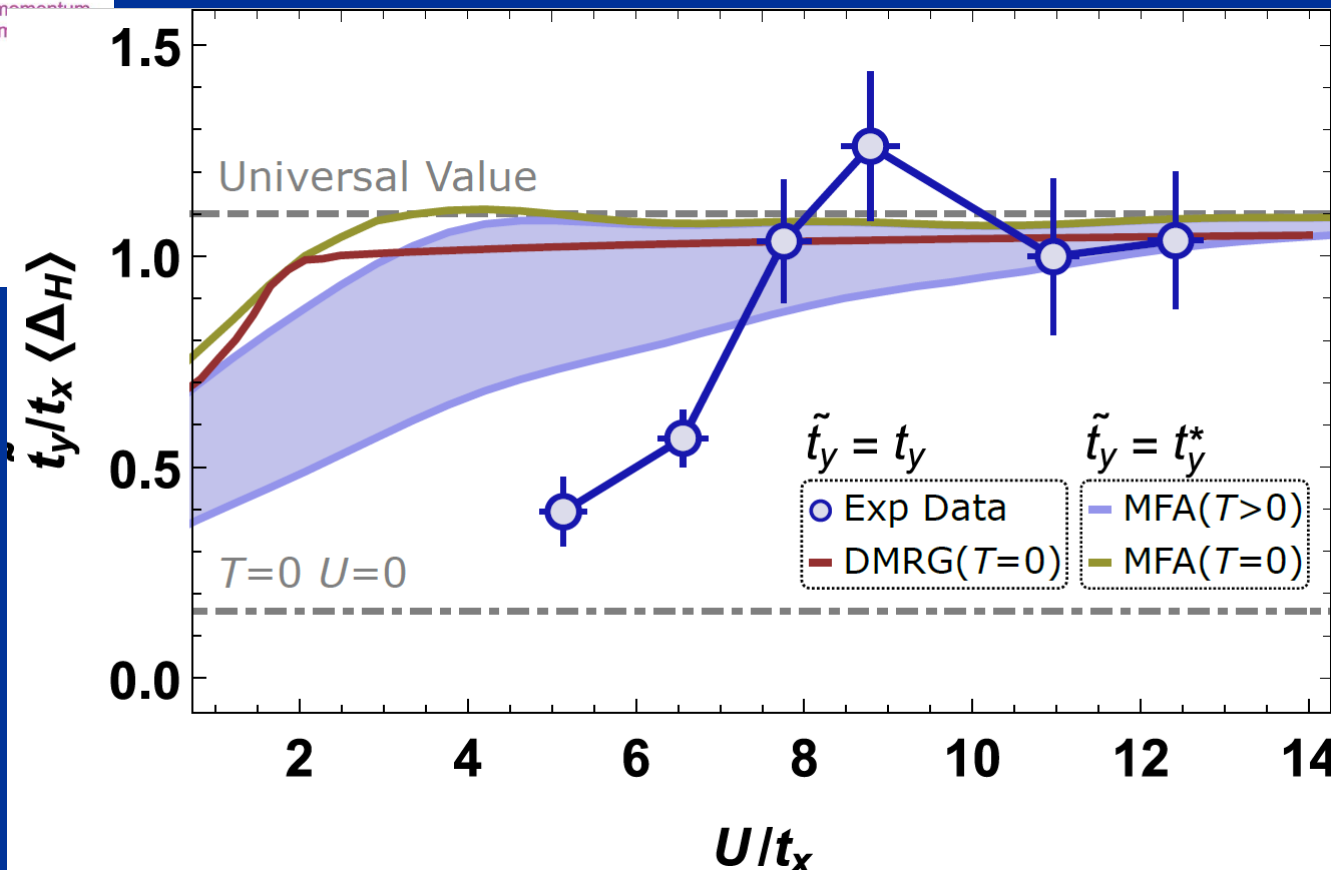
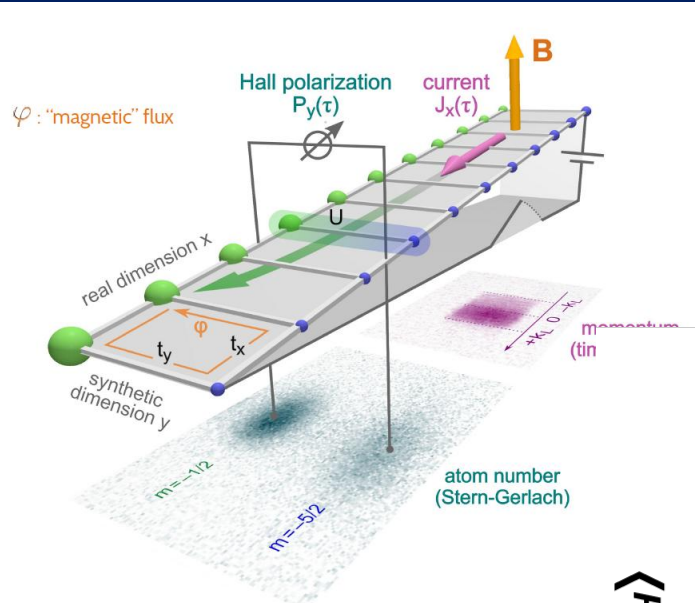
M. Lebrat, P. Grisins et al., PRX 8 011053 (2018)





Hall effect in ladders

T.-W. Zhou, G. Cappellini, D. Tusi, L. Franchi, J. Parravicini, C. Repellin, S. Greschner, M. Inguscio, TG, M. Filippone, J. Catani, L. Fallani, *Science* (2023).



Beyond Tomonaga-Luttinger liquids

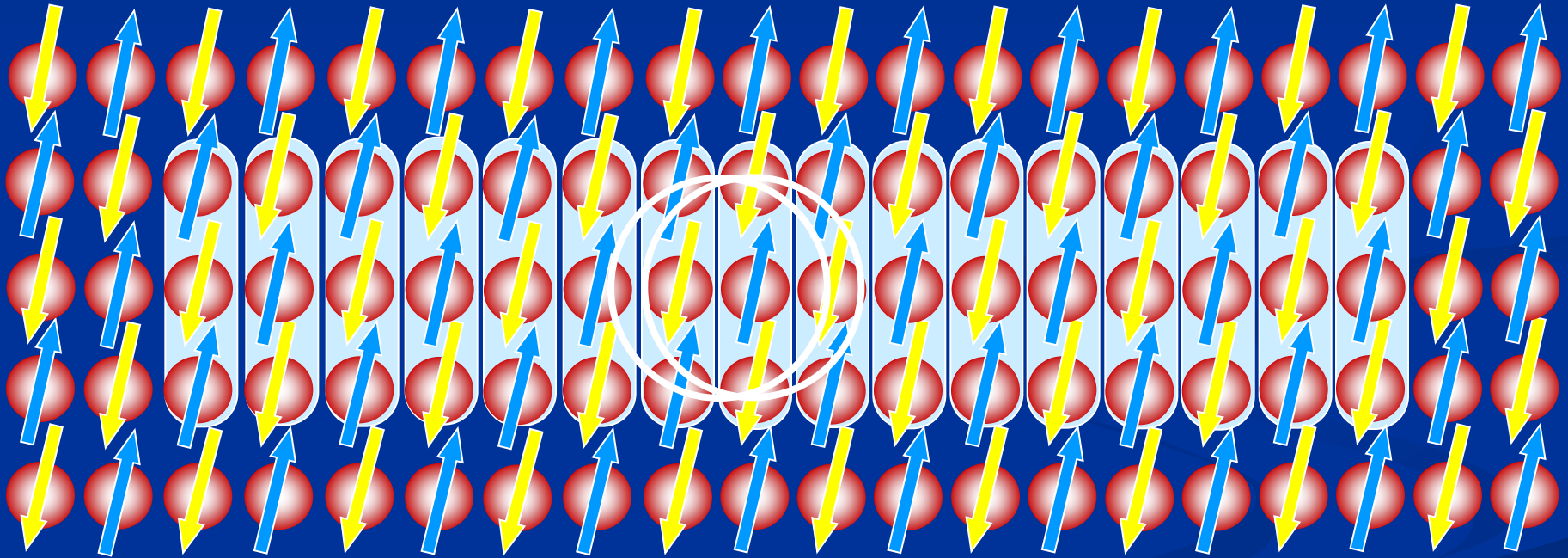
- 1D additional perturbation:
Lattice (Mott transition), disorder (Bose glass) etc.
Multicomponents, mixtures,
- New type of quantum critical points (e.g. topological)...
- Transport; Out of equilibrium situations.
- Dimensional crossover 1d – 2d/3d

Spin-Charge Separation

higher D ?

Spin

Charge

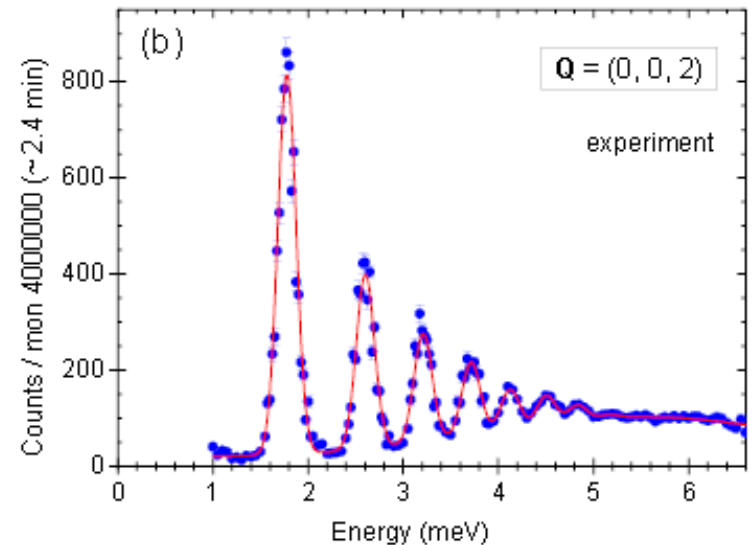
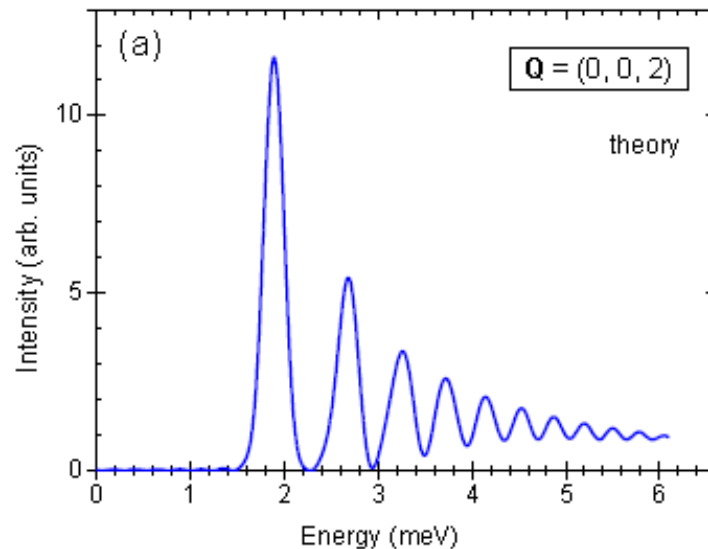
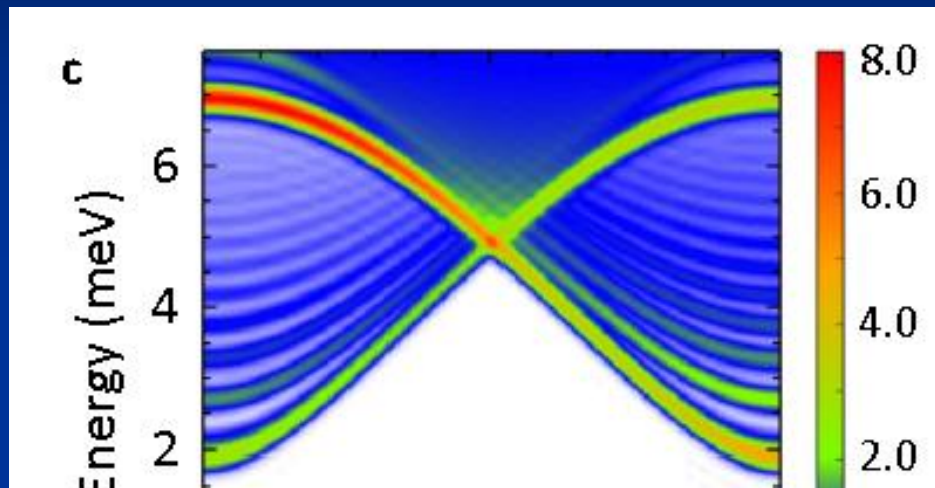
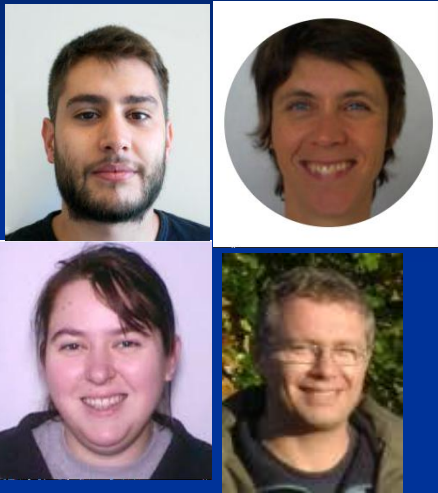


Energy increases with spin-charge separation

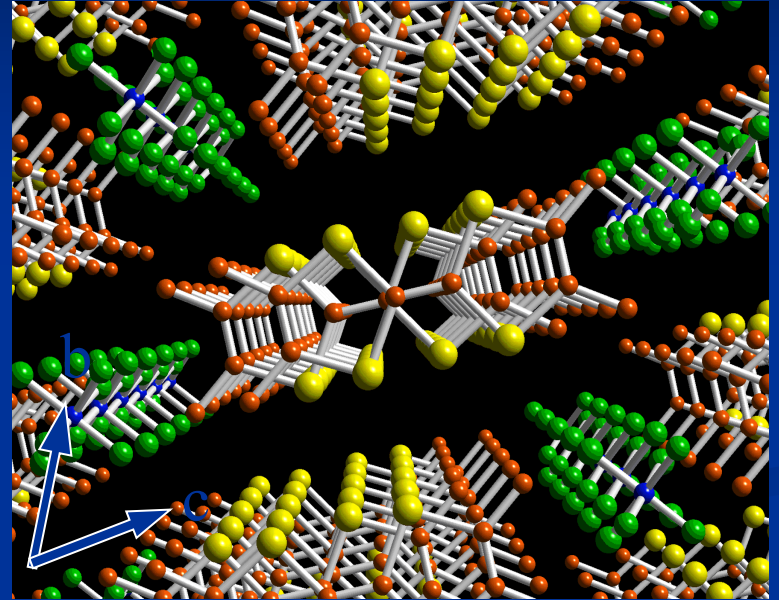
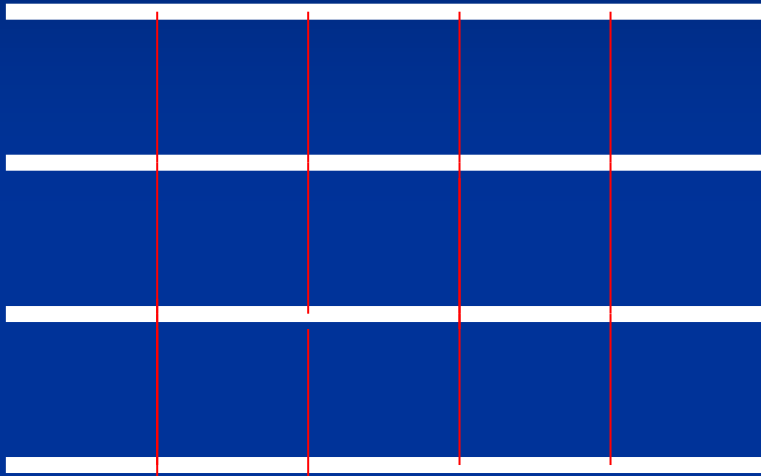
Confinement of spin-charge: « quasiparticle »

Confinement of spinons BaCoVO

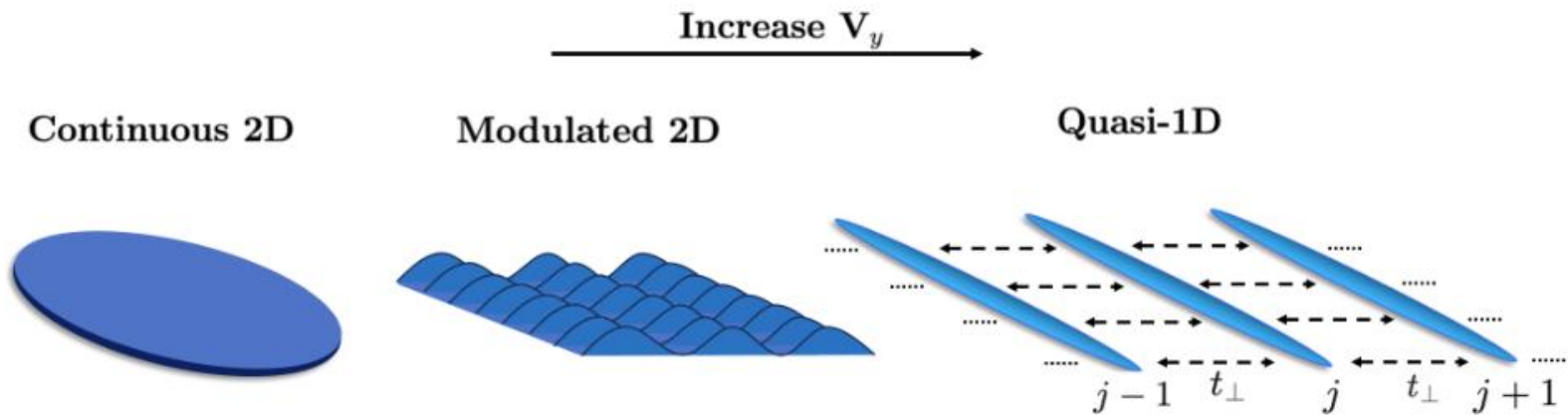
Q. Faure, S. Takayoshi, et al. Nat. Phys 14, 716 (2018)



Intermediate dimensionality



- Bosons:



2D,3D

1D

E,T

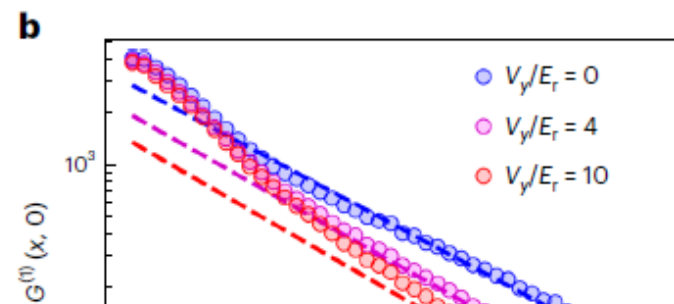
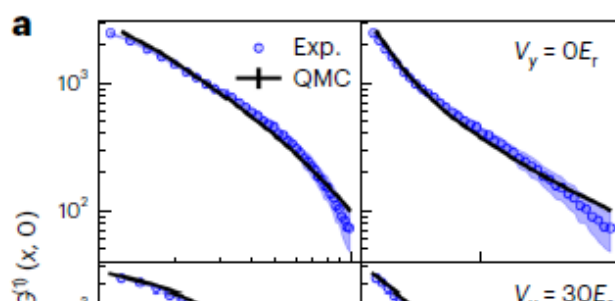
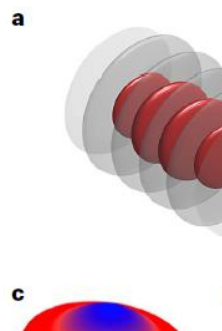
Strongly-interacting bosons at 2D-1D dimensional crossover

Hepeng Yao, Lorenzo Pizzino, Thierry Giamarchi

SciPost Phys. 15, 050 (2023) · published 7 August 2023

Observation of the 2D–1D crossover in strongly interacting ultracold bosons

Yanliang Guo^{1,3}, Hepeng Yao^{2,3}, Satwik Ramanjanappa¹, Sudipta Dhar¹, Milena Horvath¹, Lorenzo Pizzino², Thierry Giamarchi², Manuele Landini¹ & Hanns-Christoph Nägerl¹✉



SCIENCE ADVANCES | RESEARCH ARTICLE

PHYSICS

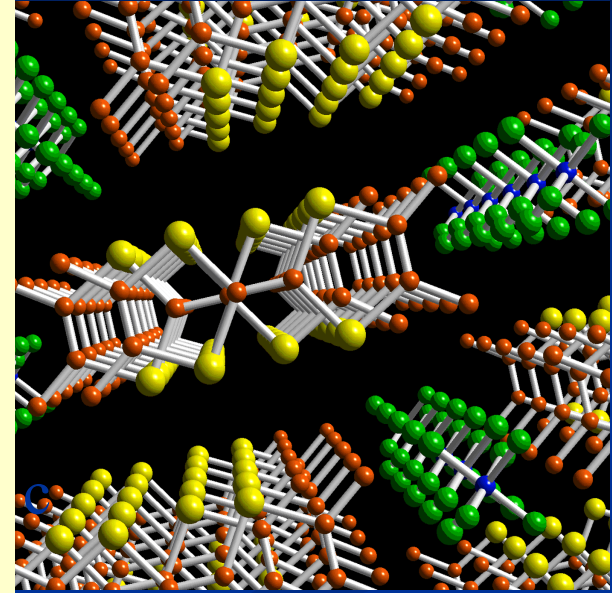
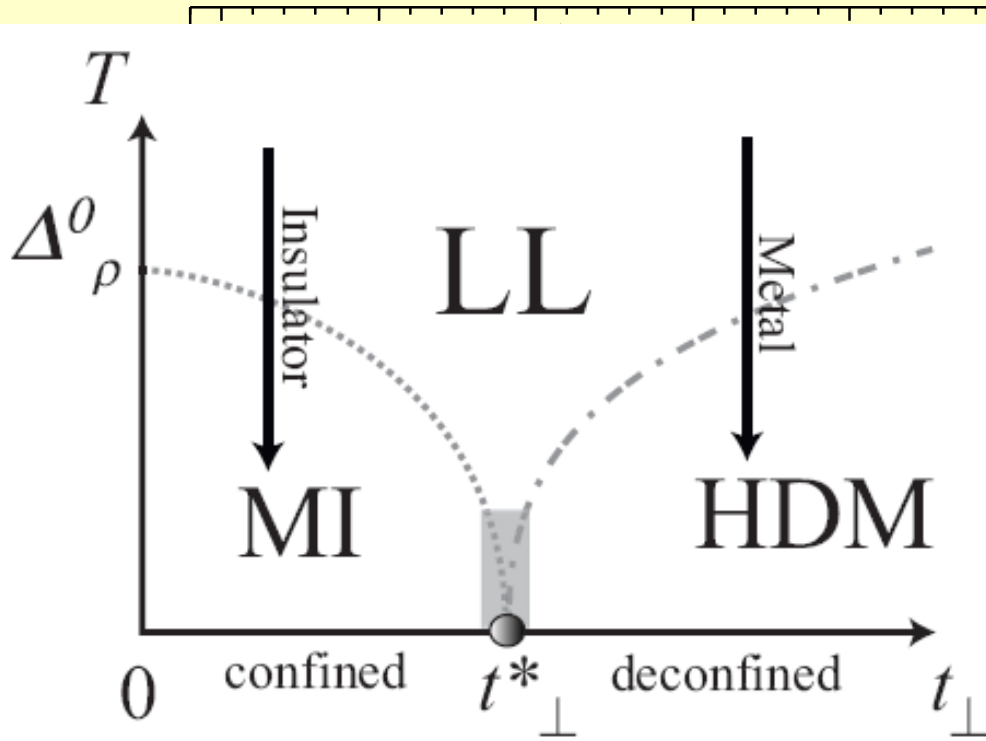
Anomalous cooling of bosons by dimensional reduction

Yanliang Guo^{1†}, Hepeng Yao^{2†}, Sudipta Dhar¹, Lorenzo Pizzino², Milena Horvath¹, Thierry Giamarchi², Manuele Landini¹, Hanns-Christoph Nägerl^{1*}

Fermions



Deconfinement



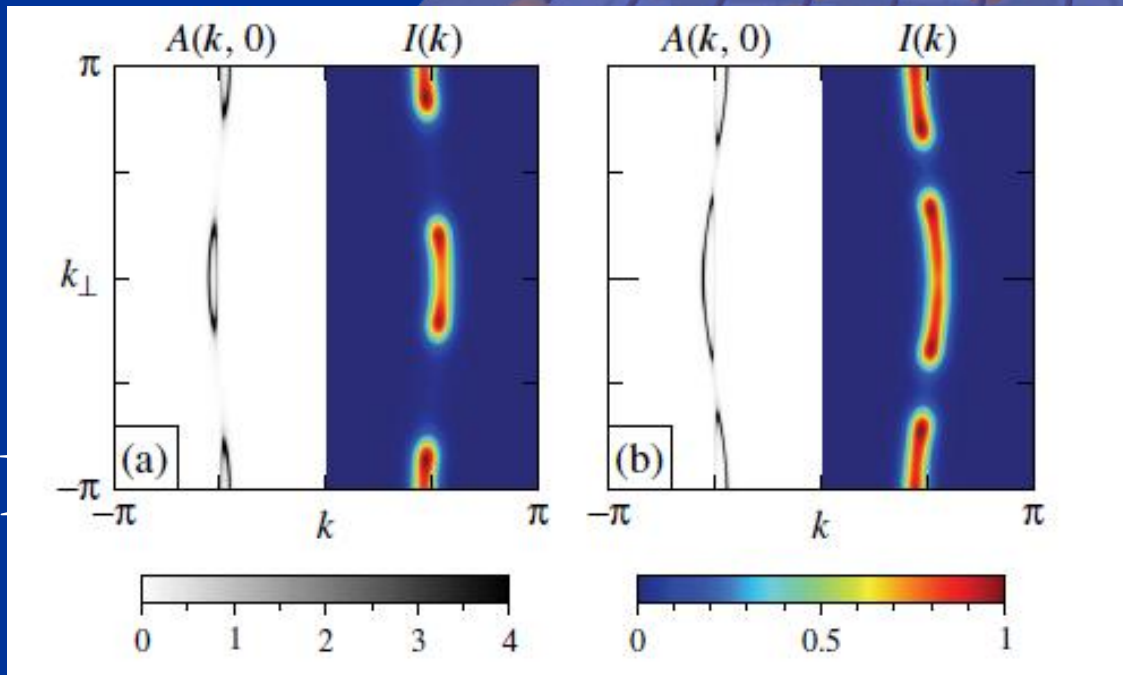
TG Chemical
Review 104 5037
(2004)

P. Auban-Senzier, D. Jérôme, C. Carcel and J.M. Fabre J de Physique IV, (2004)
A. Pashkin, M. Dressel, M. Hanfland, G. A. Kuntser, PRB 81 125109 (2010)
D. Jaccard et al., J. Phys. C, 13 L89 (2001)

A. Pashkin, M. Dressel, M. Haunfland, G. A. Kuntser, PRB 81 125109 (2010)

Self-consistent bath (DMFT)

S. Biermann, A. Georges, A. Lichtenstein, TG, PRL 87 276405 (2001)
C. Berthod et al. PRL 97, 136401 (2006)



determined



DMRG + Mean Field



Solving 2D and 3D lattice models of correlated fermions - combining matrix product states with mean field theory

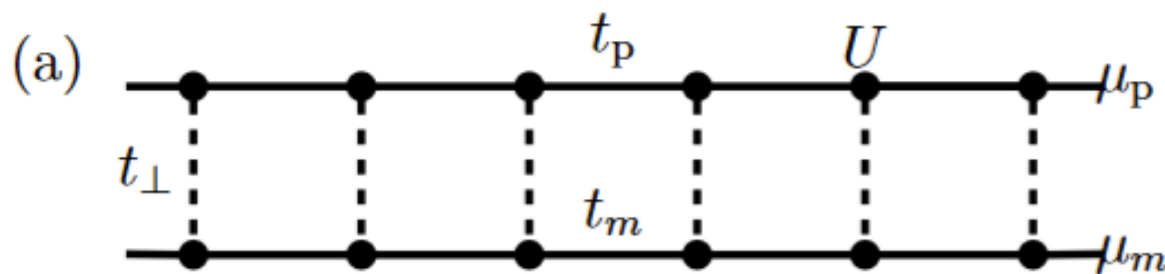
Gunnar Bollmark,¹ Thomas Köhler,¹ Lorenzo Pizzino,² Yiqi Yang,³ Johannes S. Hofmann,⁴ Hao Shi,⁵ Shiwei Zhang,⁶ Thierry Giamarchi,² and Adrian Kantian^{1,7}

Phys. Rev. X **13**, 011039 – Published 15 March 2023

1. [arXiv:2507.18707](https://arxiv.org/abs/2507.18707) [pdf, ps, other] [cond-mat.supr-con](#) [cond-mat.str-el](#)

Strong enhancements to superconducting properties of 1D systems from metallic reservoirs

Authors: J. E. Ebot, Sam Mardazad, Lorenzo Pizzino, Johannes S. Hofmann, Thierry Giamarchi, Adrian Kantian



Related directions

- Compare superfluidity with Leggett's inequalities [H. Yao, L. Pizzino]
- Other coupling of the 1D structures (e.g. triangular lattice) [X. Zhou]
- Disorder in such Q-1D geometries [G. Morpurgo, TG, PRR (2024)]

Conclusions

- 1D physics has remarkable features
- Luttinger liquid theory provides a framework to study this physics, and to go beyond
- Beautiful and challenging questions going beyond the Tomonaga-Luttinger liquids
- Requires interplay of analytical and numerical techniques (and new ideas!) to make progress
- Many experimental realizations both in condensed matter and in cold atoms