Conduction electrons in conventional and in strange metals: Fermi liquids and non-Fermi liquids

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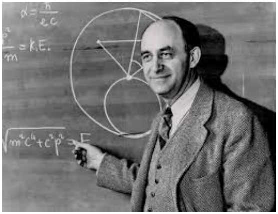
Outline

- Metals
- Fermi gas
- Fermi liquid
- Non-Fermi liquids:
  Luttinger liquids
  Quantum critical metals
  Pseudogap metals



Enrico Fermi

Conventional metals

Iron, copper, silver, gold, ...




- Excellent conductors of heat and electricity
- Electrical resistivity decreasing with temperature,
  $R(T) = R_0 + AT^2$ at low T, $R_0 > 0$ due to impurities
- Specific heat $C_V \propto T$ at low T
- Finite compressibility and finite spin susceptibility (Pauli) at low T

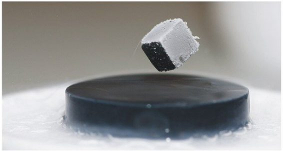
Described by "Fermi sea" of non-interacting electrons in most textbooks
(Kittel, Ashcroft & Mermin, ...)

What is the role of interactions between electrons ???

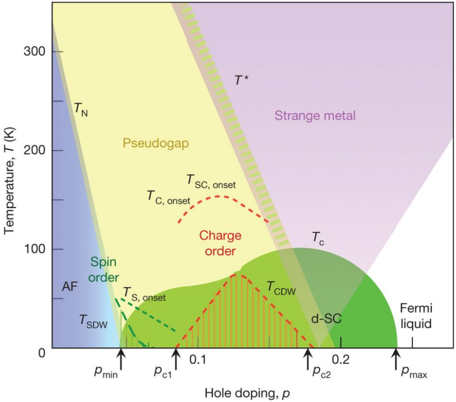
Strange metals

Exceptions from the rule:
Metals with strongly correlated electrons can be different,
e.g., $R(T) = R_0 + AT^1$ at low T

Prominent example:
Cuprate high-T_c superconductors



Keimer et al., Nature 2015



Strange metal for temperatures above T_c: bad conductor, strange temperature dependences

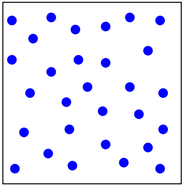
Fermi gas

Non-interacting many-fermion system, spin $\frac{1}{2}$

Single-particle basis: $\phi_{\mathbf{k}\sigma}(\mathbf{r}) = \chi_\sigma \phi_{\mathbf{k}}(\mathbf{r})$,

$\mathbf{k}$ (crystal) momentum, $\chi_\uparrow = \binom{1}{0}$, $\chi_\downarrow = \binom{0}{1}$

Single-particle energy $\epsilon_{\mathbf{k}}$, e.g. $\frac{\mathbf{k}^2}{2m}$ $\quad (\hbar = 1)$



Fermi statistics: many-particle wave function antisymmetric

$\Rightarrow$ Pauli exclusion principle: no doubly occupied single-particle states

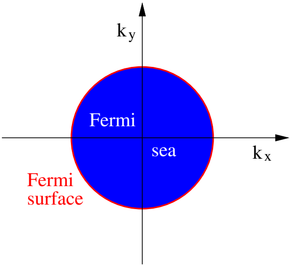
Many-particle ground state: Fermi sea

$$|\Psi_0\rangle = \prod_{\mathbf{k}, \epsilon_{\mathbf{k}} < \epsilon_F} \prod_{\sigma = \uparrow, \downarrow} a^\dagger_{\mathbf{k}\sigma} |\text{vacuum}\rangle$$

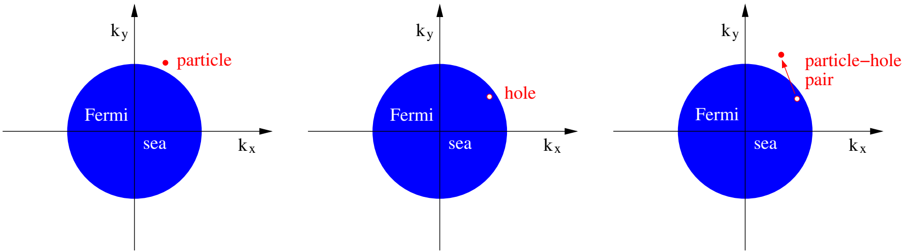
Single-particle states filled up to

Fermi energy ϵ_F

Fermi surface defined by $\epsilon_{\mathbf{k}} = \epsilon_F$



Elementary excitations:



Excitation energies:

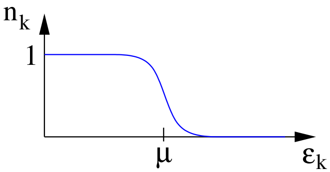
particle: $\delta E = \epsilon_{\mathbf{k}_p} - \epsilon_F$, hole: $\delta E = -(\epsilon_{\mathbf{k}_h} - \epsilon_F)$,

particle-hole pair: $\delta E = \epsilon_{\mathbf{k}_p} - \epsilon_{\mathbf{k}_h}$

Finite temperature:

Average occupation of single-particle states given by Fermi distribution

$$n_{\mathbf{k}\sigma} = f(\epsilon_{\mathbf{k}} - \mu) = \frac{1}{e^{(\epsilon_{\mathbf{k}}-\mu)/T} + 1}$$



μ chemical potential, Boltzmann constant $k_B = 1$

⇒ Thermodynamics, in particular specific heat

$C_V = \left(\frac{\partial E}{\partial T}\right)_{V,N} = \frac{\pi^2}{3} D(\epsilon_F)\, T$ for "low" temperatures, *any* space dimension d

$D(\epsilon_F)$ density of states at Fermi energy,

$\propto$ mass m for free electrons with $\epsilon_{\mathbf{k}} = \frac{\mathbf{k}^2}{2m}$

Chemical potential $\mu = \epsilon_F + \mathcal{O}(T^2)$ at low temperatures

Fermi liquid

Fermi liquid theory:

Low-energy ($\ll \epsilon_F$) physics of interacting Fermi systems in normal (not symmetry-broken) state.

Developed by Lev Landau in 1956-1958.

Not restricted to weak interactions!

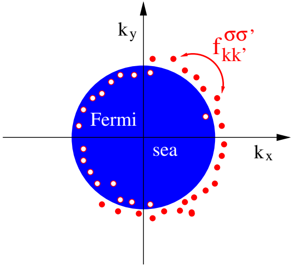
Valid for liquid ^{3}He and most metals.

Phenomenologic theory – microscopic derivation.

Phenomenologic formulation:

Low-energy excitations are superposition of fermionic "quasi-particles" (including holes) with same quantum numbers $(\mathbf{k}, \sigma)$ and statistics as for Fermi gas.

Excited states characterized by
quasi-particle distribution $\delta n_{\mathbf{k}\sigma}$,
$\delta n_{\mathbf{k}\sigma} \geq 0$ for quasi-particles,
$\delta n_{\mathbf{k}\sigma} \leq 0$ for quasi-holes.



Total excitation energy:

$$\delta E[\delta n_{\mathbf{k}\sigma}] = \sum_{\mathbf{k}\sigma} (\epsilon^*_{\mathbf{k}} - \mu) \delta n_{\mathbf{k}\sigma} + \frac{1}{2V} \sum_{\mathbf{k}\sigma} \sum_{\mathbf{k}'\sigma'} f^{\sigma\sigma'}_{\mathbf{k}\mathbf{k}'} \delta n_{\mathbf{k}\sigma} \delta n_{\mathbf{k}'\sigma'}$$

$\epsilon^*_{\mathbf{k}}$ energy of a single quasi-particle (differs from "bare" energy $\epsilon_{\mathbf{k}}$)

$f^{\sigma\sigma'}_{\mathbf{k}\mathbf{k}'}$ quasi-particle interaction

Parametrization of $\epsilon^*_{\mathbf{k}}$ and f-function:

Only low-energy excitations, with $\mathbf{k}$ close to Fermi surface, contribute.

Quasi-particle energy can be linearized for momenta near Fermi surface.

E.g., for isotropic system with Fermi momentum k_F,

$\epsilon^*_{\mathbf{k}} \approx \epsilon_F + v^*_F(|\mathbf{k}| - k_F)$ with Fermi velocity $v^*_F = k_F / m^*$

Quasi-particle interaction, symmetric and antisymmetric components:

$$f^c_{\mathbf{kk}'} = \frac{1}{2}(f^{\sigma\sigma}_{\mathbf{kk}'} + f^{\sigma,-\sigma}_{\mathbf{kk}'}), \qquad f^s_{\mathbf{kk}'} = \frac{1}{2}(f^{\sigma\sigma}_{\mathbf{kk}'} - f^{\sigma,-\sigma}_{\mathbf{kk}'})$$

For isotropic system, dependence only on angle θ between $\mathbf{k}$ and $\mathbf{k}'$:

$$f^{c/s}_{\mathbf{kk}'} \approx f^{c/s}_{\mathbf{k}_F\mathbf{k}'_F} = f^{c/s}(\theta) = \sum_{\ell=0}^{\infty} f^{c/s}_\ell P_\ell(cos\theta)$$

last equation for 3D systems

Landau parameters $F^{c/s}_\ell = D^*(\epsilon_F) f^{c/s}_\ell$ dimensionless numbers

Simple physical properties:

Specific heat

$$C_V = \frac{\pi^2}{3} D^*(\epsilon_F)\, T$$

linear in T as for Fermi gas

prefactor usually enhanced, since $D^*(\epsilon_F) > D(\epsilon_F)$

Compressibility

$$\kappa = -\frac{1}{V}\frac{\partial V}{\partial P} = \frac{1}{n^2}\frac{\partial n}{\partial \mu} = \frac{1}{n^2}\frac{D^*(\epsilon_F)}{1+F_0^c}$$

usually reduced by $F_0^c > 0$

Spin susceptibility

$$\chi = \mu_B^2 \frac{D^*(\epsilon_F)}{1+F_0^s}$$

usually enhanced by $F_0^s < 0$

ferromagnetic instability for $F_0^s \to -1$

Enhancement of DOS can be huge, factor 1000 in "heavy fermion systems"

$\Rightarrow$ *HEAVY METAL*

Microscopic theory (Quantum field theory)

Quasi-particles related to poles in Green-function (propagator)

$$G(\mathbf{k},\omega) = \frac{Z_{\mathbf{k}}}{\omega - (\epsilon^*_{\mathbf{k}} - \mu) + i\gamma_{\mathbf{k}}} + G_{\text{reg}}(\mathbf{k},\omega)$$

$Z_{\mathbf{k}} < 1$ quasi-particle weight
$\gamma_{\mathbf{k}}$ quasi-particle decay rate

Quasi-particle decay rate $\gamma_{\mathbf{k}}$ of order $(\epsilon^*_{\mathbf{k}} - \epsilon_F)^2/\epsilon_F$ and T^2/ϵ_F,
that is, $\gamma_{\mathbf{k}} \ll |\epsilon^*_{\mathbf{k}} - \epsilon_F|$ for $\mathbf{k}$ near Fermi surface
$\Rightarrow$ quasi-particles "asymptotically stable"

Fermi surface still sharply defined by discontinuity of momentum distribution $\langle n_{\mathbf{k}} \rangle$ at $T = 0$.

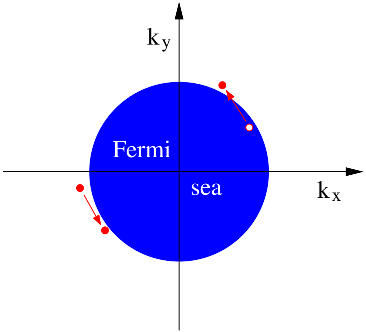
Contribution from electron-electron scattering to electrical resistivity $\rho_{\text{e-e}}(T) \propto T^2$ (requires umklapp scattering)

Quasi-particle interaction obtained from two-particle interaction vertex in forward scattering limit

Stability of quasi particles

Decay of quasiparticles limited by Fermi sea and Pauli principle

$\Rightarrow \gamma_{\mathbf{k}} \propto (\epsilon^*_{\mathbf{k}} - \epsilon_F)^2 / \epsilon_F$



Are Fermi liquids stable?

At low temperatures, instability toward ordered state may occur.

Kohn & Luttinger 1965:

There is always a Cooper instability leading to a superfluid state, even for purely repulsive (bare) interactions and without phonons!

Reason: Logarithmic divergence of two-particle vertex for vanishing total momentum, $\mathbf{k}_1 + \mathbf{k}_2 = 0$, and attractive *effective* interactions.

However, critical temperature T_c usually too tiny to be observed.

Fermi liquid theory valid in temperature window $T_c < T \ll \epsilon_F$.

Notable exception: High-T_c superconductivity

Rigorous Fermi liquid theory

Phil Anderson 1990: No Fermi liquid in two spatial dimensions!

Impressive mathematically rigorous work on interacting Fermi systems with Fermi liquid behavior developed since the 1990s:

Benfatto & Gallavotti, Mastropietro
Feldman, Knörrer, Magnen, Rivasseau, Salmhofer, Trubowitz

⇒ 2D Fermi liquids exist

Major tool: renormalization group

No surprises, Landau was right!
(Anderson was wrong)

⇒ Renormalization group also suitable tool for computations
wm, Salmhofer, Honerkamp, Meden, Schönhammer, Rev. Mod. Phys. 2012

Disordered Fermi liquids

Disorder in non-interacting systems leads to diffusive motion and localization effects.

Combined effects of disorder and interactions in Fermi systems captured by Fermi liquid theory with scale dependent parameters.

Castellani, Di Castro, et al. 1984 - 1986

Non-Fermi liquids

Fermi liquids + non-Fermi liquids = all liquids ?

Tertium datur : Water is neither Fermi liquid, nor non-Fermi liquid!

Boris Altshuler:
What is the difference between Fermi liquid and non-Fermi liquid?
The difference is the same as between bananas and non-bananas.

Common definition:
A normal (non-symmetry broken) metallic (not fully gapped) state without asymptotically stable fermionic quasi-particles.

$\Rightarrow$ *Some* properties deviate from Fermi liquid behavior.

- Luttinger liquids
- Quantum critical metals
- Pseudogap metals $\longrightarrow$ Talk on Monday @ Sapienza

Luttinger liquids

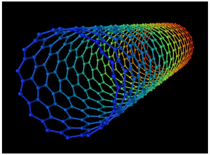
Interaction effects in one-dimensional (1D) systems stronger than in higher dimensions

Fermi liquid theory based on asymptotically stable fermionic quasi-particles not valid in 1D

Instead, normal metallic (no charge gap) one-dimensional interacting Fermi systems are generically Luttinger liquids

Tomonaga, Luttinger, Lieb & Mattis, Solyom, Haldane, ...

- Chemical compounds containing atom chains
- Quantum wires (in heterostructures)
- Carbon nanotubes
- Edge states of 2D topological insulators



Textbook: Thierry Giamarchi: Quantum physics in one dimension

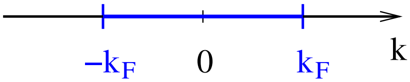
One-dimensional Fermi gas:

Dispersion relations:

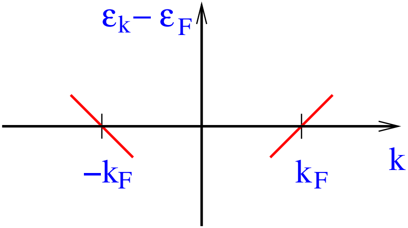
$\epsilon_k = k^2/2m$ (low carrier density)

$\epsilon_k = -2t \cos k$ (tight binding)

"Fermi surface": 2 points $\pm k_F$



Dispersion near Fermi points:



approx. linear:

$\epsilon_k - \epsilon_F \approx v_F \left(|k| - k_F \right)$

Interactions ⇒ Luttinger liquid

- No fermionic quasi-particles
- Bosonic elementary excitations:
  collective charge/spin density oscillations,
  with linear dispersion $\omega_q^c = u_c|q|$, $\omega_q^s = u_s|q|$
  ⇒ Specific heat $C_V \propto T$ as in a Fermi liquid!
- Compressibility $\kappa = \frac{1}{n^2}\frac{D^*(\epsilon_F)}{1+F_0^c}$ as in a Fermi liquid
- Spin susceptibility $\chi = \mu_B^2 \frac{D^*(\epsilon_F)}{1+F_0^s}$ as in a Fermi liquid

Finite compressibility and spin susceptibility guaranteed by conservation laws (wm & Di Castro 1993)

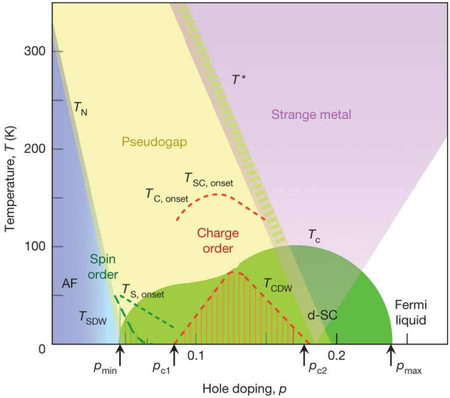
- Power-laws with non-universal exponents:

  Tunneling density of states $D_t(\epsilon) \propto |\epsilon - \epsilon_F|^\eta$ for $\epsilon \to \epsilon_F$, vanishes at Fermi energy

  Density correlation function $N(q) \propto |q - 2k_F|^{-\alpha}$ for $q \to 2k_F$
  $\Rightarrow$ enhanced back-scattering from impurities

Quantum critical metals

Strange metal in CuO_2 high temperature superconductors



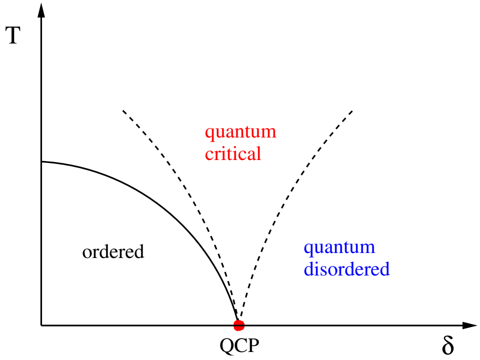
- antiferromagnetism in undoped compounds
- d-wave superconductivity at sufficient doping
- "pseudo gap"
- "strange metal" in normal phase above highest T_c

Keimer et al., *Nature* 2015

Strange metal non-Fermi liquid with resistivity $R(T) \propto T$
and other unusual properties $\Leftrightarrow$ quantum criticality ?

Quantum critical metals

Quantum phase transition: Phase transition at $T = 0$ driven by control parameter δ (pressure, density, ...)



If order parameter vanishes *continuously* at transition: quantum critical point.

Quantum fluctuations lead to unusual physical properties.

Quantum phase transitions in metals:
non-Fermi liquid behavior in quantum critical regime

Examples for QCPs in metals:

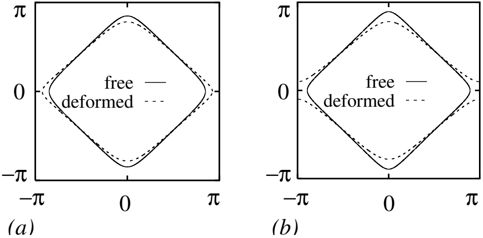
- Magnetic, especially antiferromagnetic order
  (Chubukov, Schmalian, Metlitski, Sachdev, S.S. Lee, ...)
- Charge order
  (Grilli, Castellani, Di Castro, Caprara, Lorenzana ...; Varma)
- Nematic order
  (Kivelson, Fradkin, wm, Chubukov, S.S. Lee, Metlitski, Sachdev, ...)

Most interest in two-dimensional systems: strong fluctuations

Nematic order from d-wave Pomeranchuk instability

Quasi-particle interaction $f^c_{\mathbf{kk}'}$ with strongly attractive d-wave component

$\Rightarrow$ d-wave Pomeranchuk instability



Spontaneous breaking of tetragonal symmetry

Halboth & wm 2000

Yamase & Kohno 2000

Order parameter $n_d = \sum_{\mathbf{k}} d_{\mathbf{k}} \langle n_{\mathbf{k}} \rangle$ where $d_{\mathbf{k}} = \cos k_x - \cos k_y$

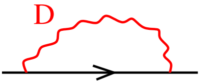
Realization of "nematic" electron liquid ($\rightarrow$ Kivelson et al. 1998)

Mechanism for non-Fermi liquid behavior

Electrons scattered by quantum critical fluctuations

At QCP divergent order parameter susceptibility

$\Rightarrow$ singular fluctuation propagator, e.g., $D(\mathbf{q}, \omega) = \dfrac{1}{\mathbf{q}^2 + i\omega}$

$\Rightarrow$ Large electron self-energy $\Sigma =$ 

$\Sigma(\mathbf{k}_F, \omega) \propto \omega^\alpha$ with exponents $\alpha < 1$ $\Rightarrow$ non-Fermi liquid

Complete theory difficult:
no natural small parameter, secondary instabilities

T-linear resistivity from quantum critical fluctuations?

Two major obstacles:

- Fast decay of quasiparticles does NOT imply fast decay of current: Need large momentum transfers and momentum-non-conserving processes (e.g. umklapp).
- "Cold" parts of the Fermi surface (not directly coupled to critical fluctuations) short circuit current and thus dominate conductivity.

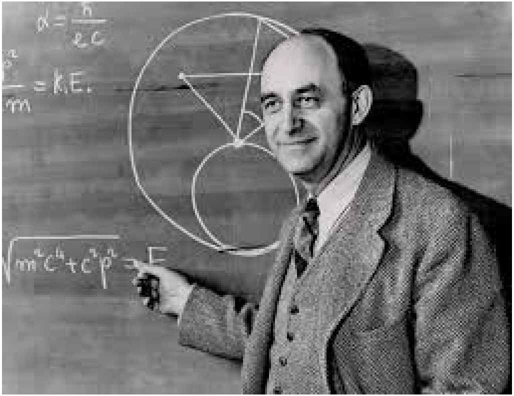
Example: Nematic quantum criticality: Dell'Anna & wm, PRL 2007

- Quasiparticle decay rate $\tau^{-1} \propto T^{1/2}$
- Transport decay rate $\tau_{\mathrm{tr}}^{-1} \propto d_{\mathbf{k}}^2 T^{4/3}$
- Resistivity $R(T) \propto T^{5/3}$ ("cold spots" on BZ diagonal)

Short circuit effect even stronger for magnetic or charge density wave QCPs, where most of the Fermi surface is "cold"!

Summary

- In spite of sizable interactions, the electronic properties of most metals are well described by Fermi liquid theory, with elementary excitations as in a Fermi gas.
- Non-Fermi liquid behavior in strange metals requires special circumstances, such as confinement to one dimension (Luttinger liquid) or vicinity of a quantum critical point.



Thank you for your attention!