Quantum entanglement of mobile fermions

Fermi Legacy in Low Energy Physics

Celebrating the 100th anniversary of Enrico Fermi's pioneering paper

"Sulla quantizzione del gas perfecto monoatomic"

Academia Nazionale dei Lincei, Rome

February 5, 2026

Subir Sachdev

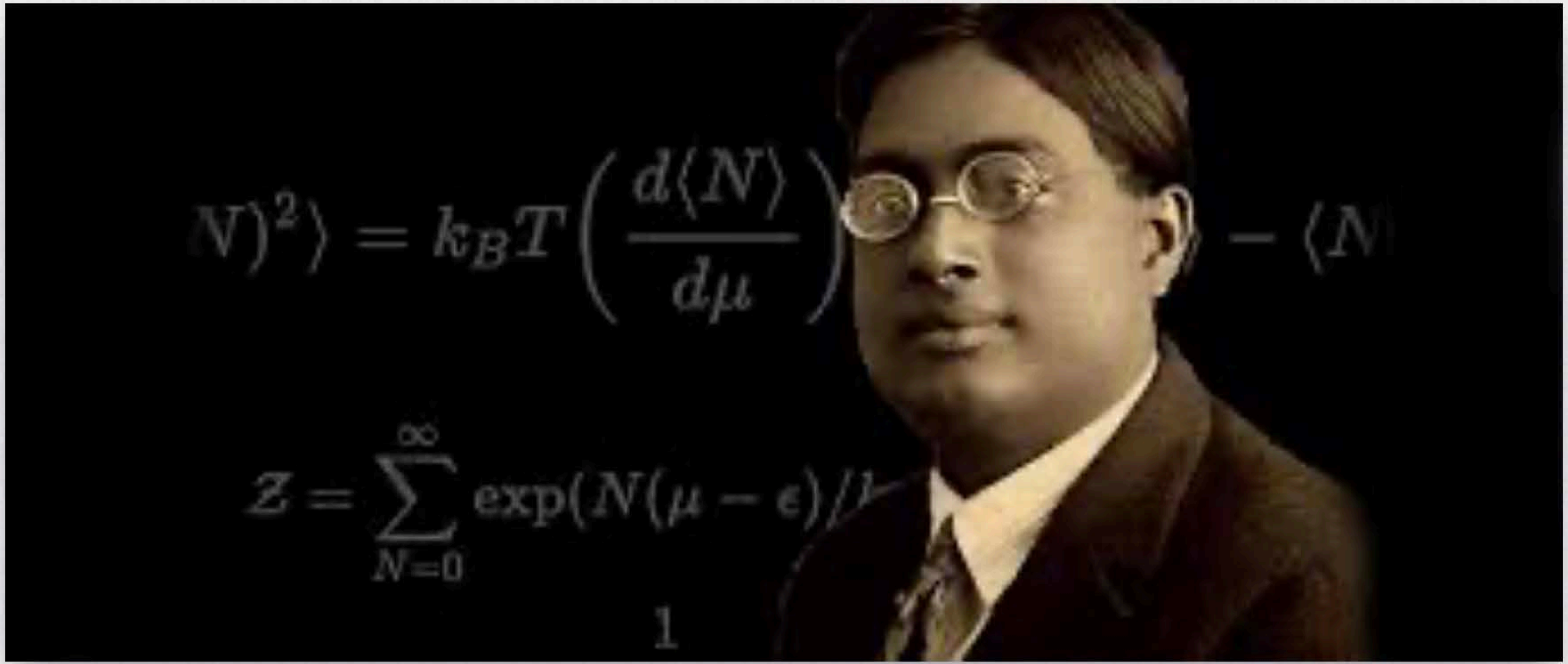
PHYSICS

HARVARD

BoseStat@100: Centenary of Bose Statistics

Links

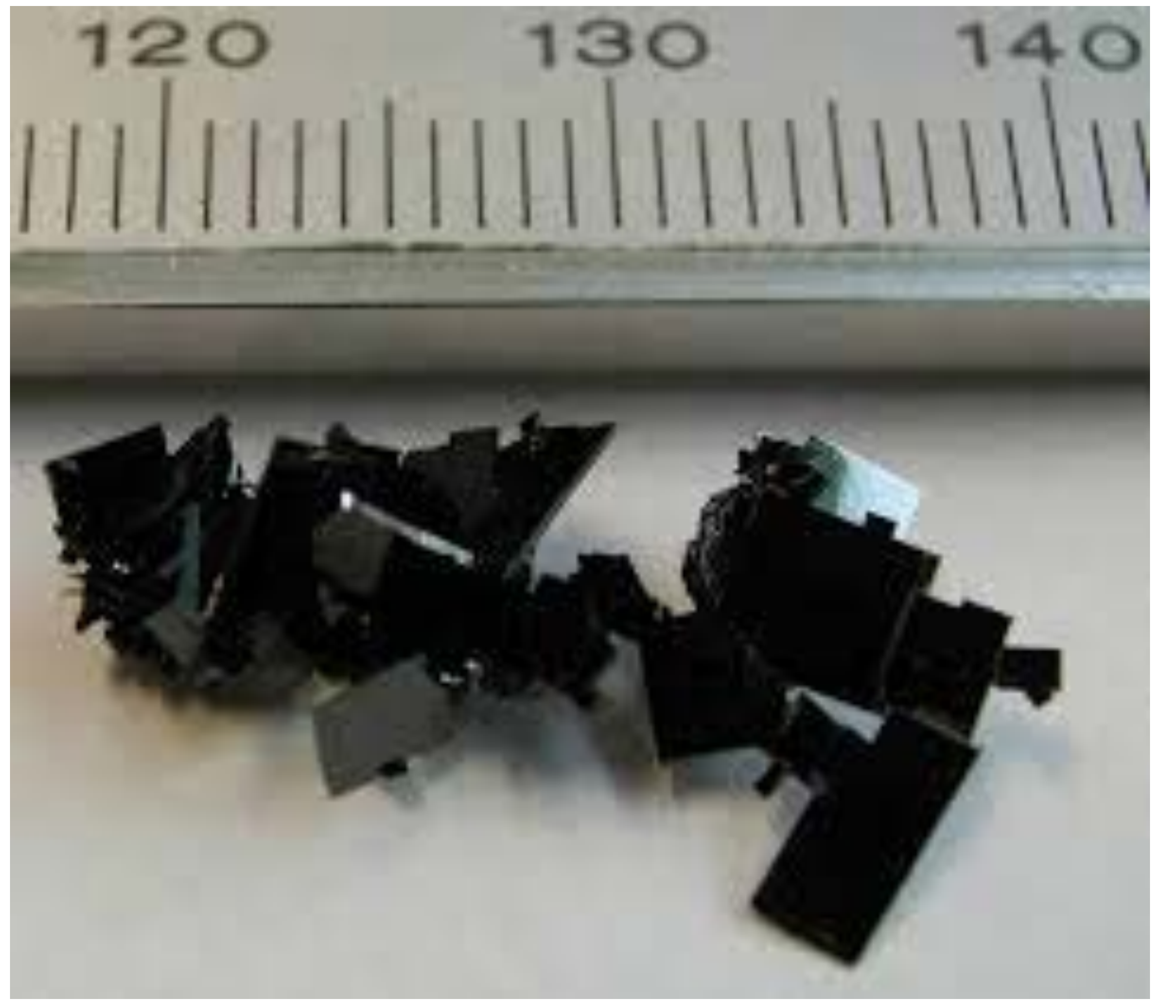
- Home
- Committees
- International Conferences
  - Photonics, Quantum Information, and Quantum Communication
    - **Jan 29 - Feb 02, 2024**
    - Photo gallery
    - Media coverage
    - Youtube video
  - Women in Quantum Science and technologies
    - **July 17 -July 19, 2024**
    - Photo gallery
    - Media coverage
    - Youtube video
  - Bose-Einstein Condensation, Superconductivity, Superfluidity, and Quantum Magnetism
    - **Nov 12 - Nov 16, 2024**
    - Photo gallery
    - Media coverage

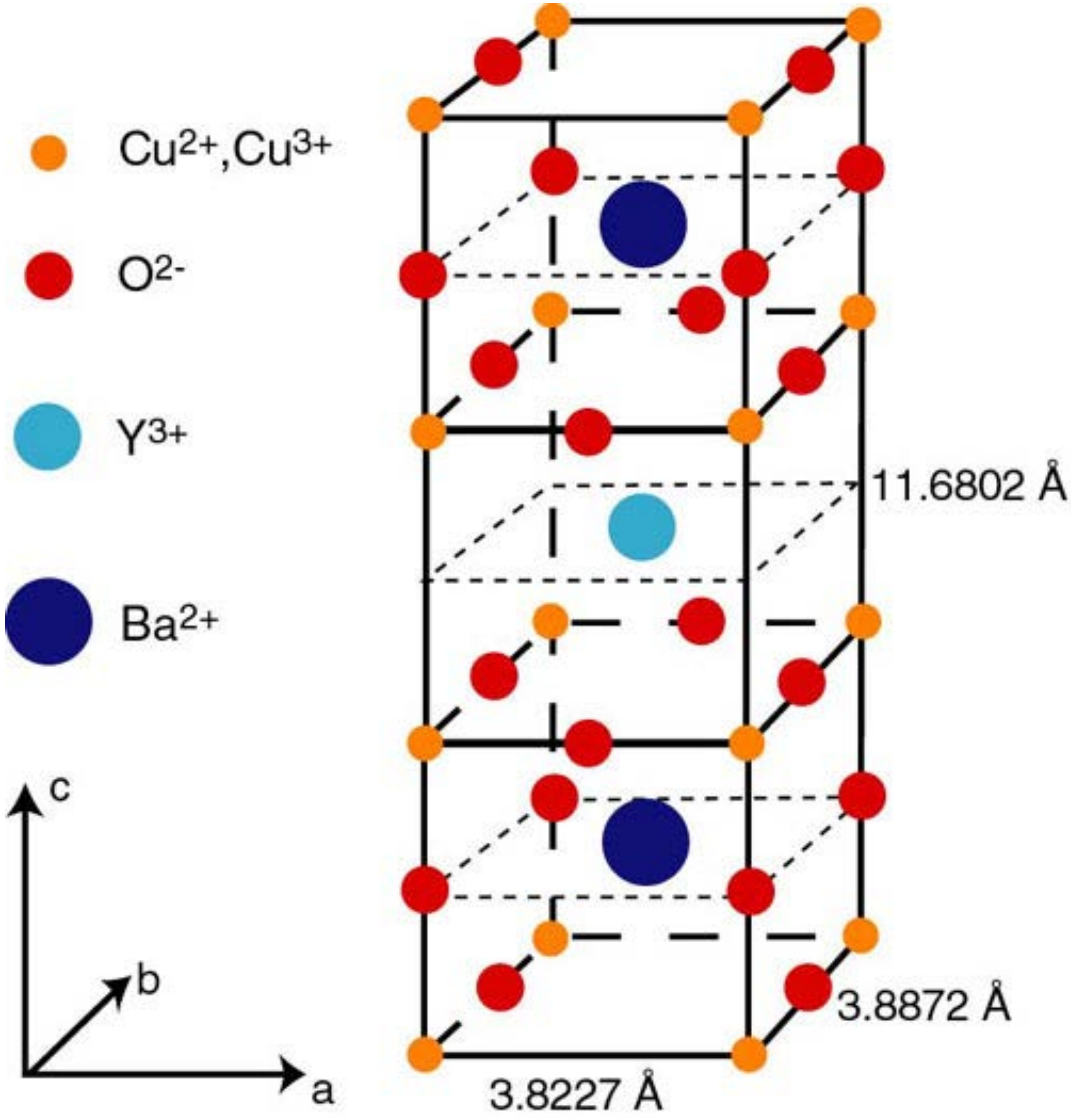


Welcome to BoseStat@100!

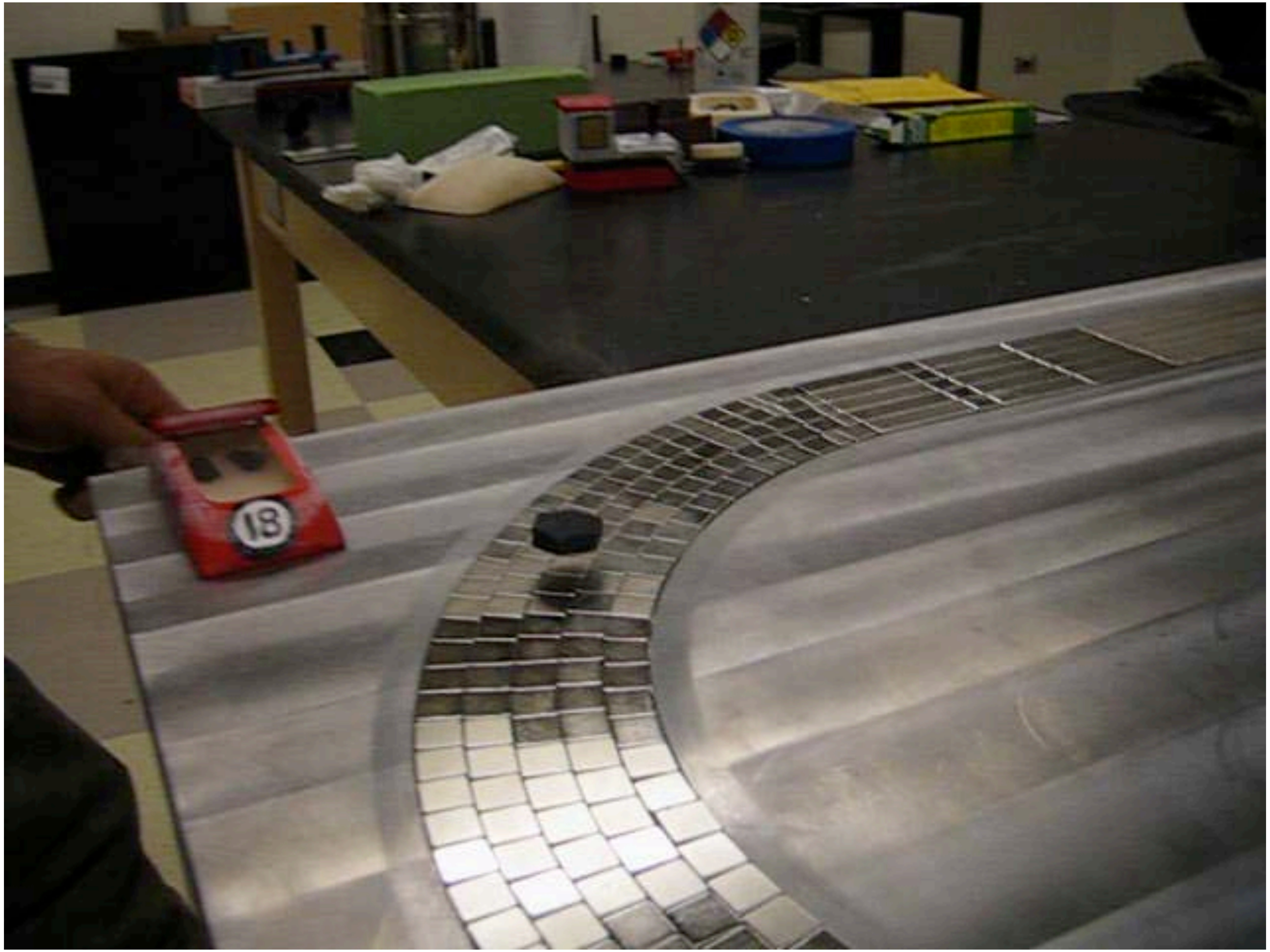
Satyendra Nath Bose was one of the founding fathers of quantum mechanics, the most successful description of the physical world. His pioneering work on quantum statistics has paved the way for development of modern quantum technologies including Bose-Einstein condensation, quantum

Cuprate
high temperature
superconductors





$YBa_2Cu_3O_{6+x}$



Nd-Fe-B magnets, YBaCuO superconductor

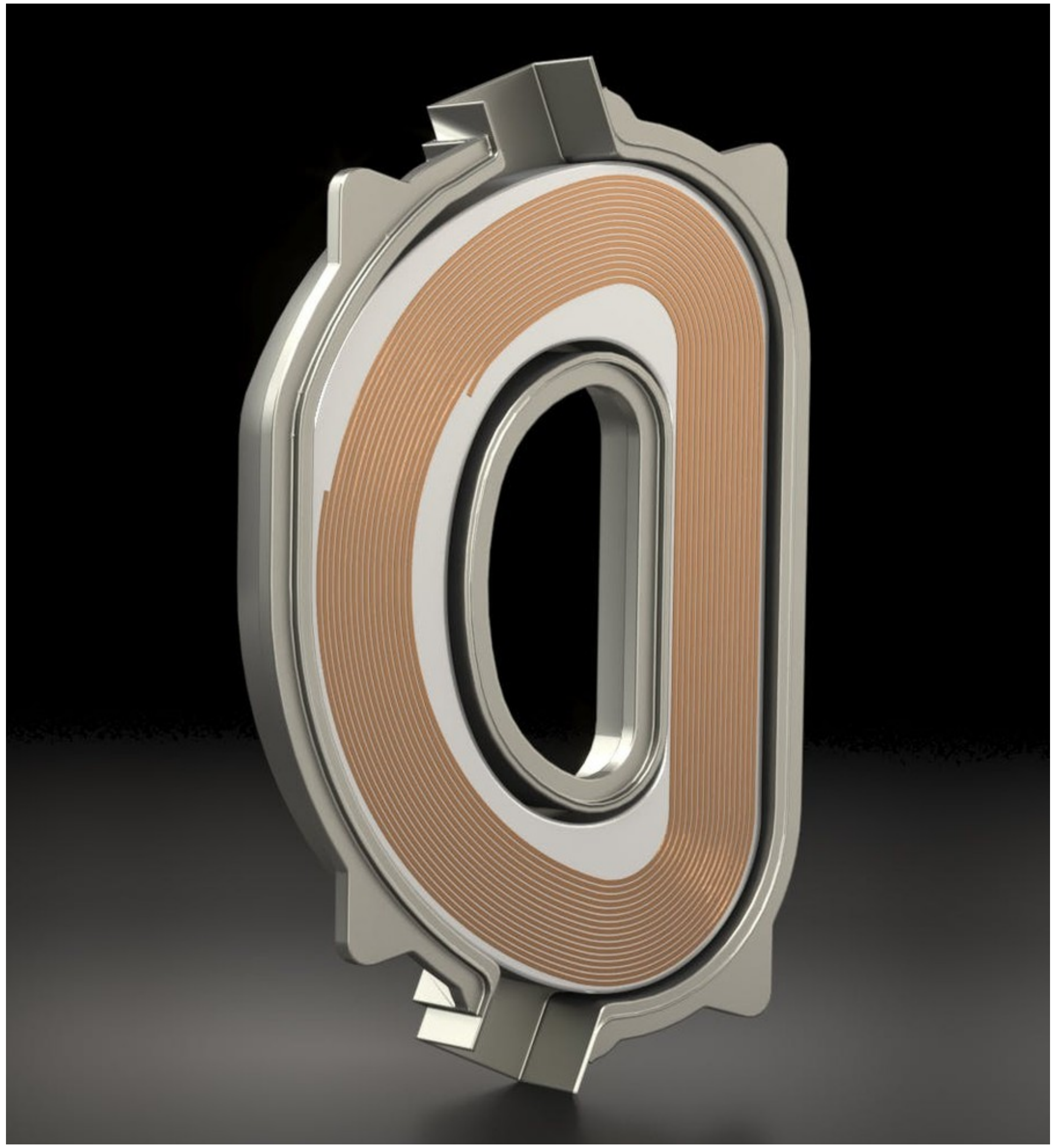
Julian Hetel and Nandini Trivedi, Ohio State University

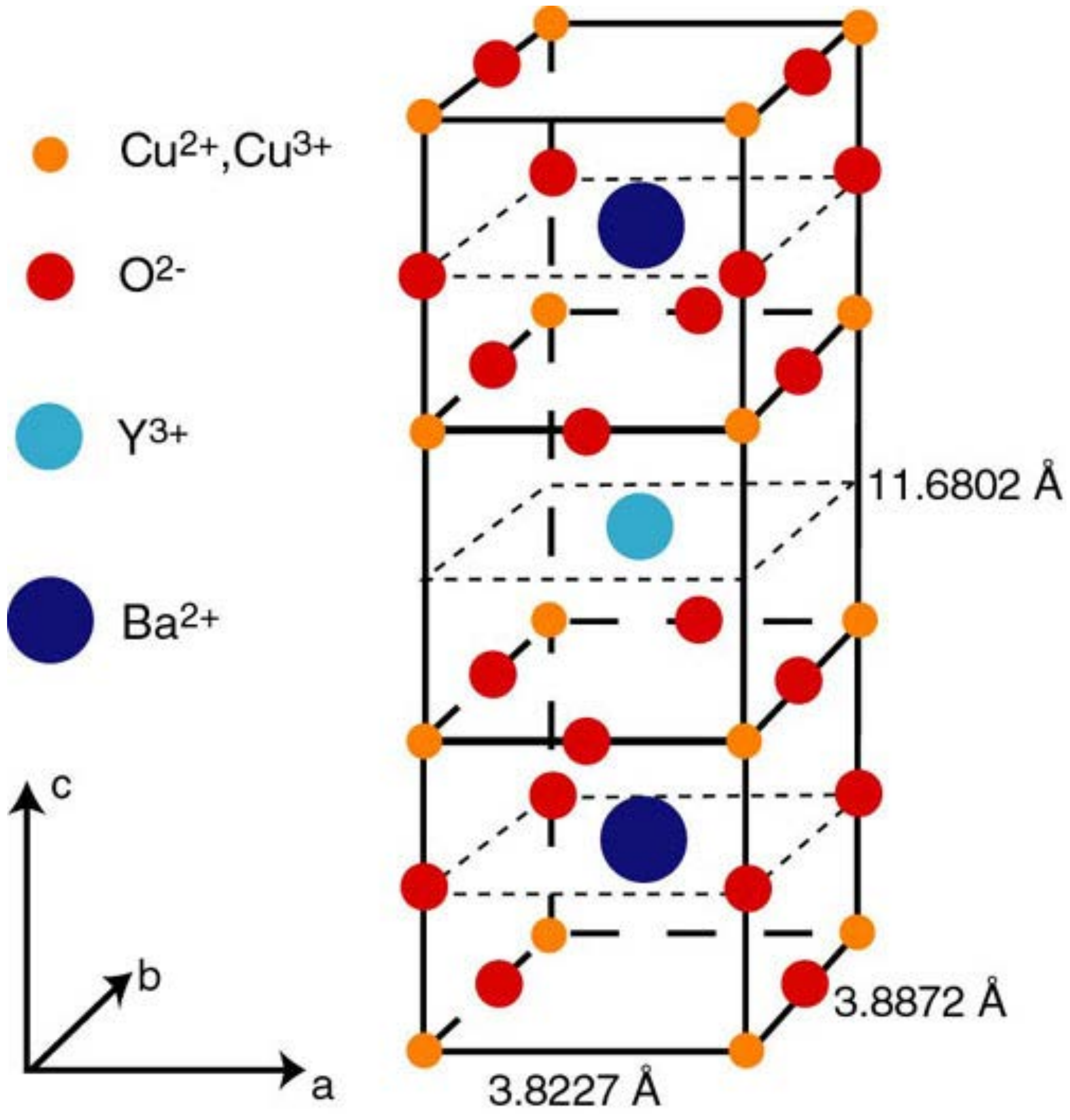
HTS Magnets: Enabling Technology

The surest path to limitless, clean, fusion energy

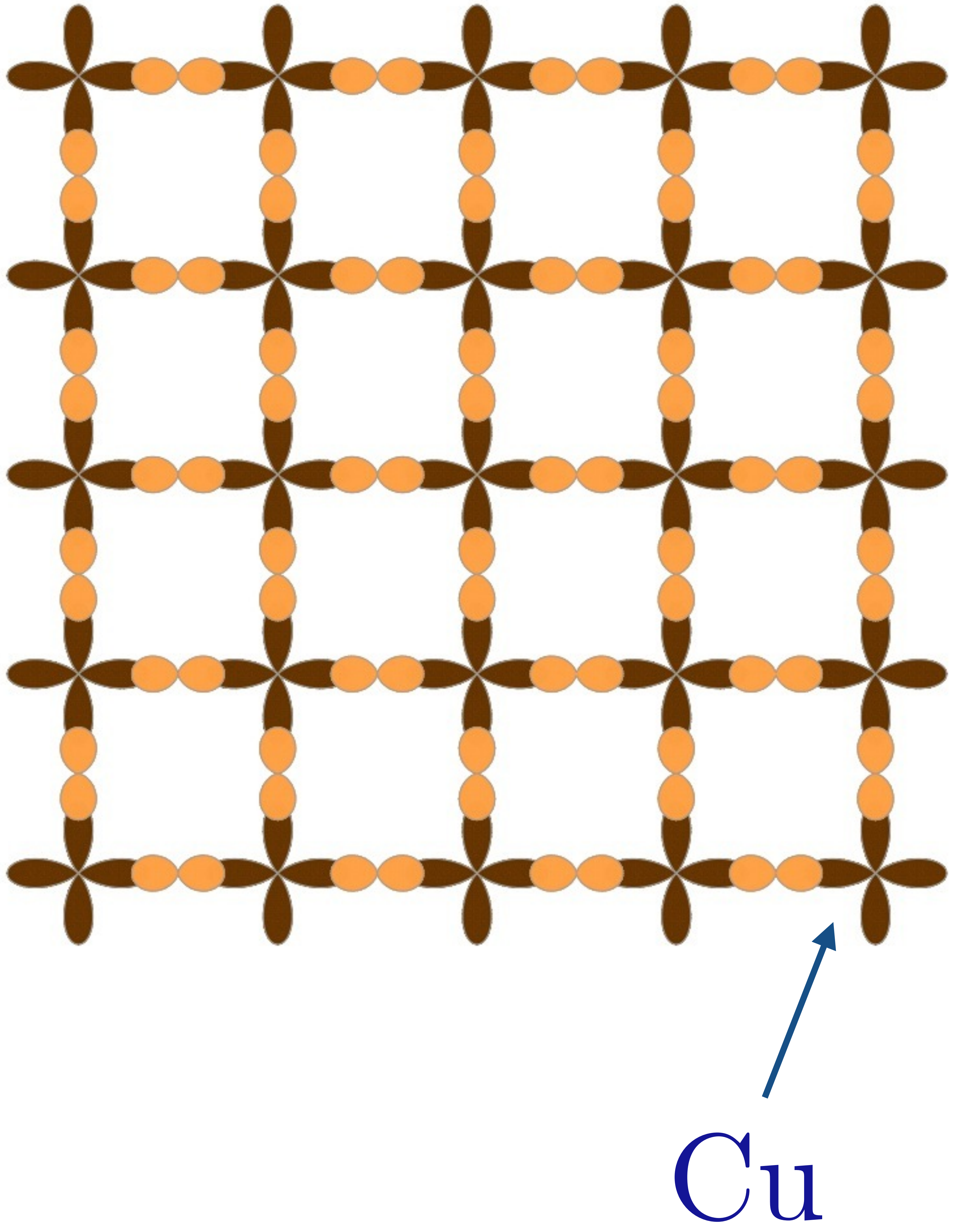
YBCO magnets allow for smaller, faster, and less expensive tokamaks for plasma fusion

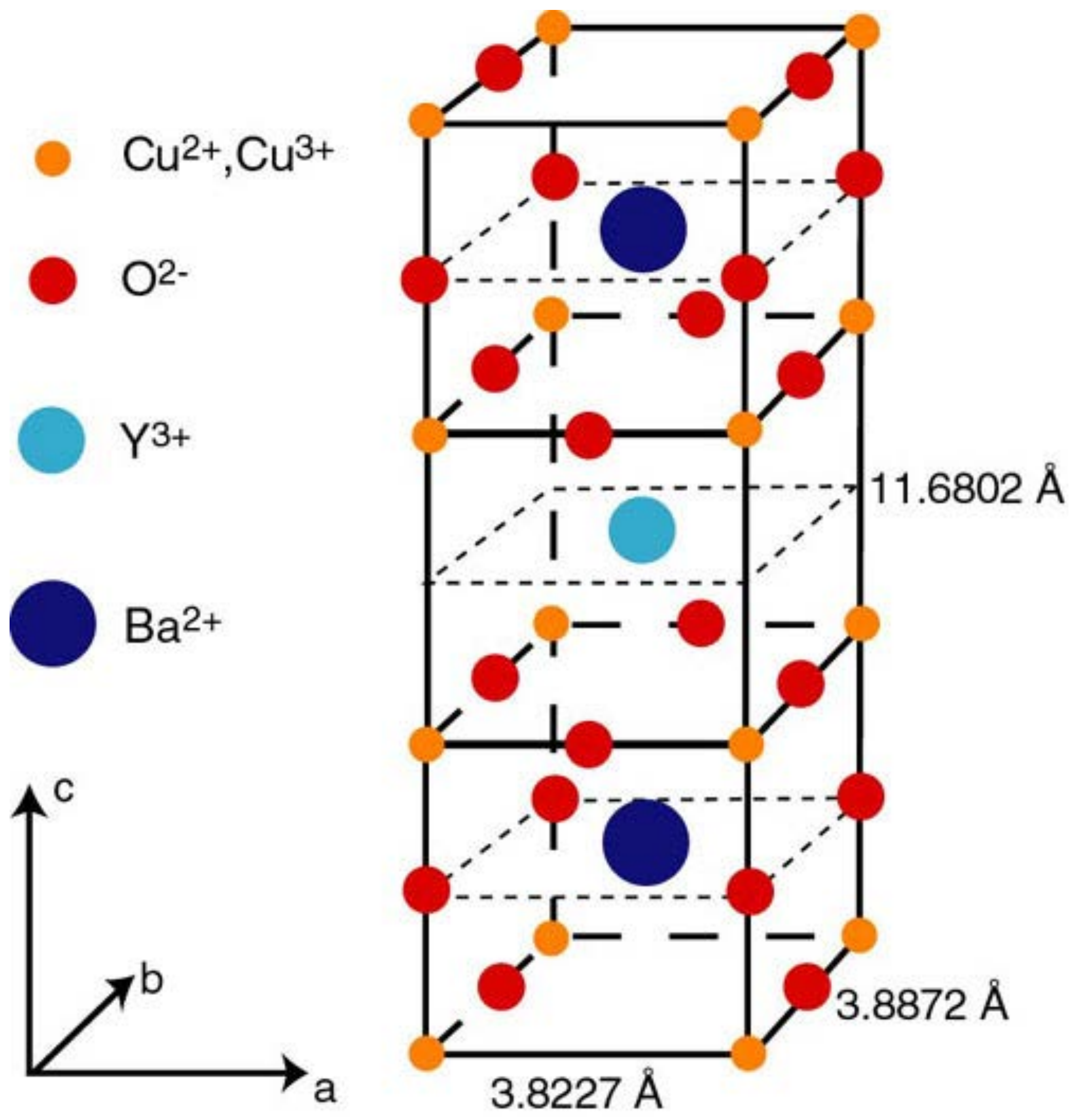
Commonwealth Fusion Systems



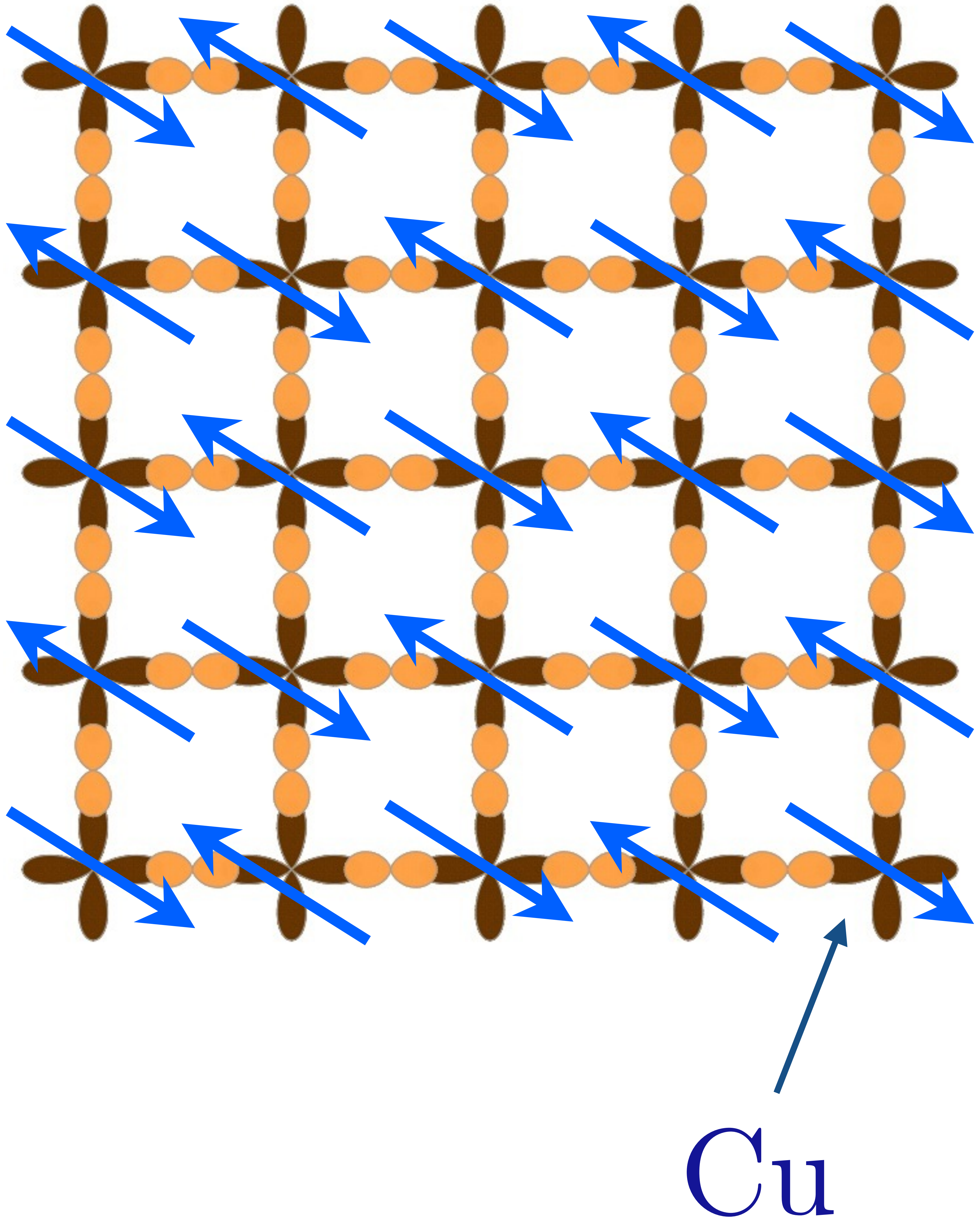


$$YBa_2Cu_3O_{6+x}$$





$$\mathrm{YBa_2Cu_3O_{6+x}}$$



$$\mathcal{H} = J \sum_{\langle ij \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

$S = 1/2$ on each site

$$|\uparrow\rangle, \ |\uparrow\rangle$$

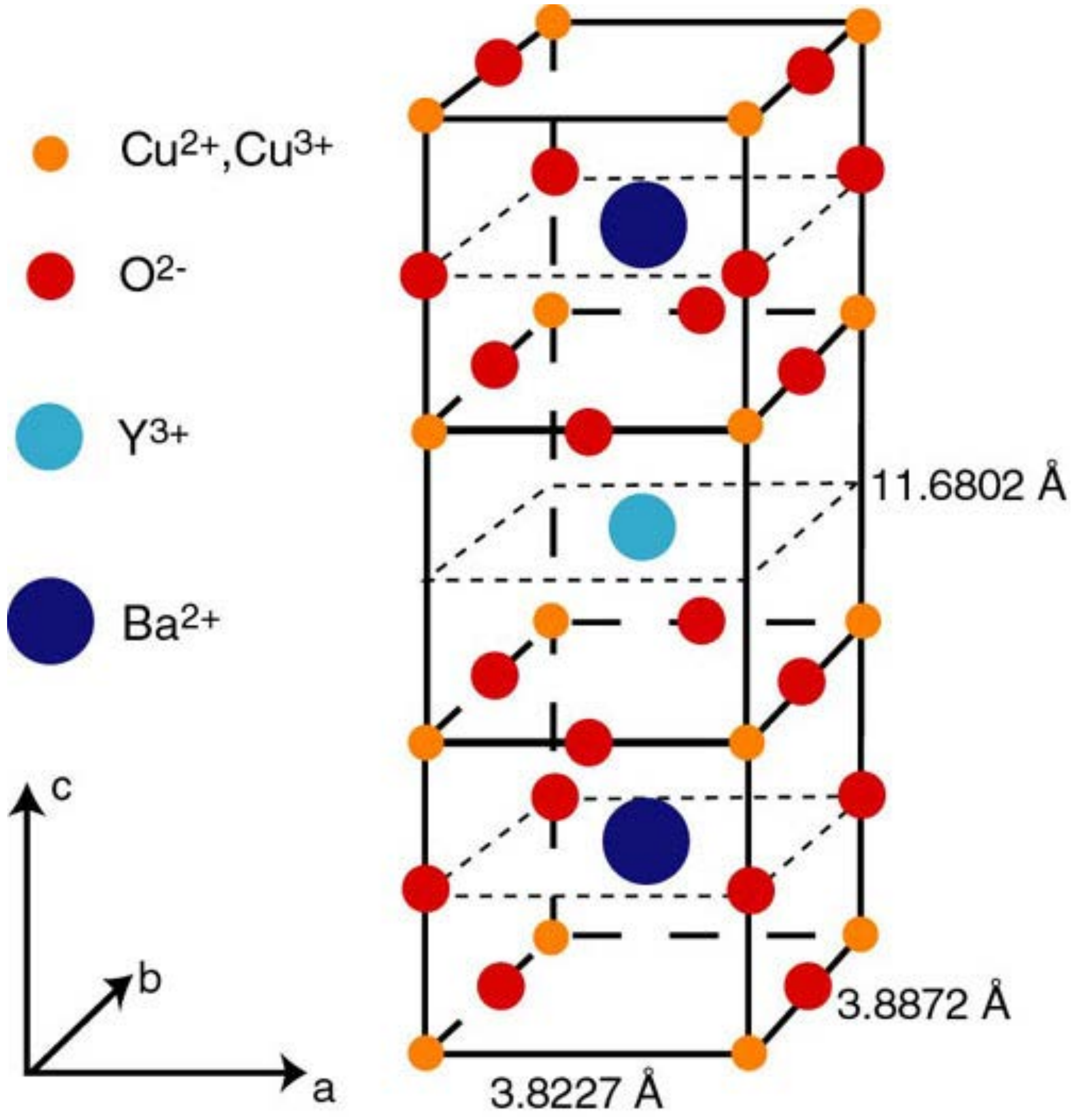
$$S_z \, |\uparrow\rangle = (1/2) \, |\uparrow\rangle$$

$$S_z \, |\downarrow\rangle = -(1/2) \, |\downarrow\rangle$$

$$(S_x + iS_y) \, |\downarrow\rangle = |\uparrow\rangle$$

$$(S_x - iS_y) \, |\uparrow\rangle = |\downarrow\rangle$$

Insulating antiferromagnet with one electron per site



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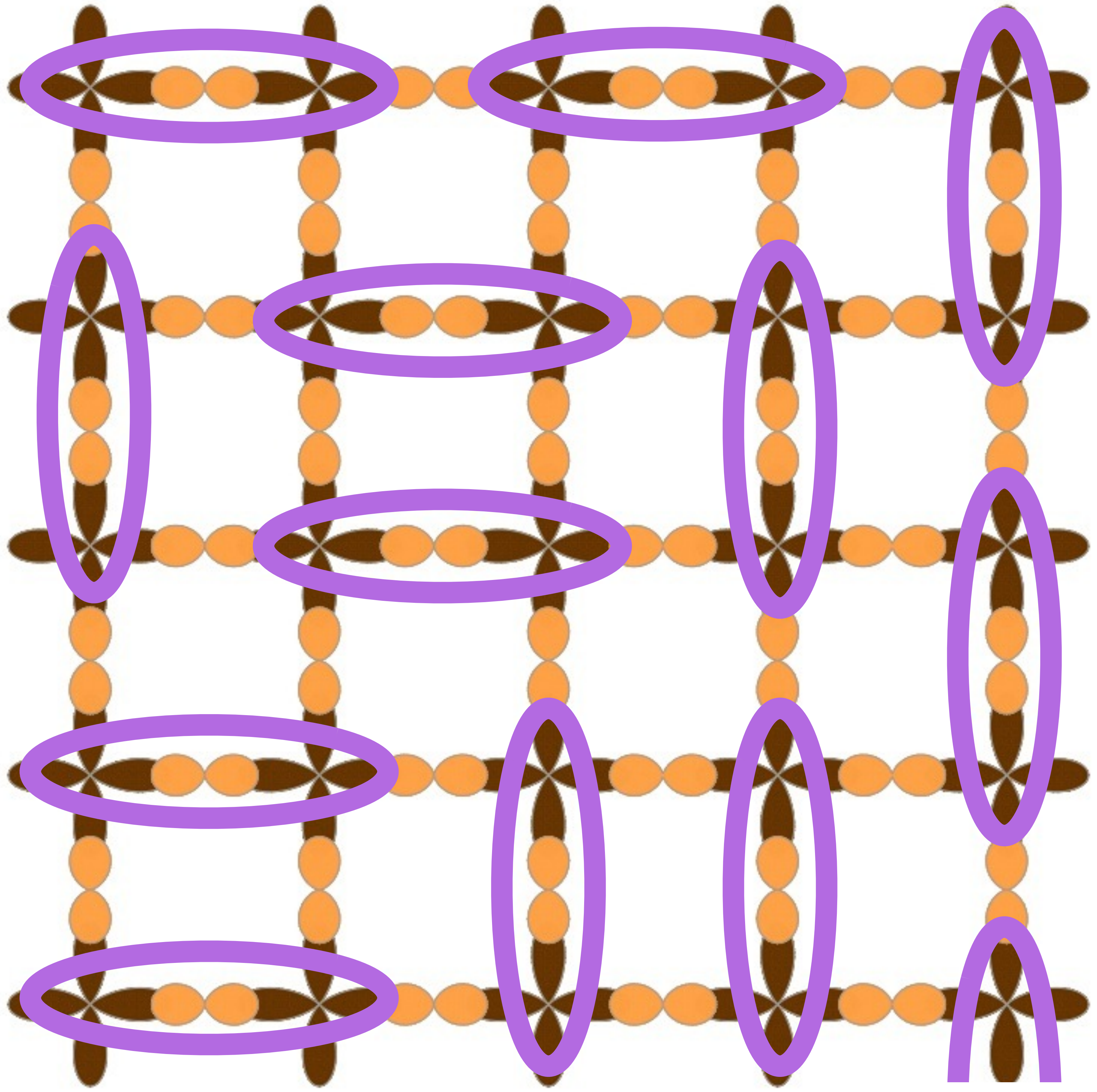
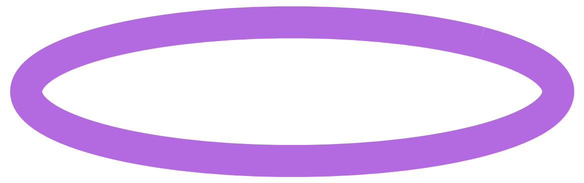


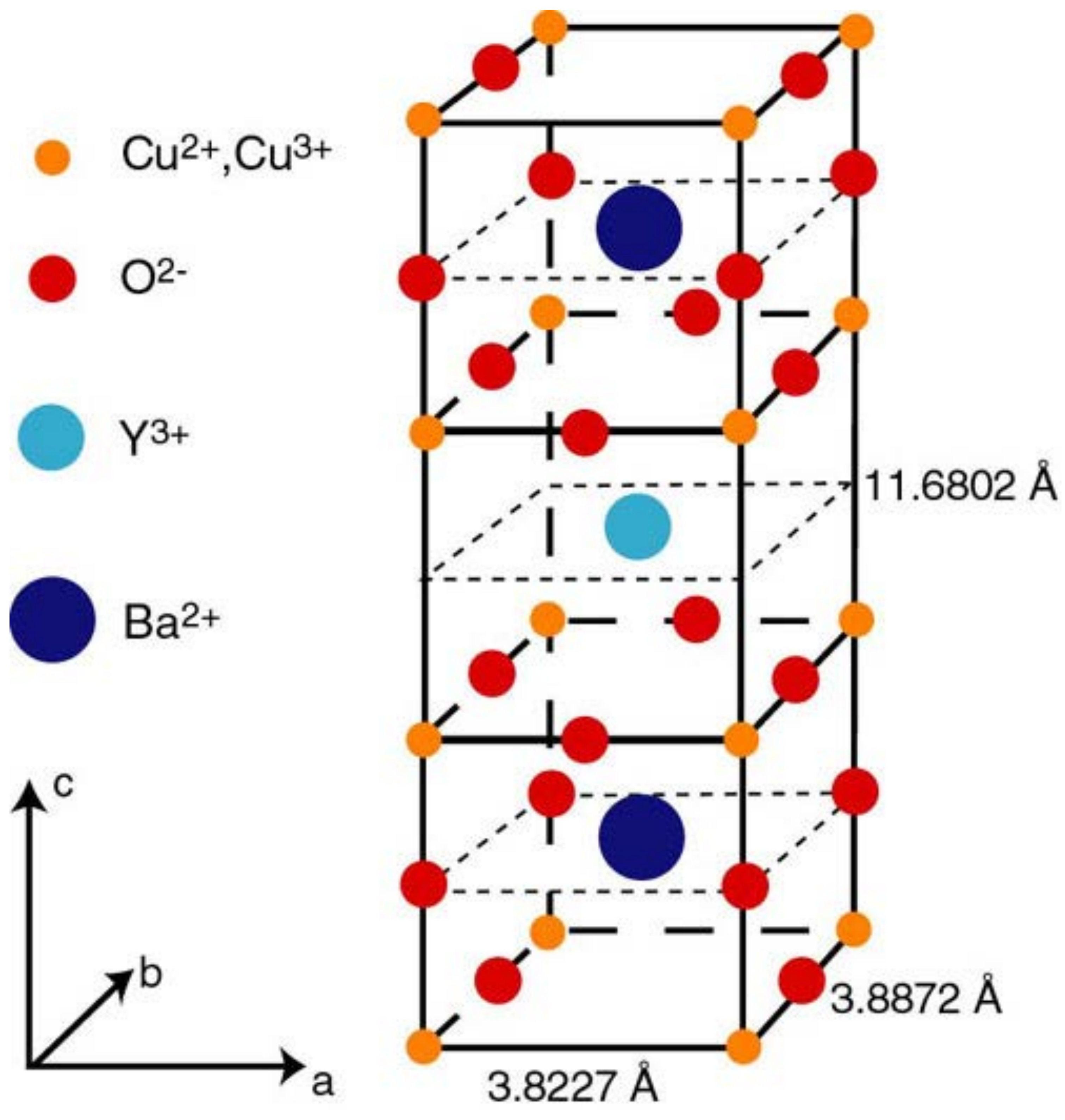
$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} \, |\mathcal{D}\rangle$$

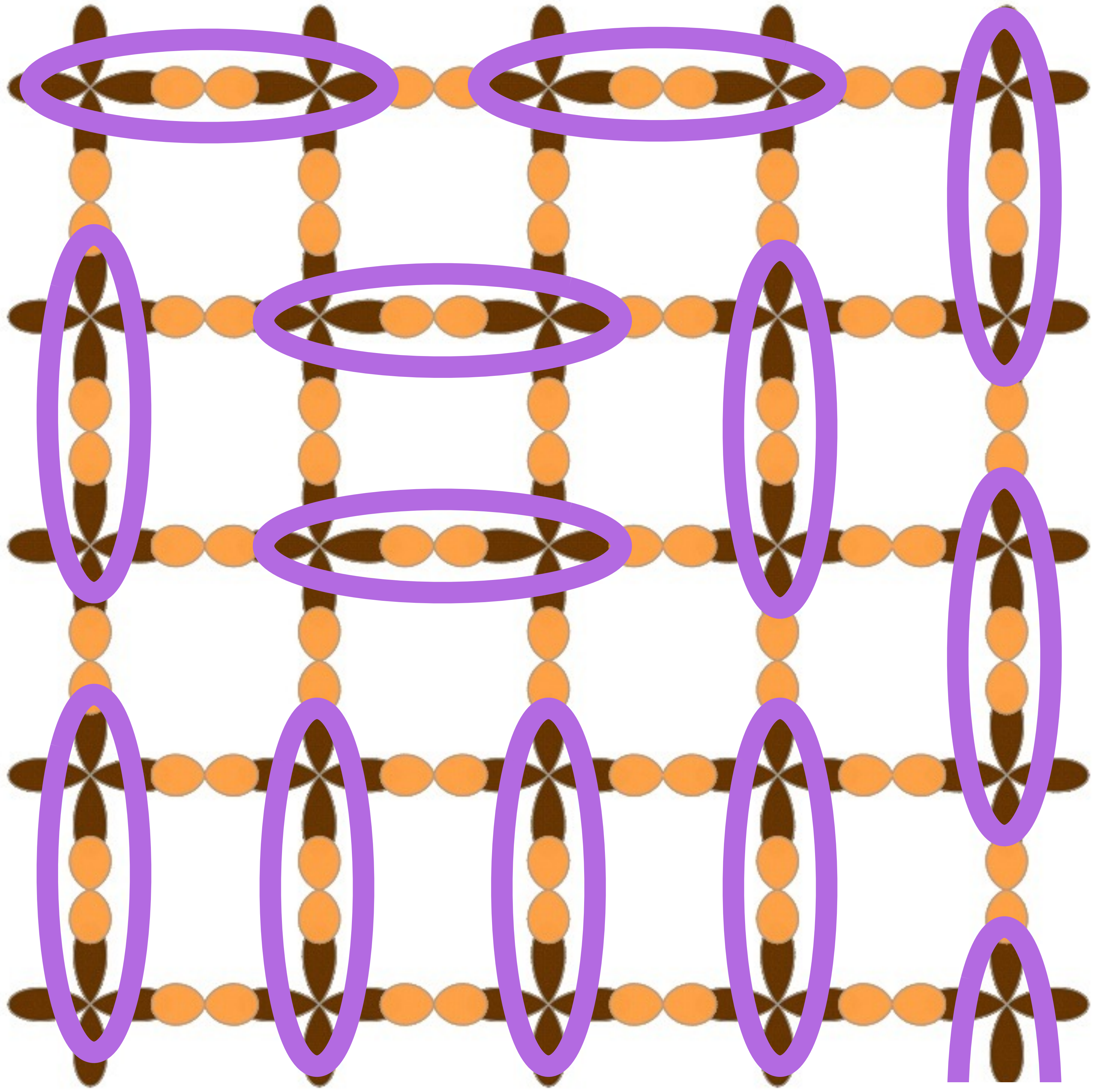
$\mathcal{D} \to$ dimer covering of lattice

 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

P.W. Anderson (1987): The key to high temperature superconductivity is the formation of a "resonating valence bond state".

A quantum spin liquid with many-boson (spins on Cu) **entanglement**





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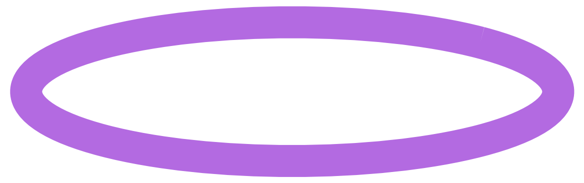 $= \frac{1}{\sqrt{2}} \left(| \uparrow\downarrow\rangle - | \downarrow\uparrow\rangle \right)$

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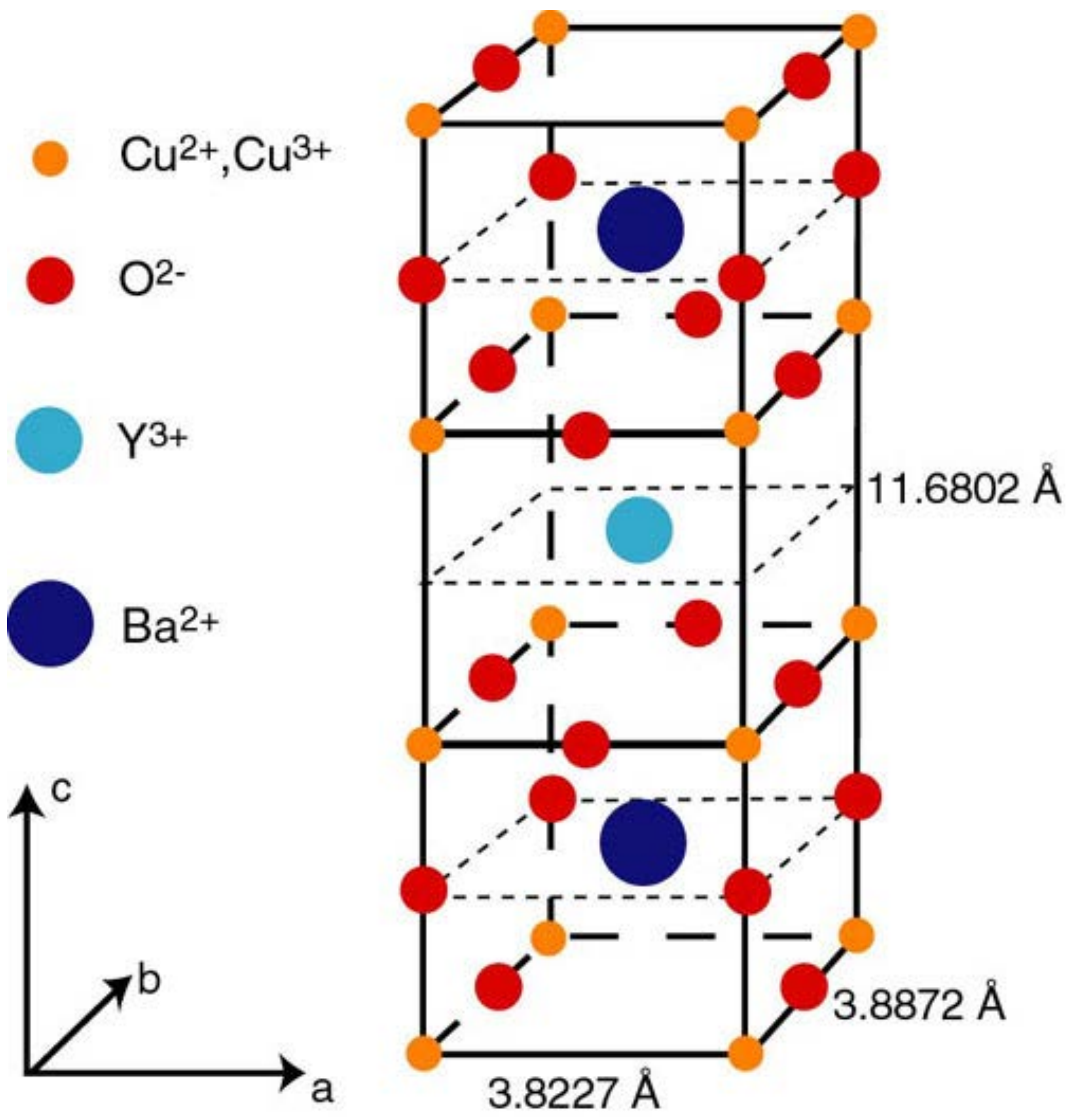

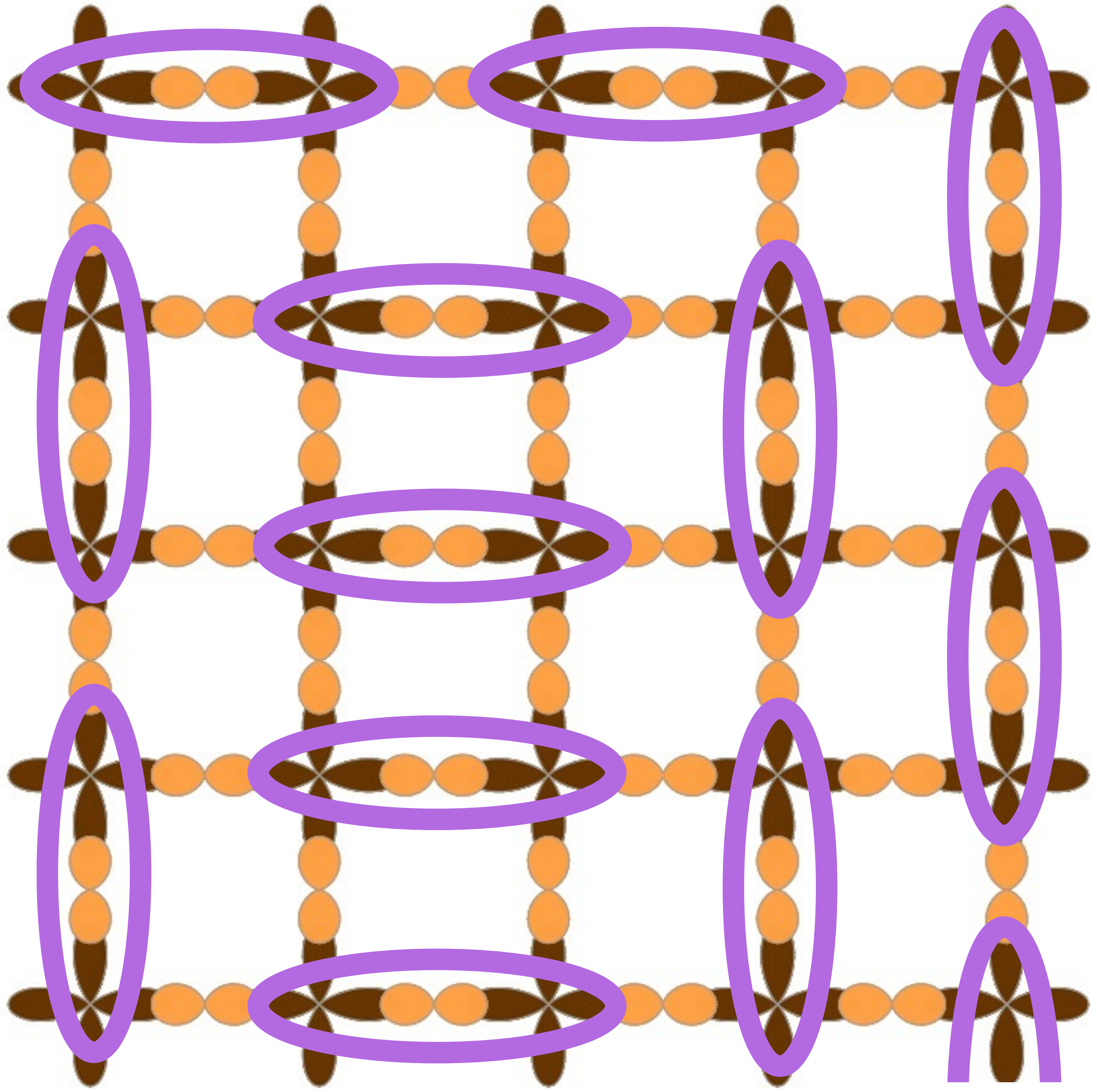

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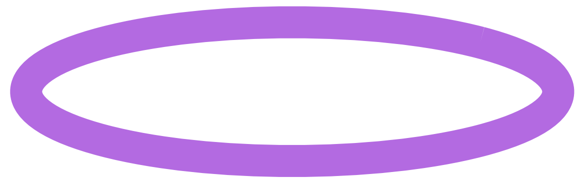 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

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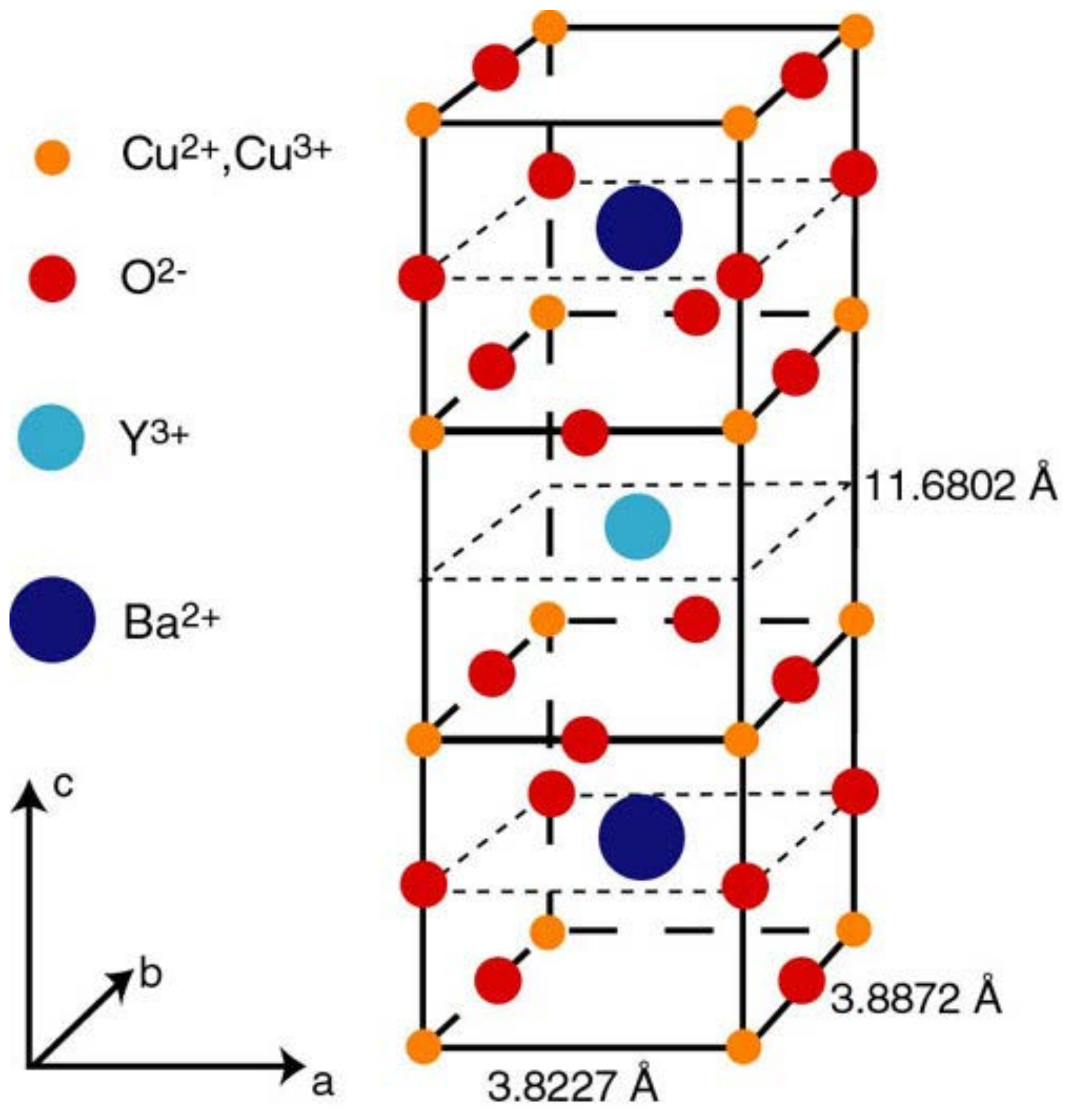

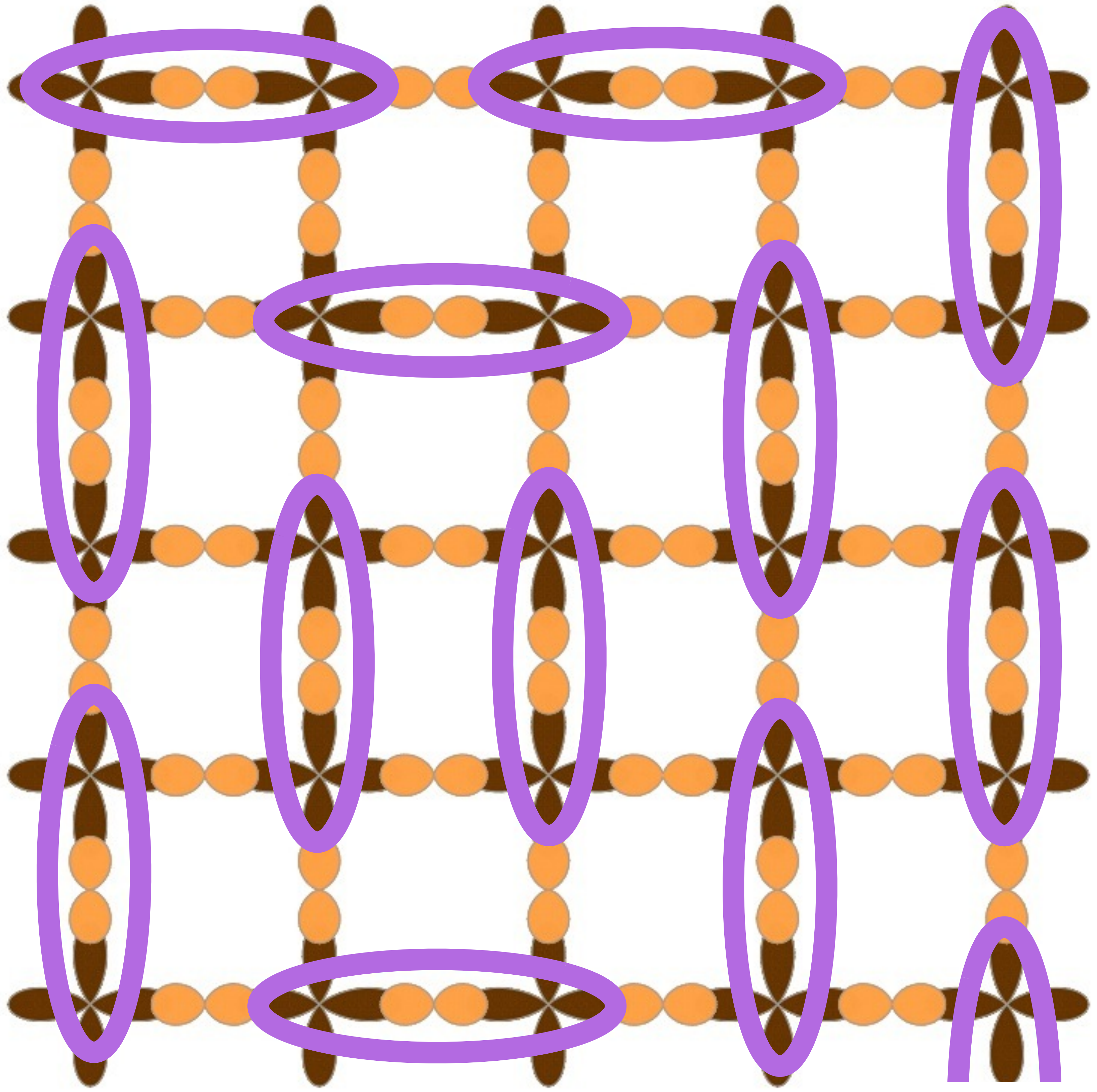

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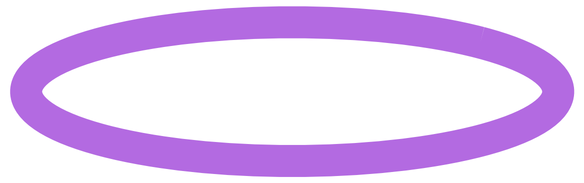 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

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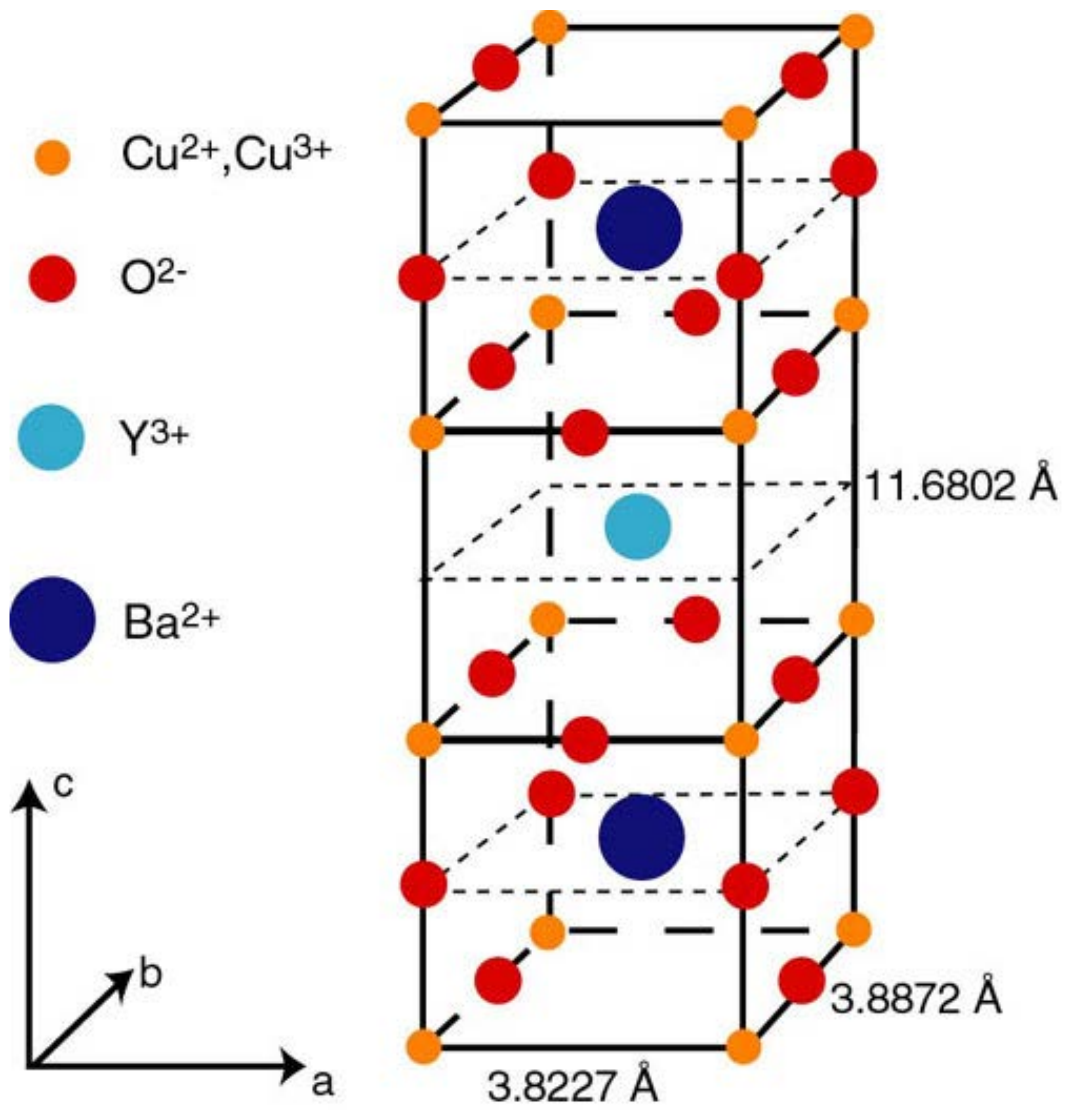

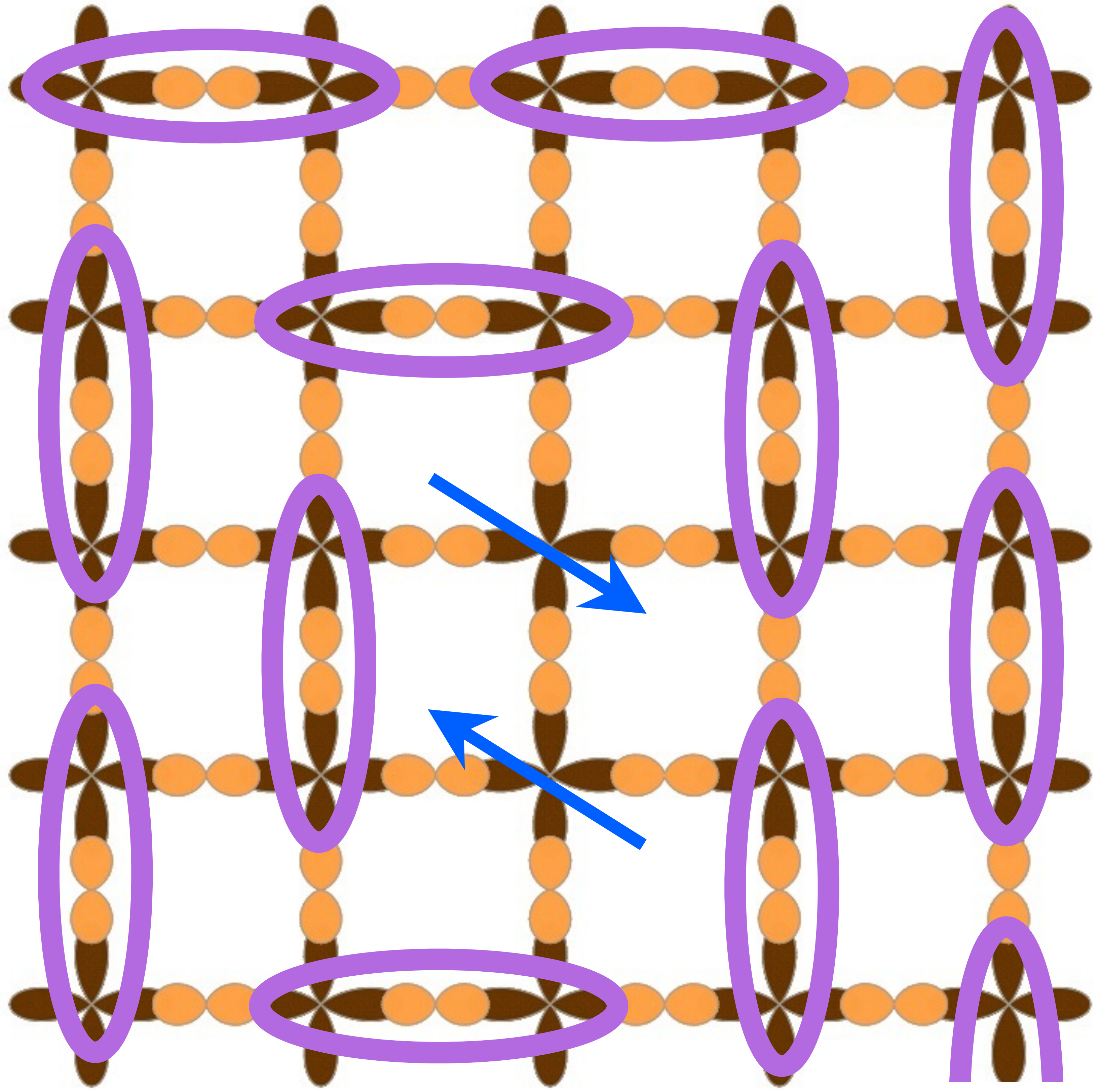

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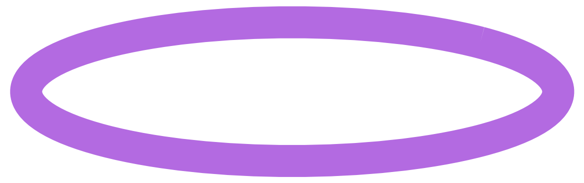
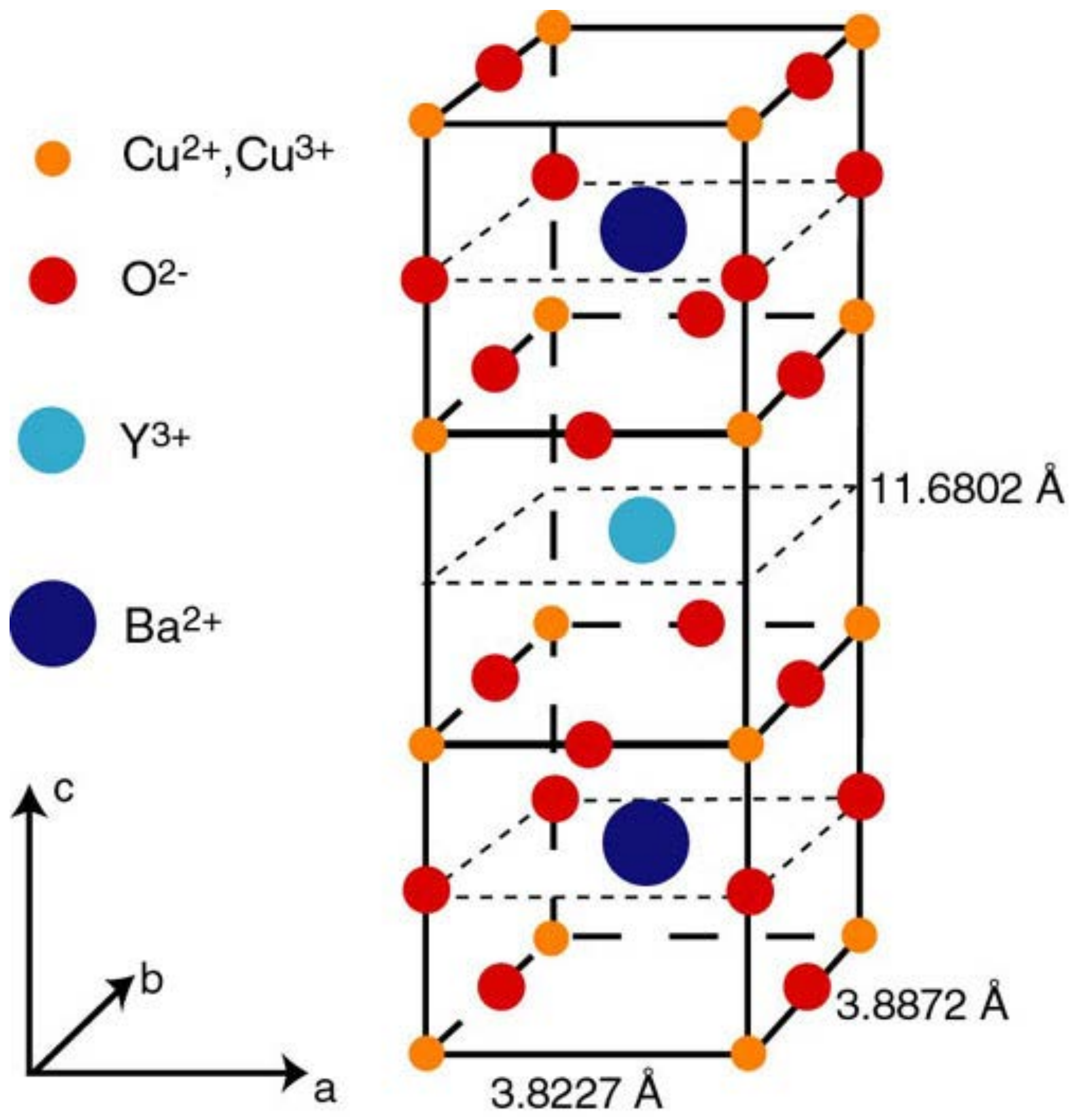
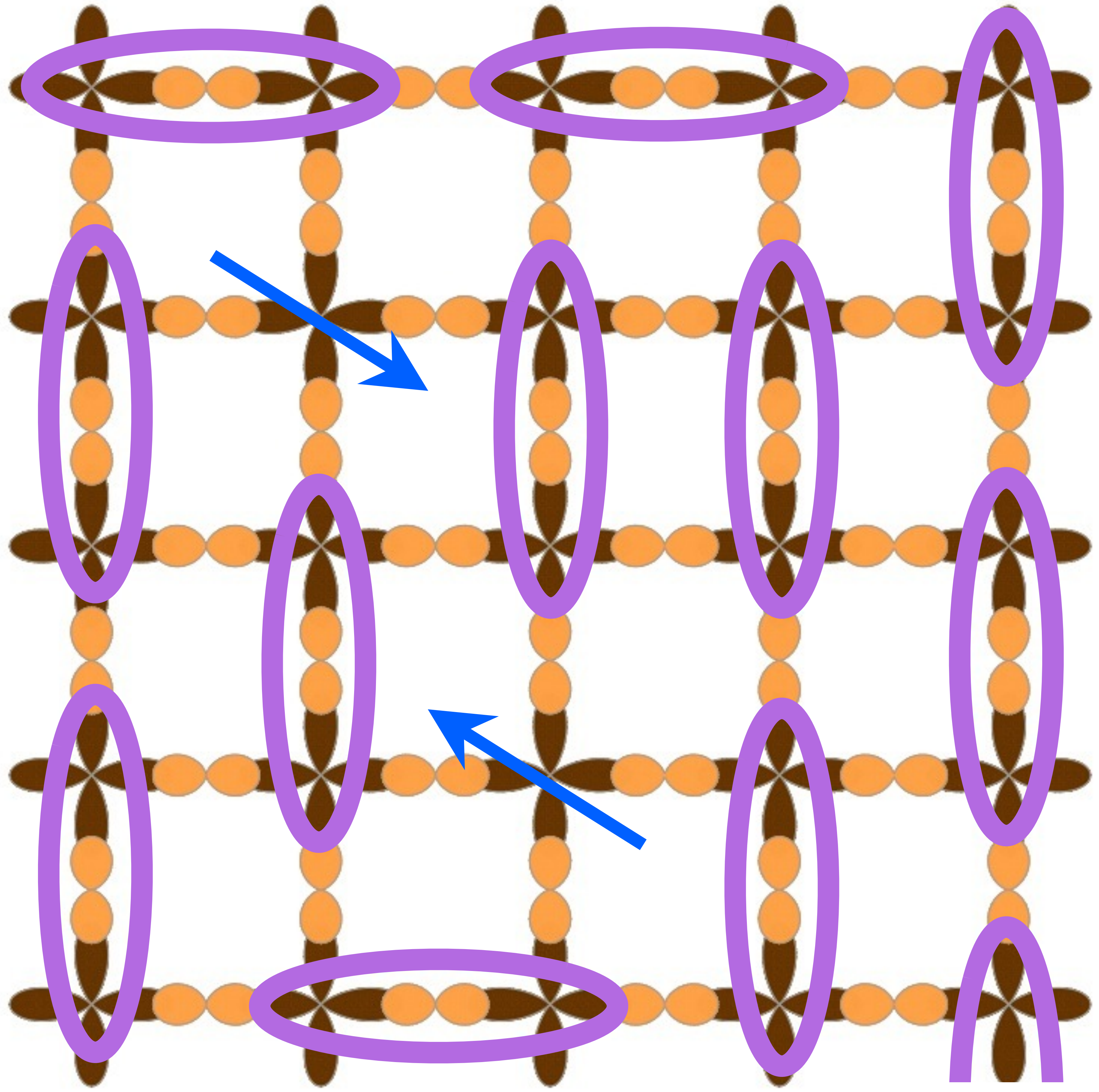
 $= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

Key feature: fractionalization. Excitations are particle-like, but cannot be created by local operators: they are classified under distinct superselection/anyon sectors.





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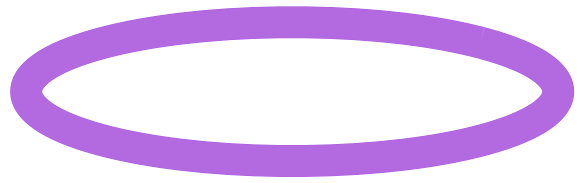 $= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

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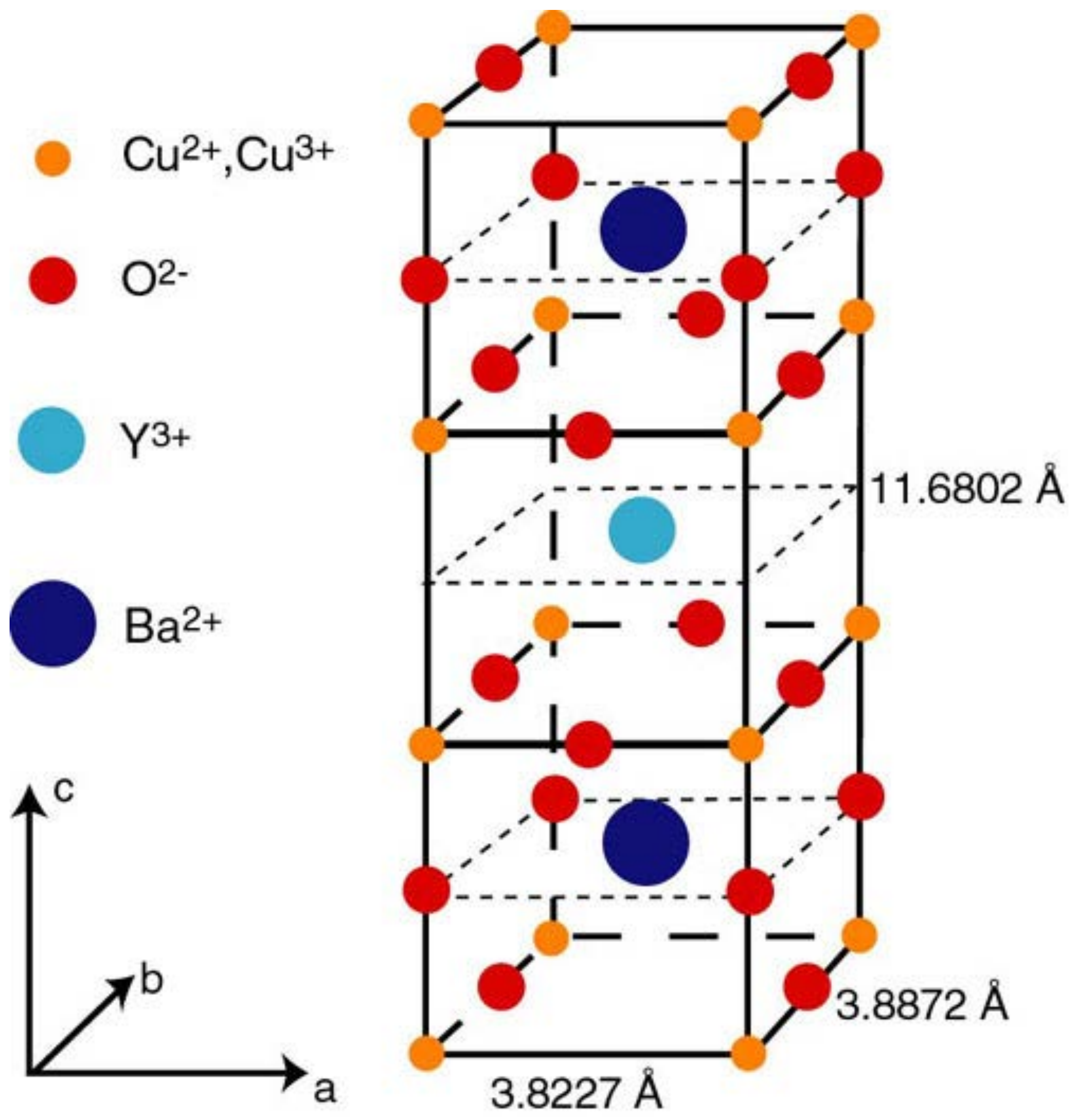

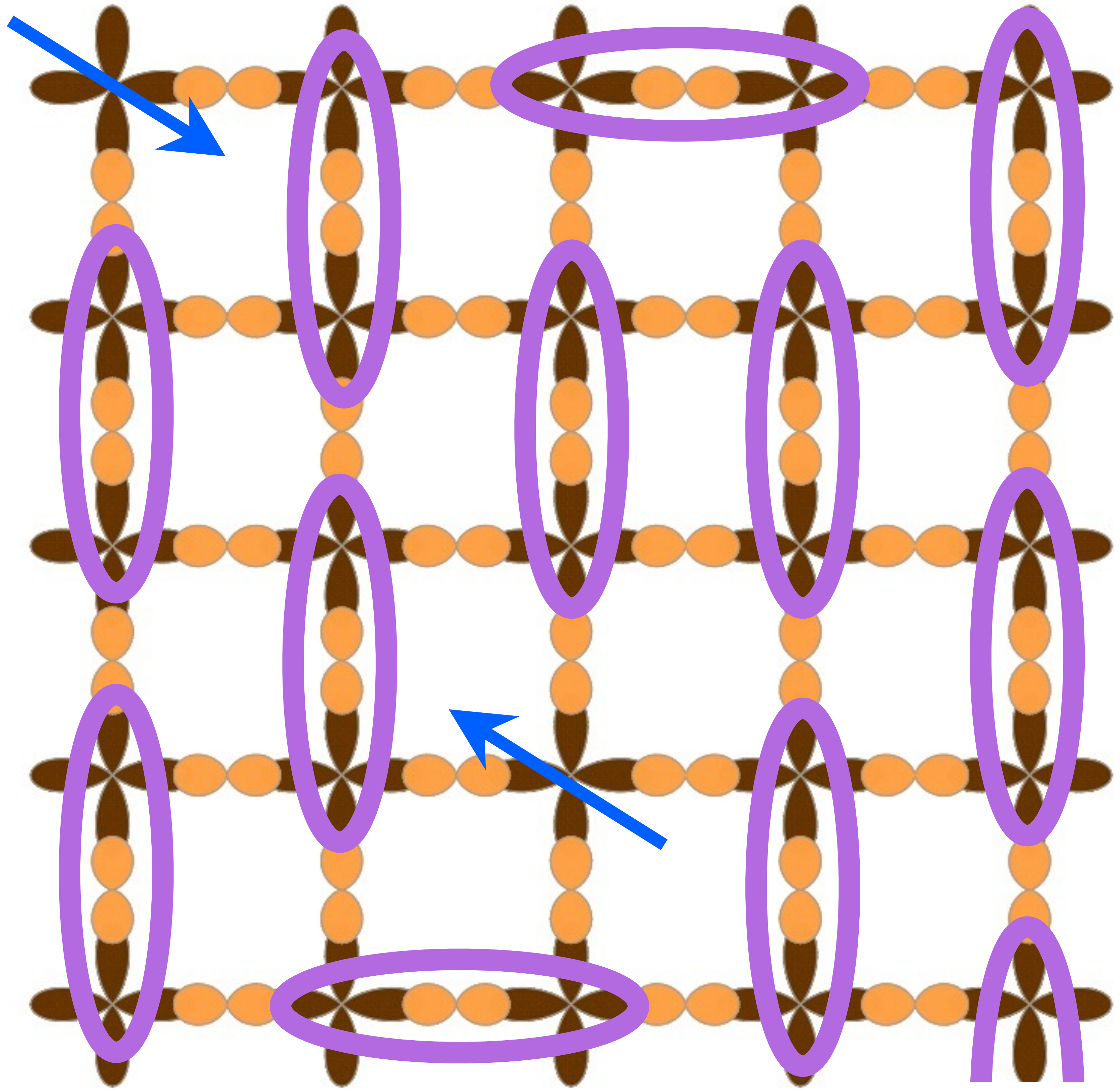

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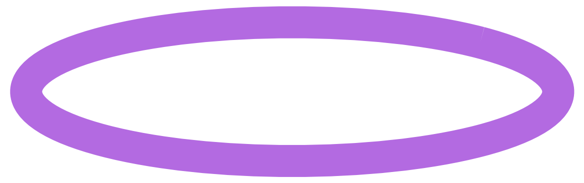 $= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

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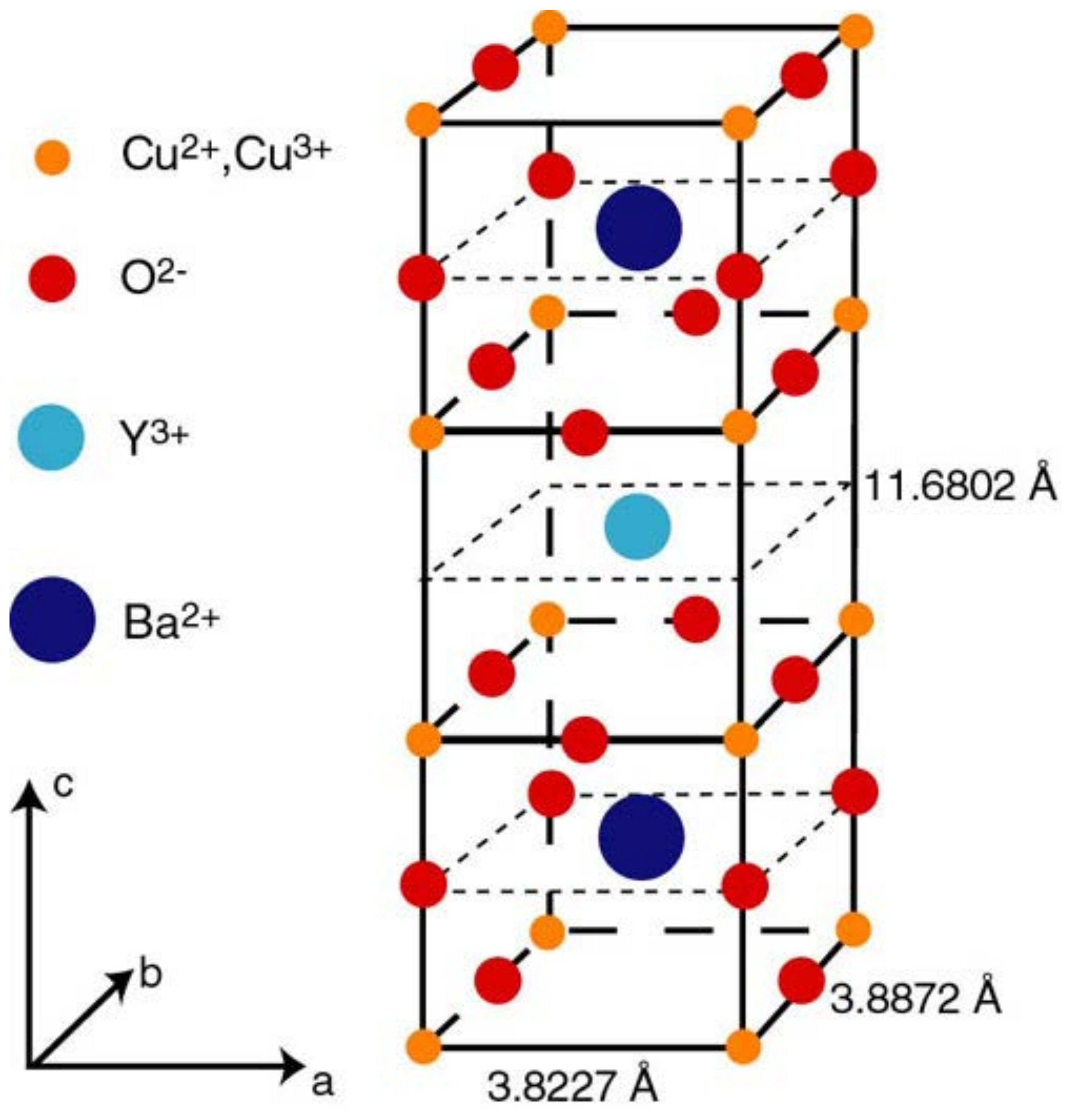
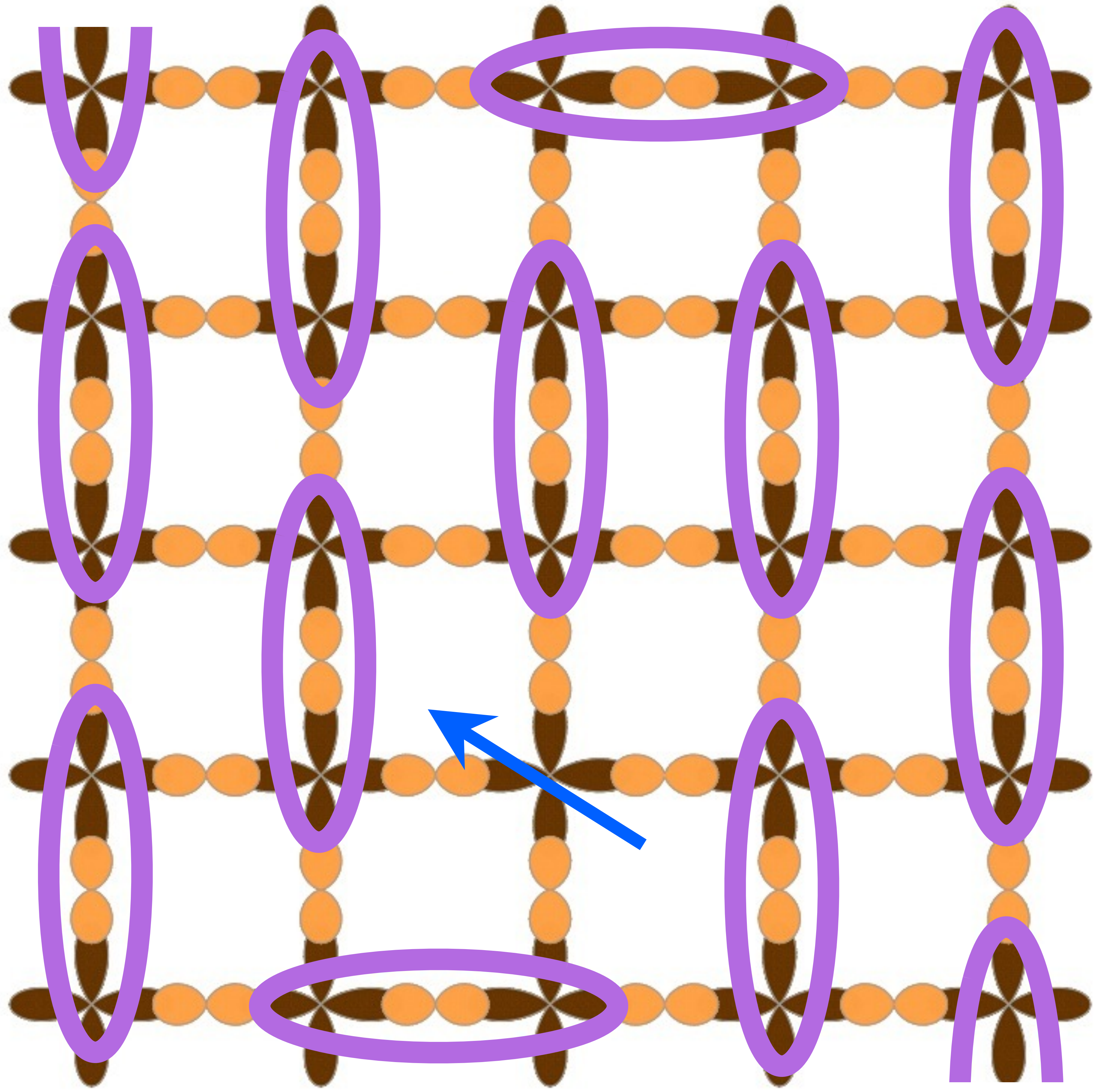
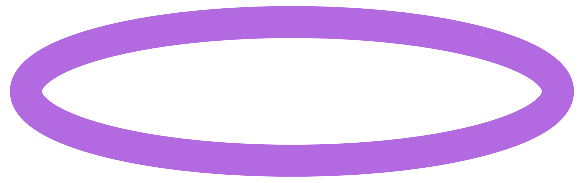




Spin S=1/2, charge neutral spinon

$$\mathrm{YBa_2Cu_3O_{6+x}}$$

 $= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

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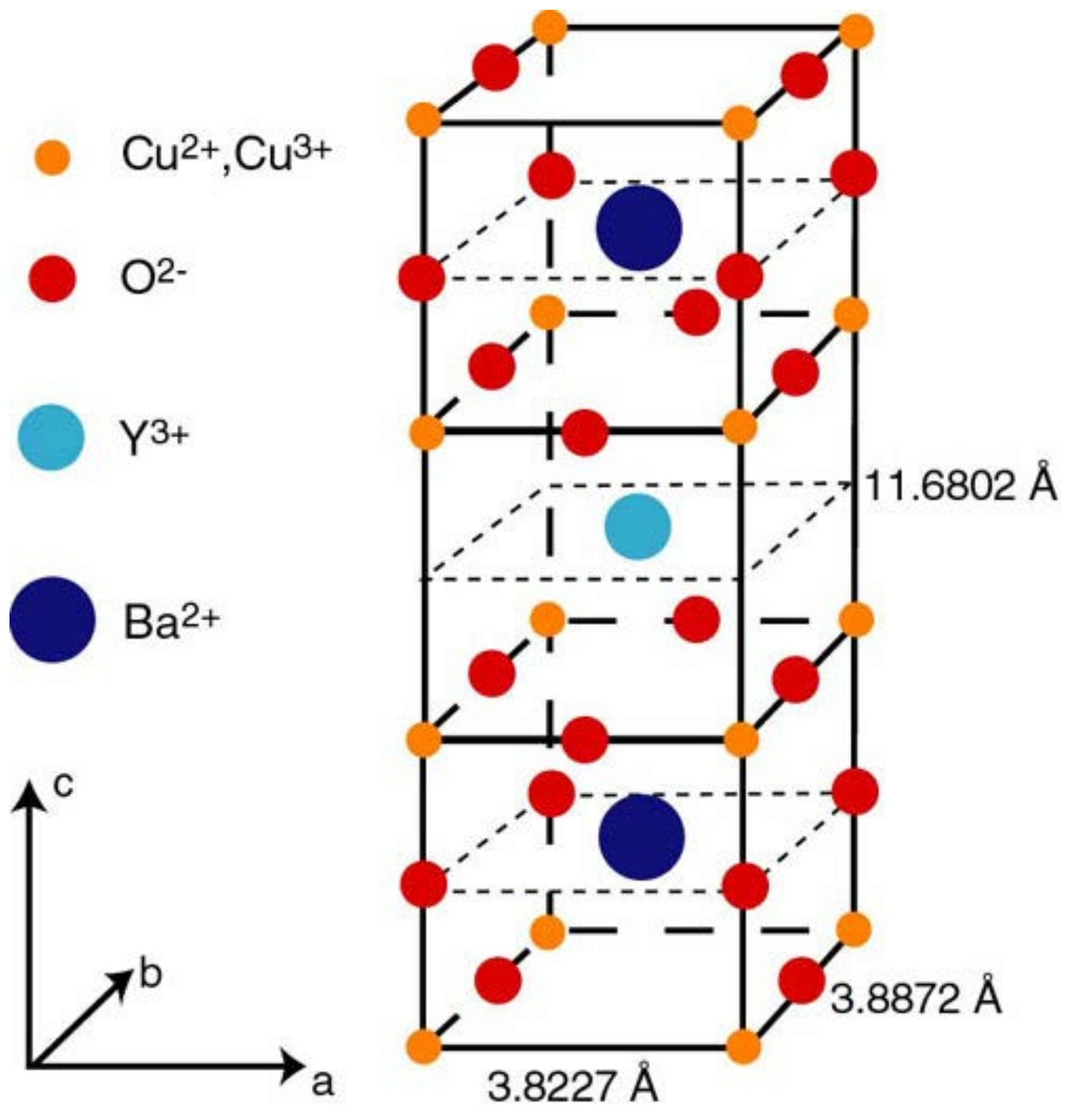

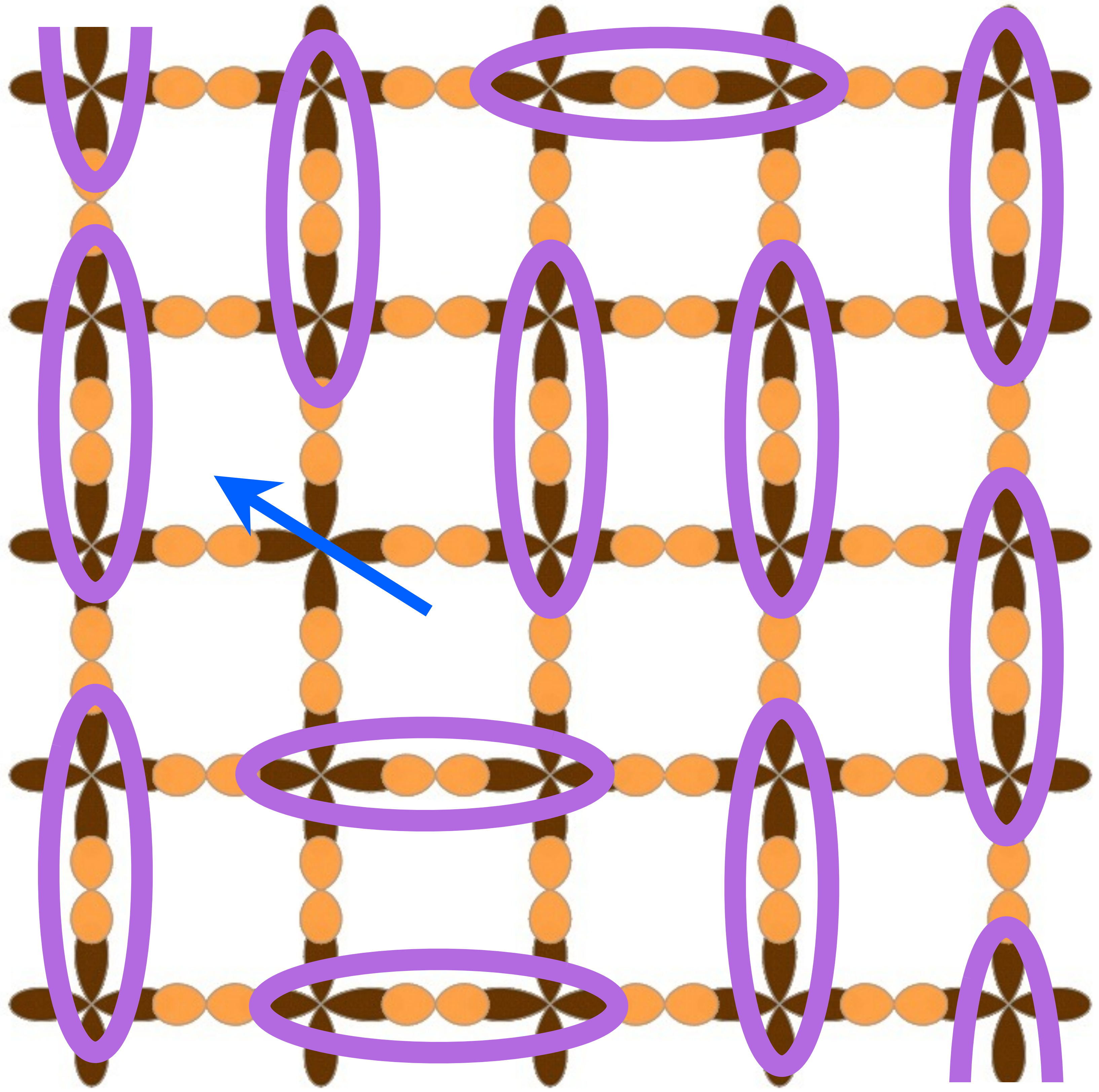

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$$\bigcirc = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

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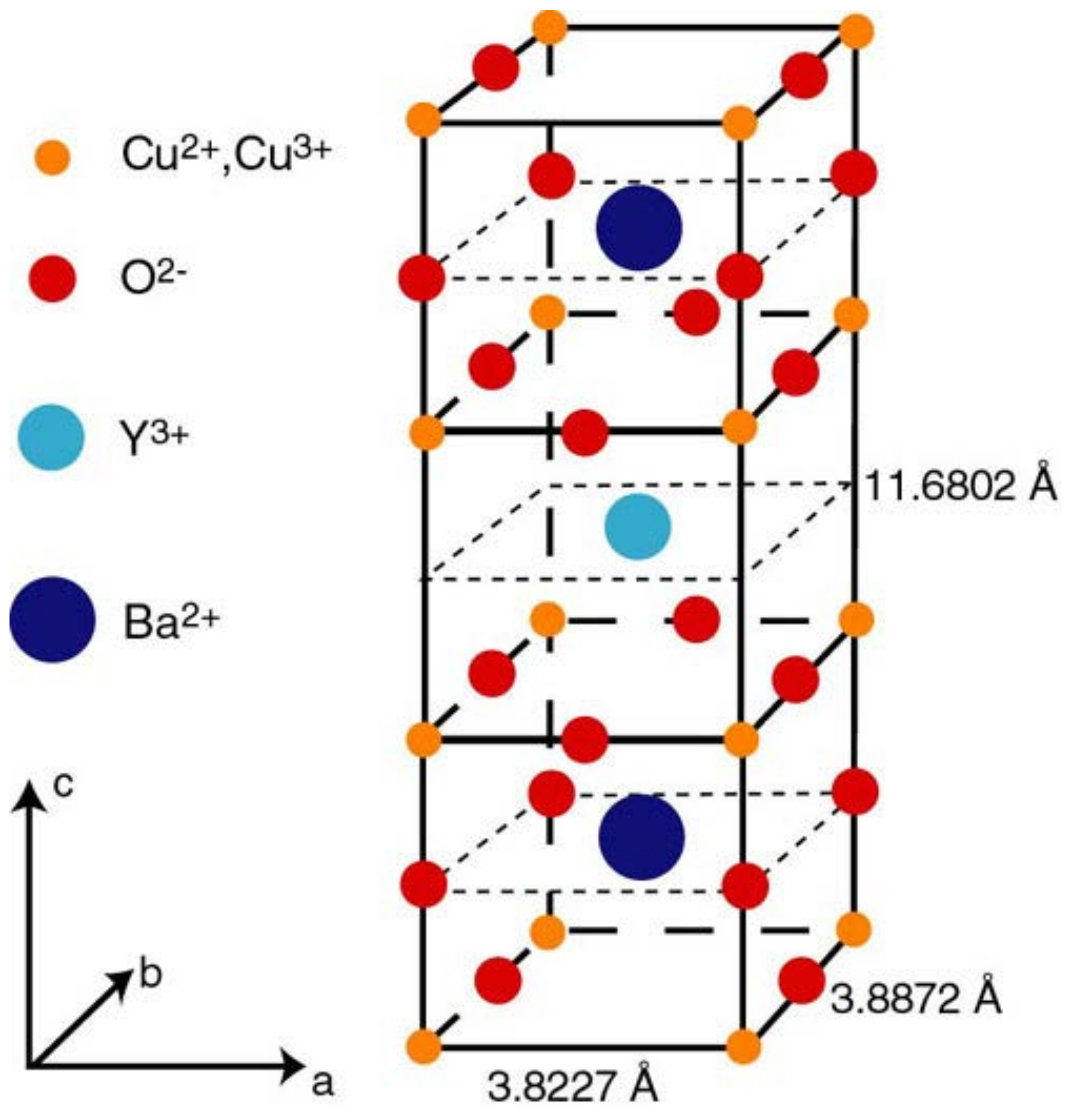

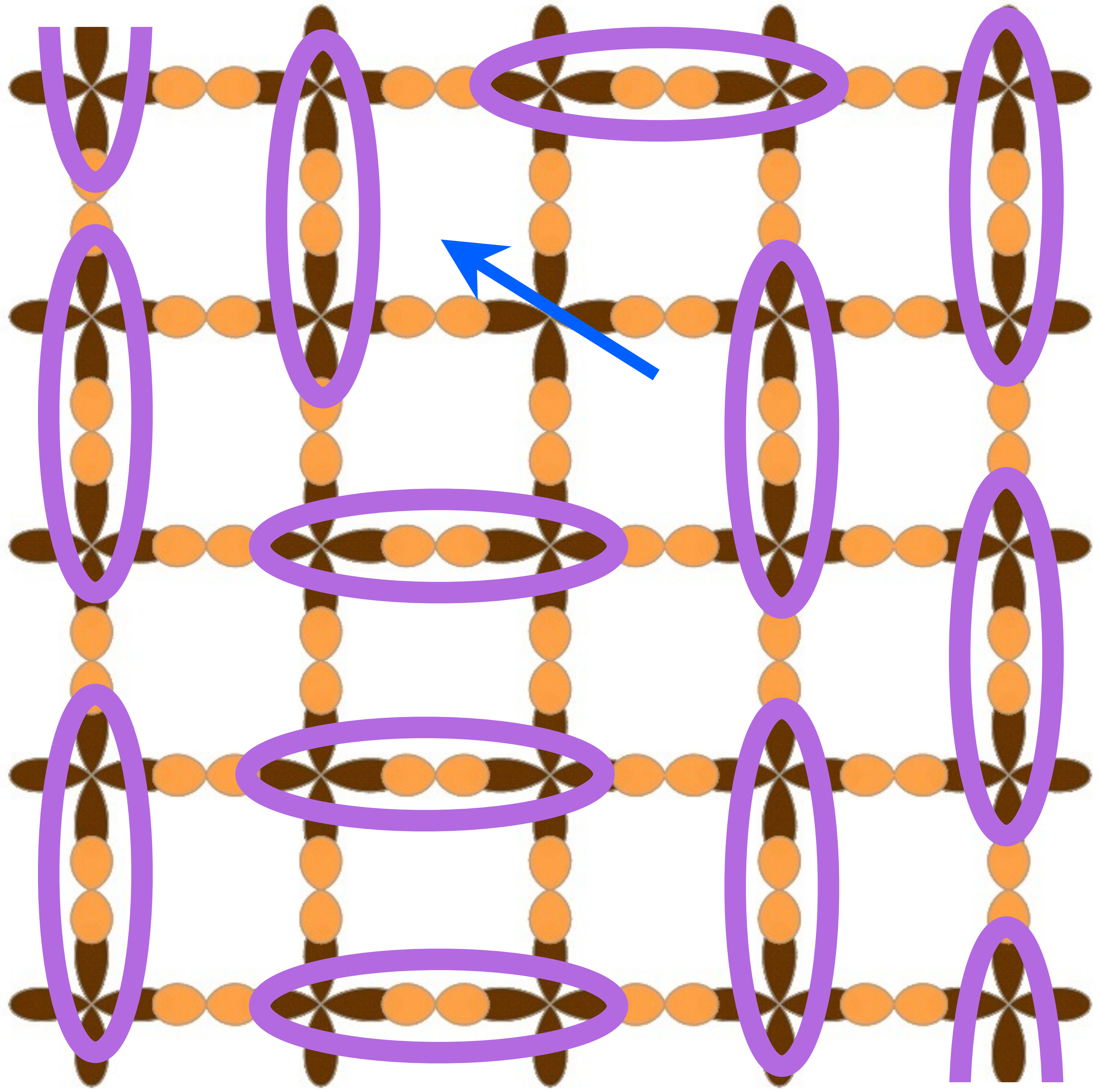

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$$YBa_2Cu_3O_{6+x}$$

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Quantum entanglement of mobile fermions

1. Fractionalized Fermi liquids (FL*)

   - Entanglement of a (critical or gapped) quantum spin liquid coexisting with electronic Landau quasiparticles. Charge is carried by ordinary electrons, but there are also fractionalized (anyonic) spin excitations.
   - Applies to cuprate pseudogap (predictions consistent with recently observed Yamaji effect)

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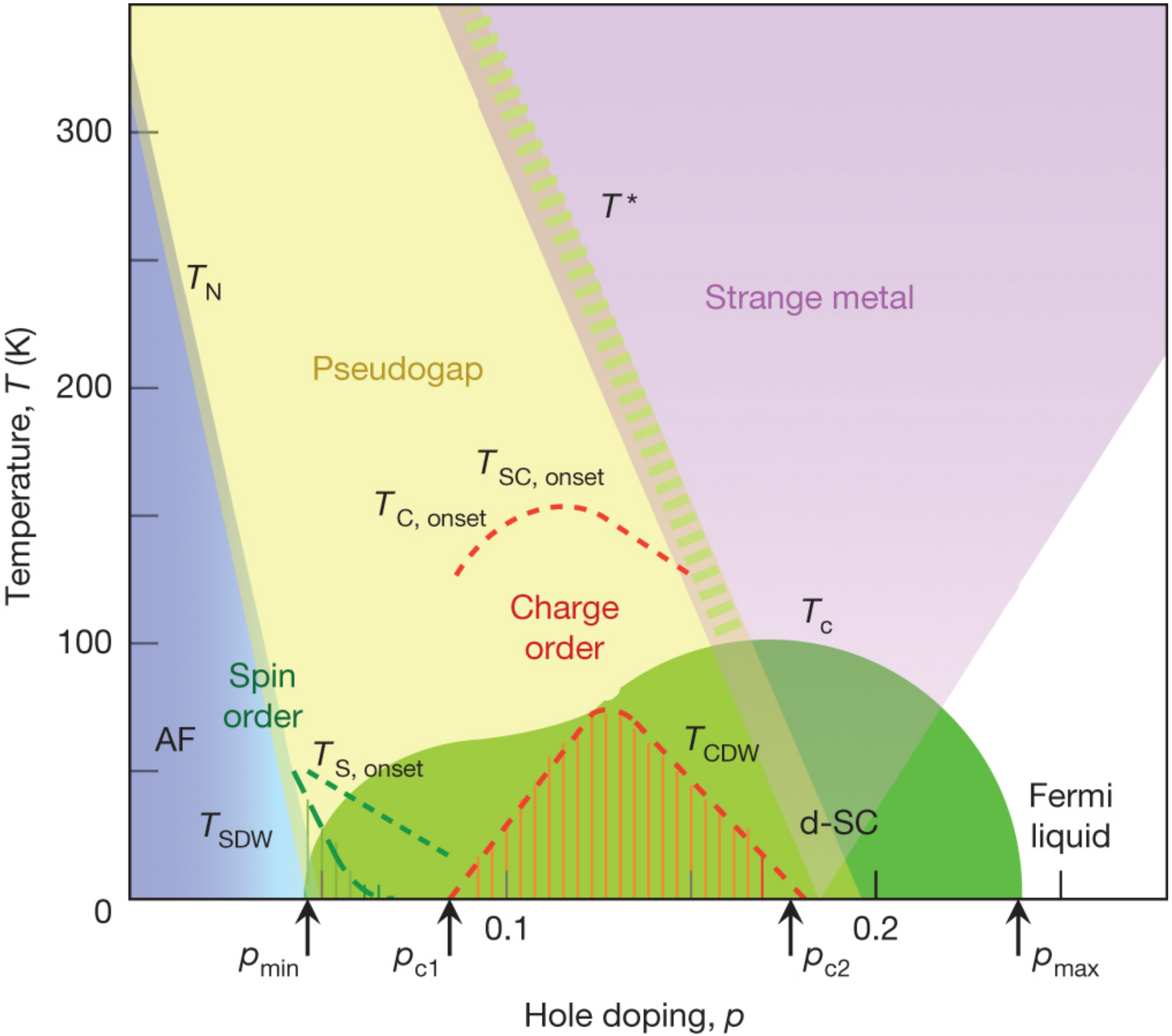
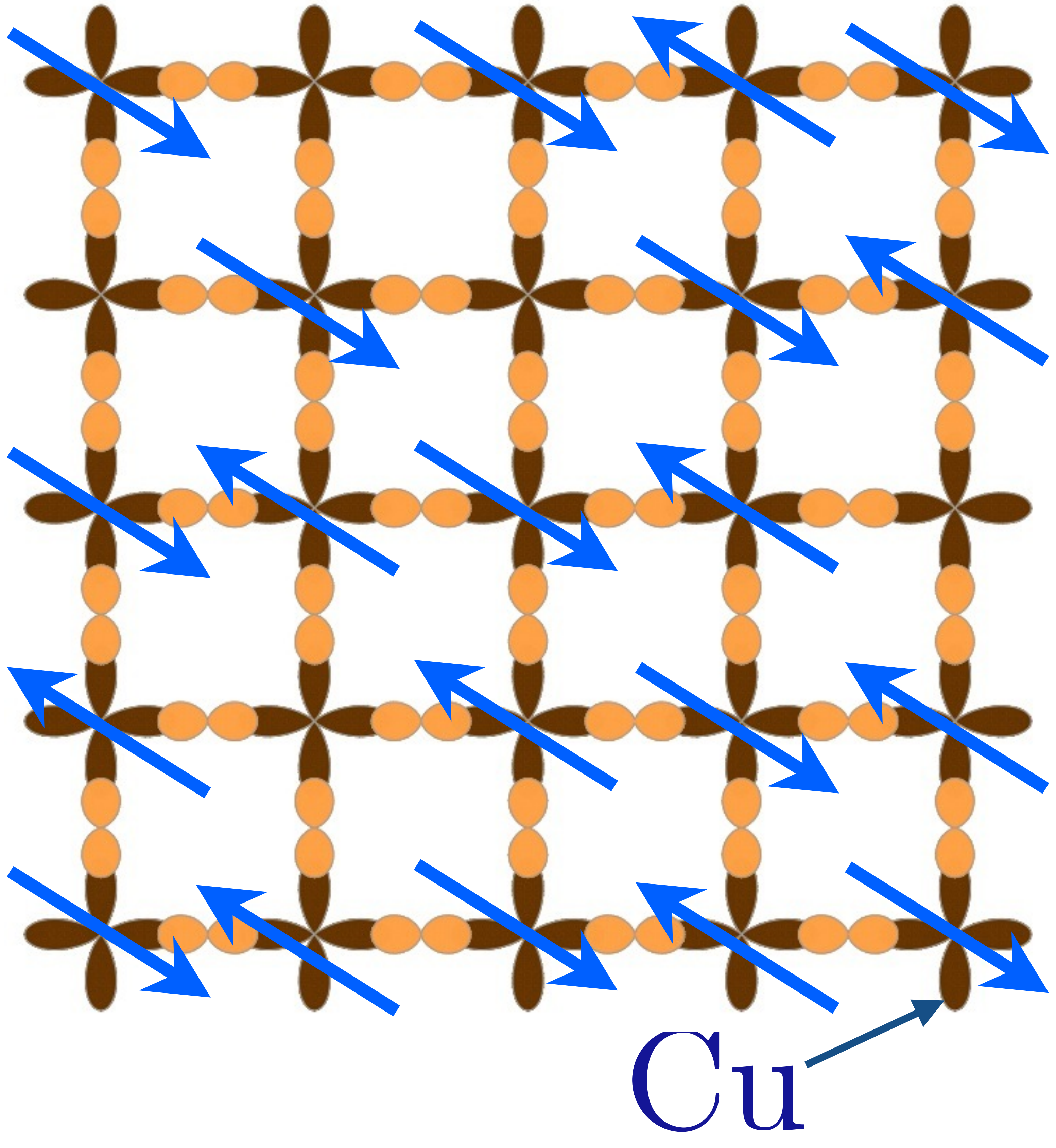
2. Sachdev-Ye-Kitaev (SYK) liquids

    - Complex entanglement of a compressible state with no quasiparticles.
    - Universal theory of strange metals (and generic charged black holes in asymptotically flat 3+1 dimensional space).

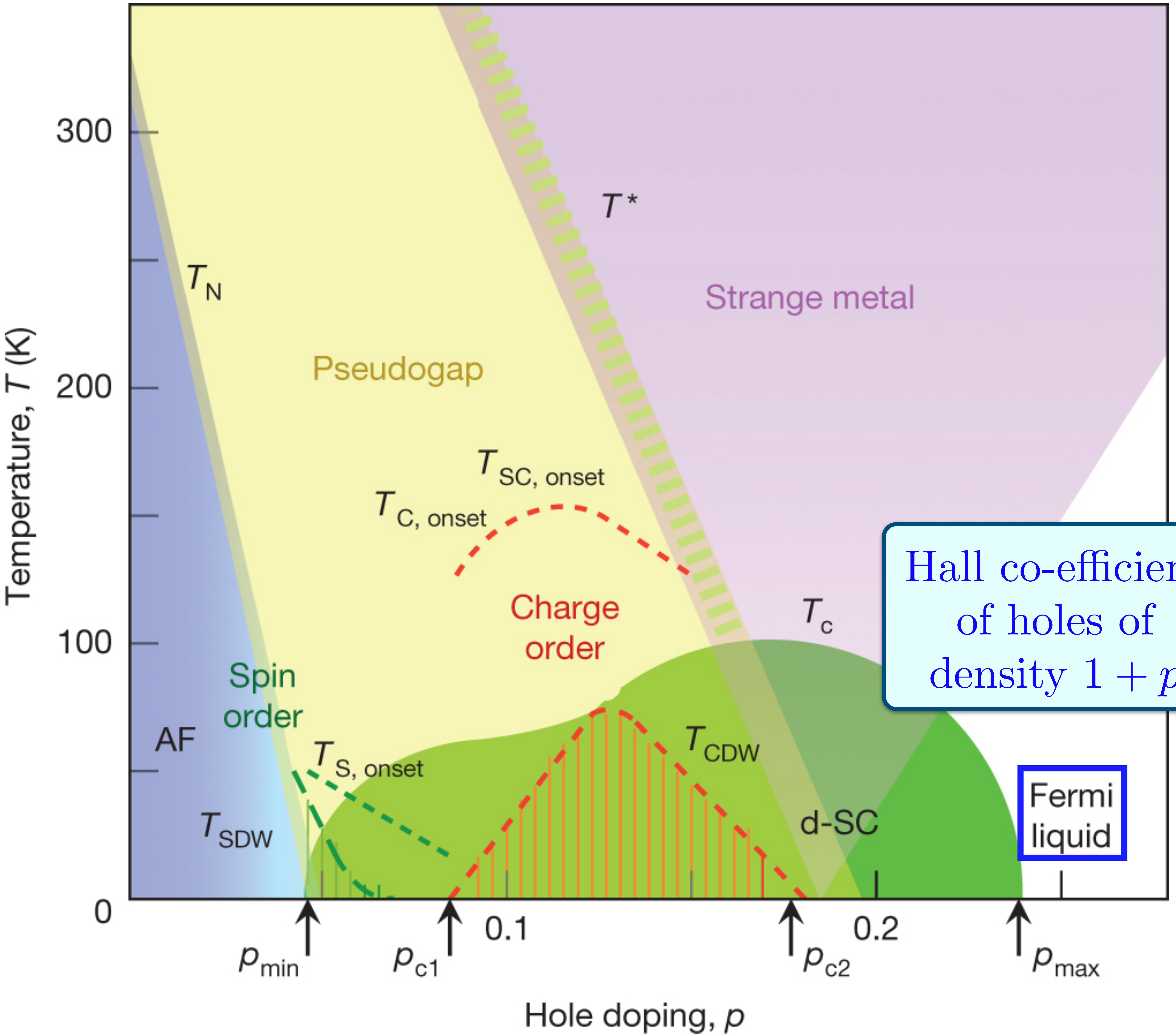
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)





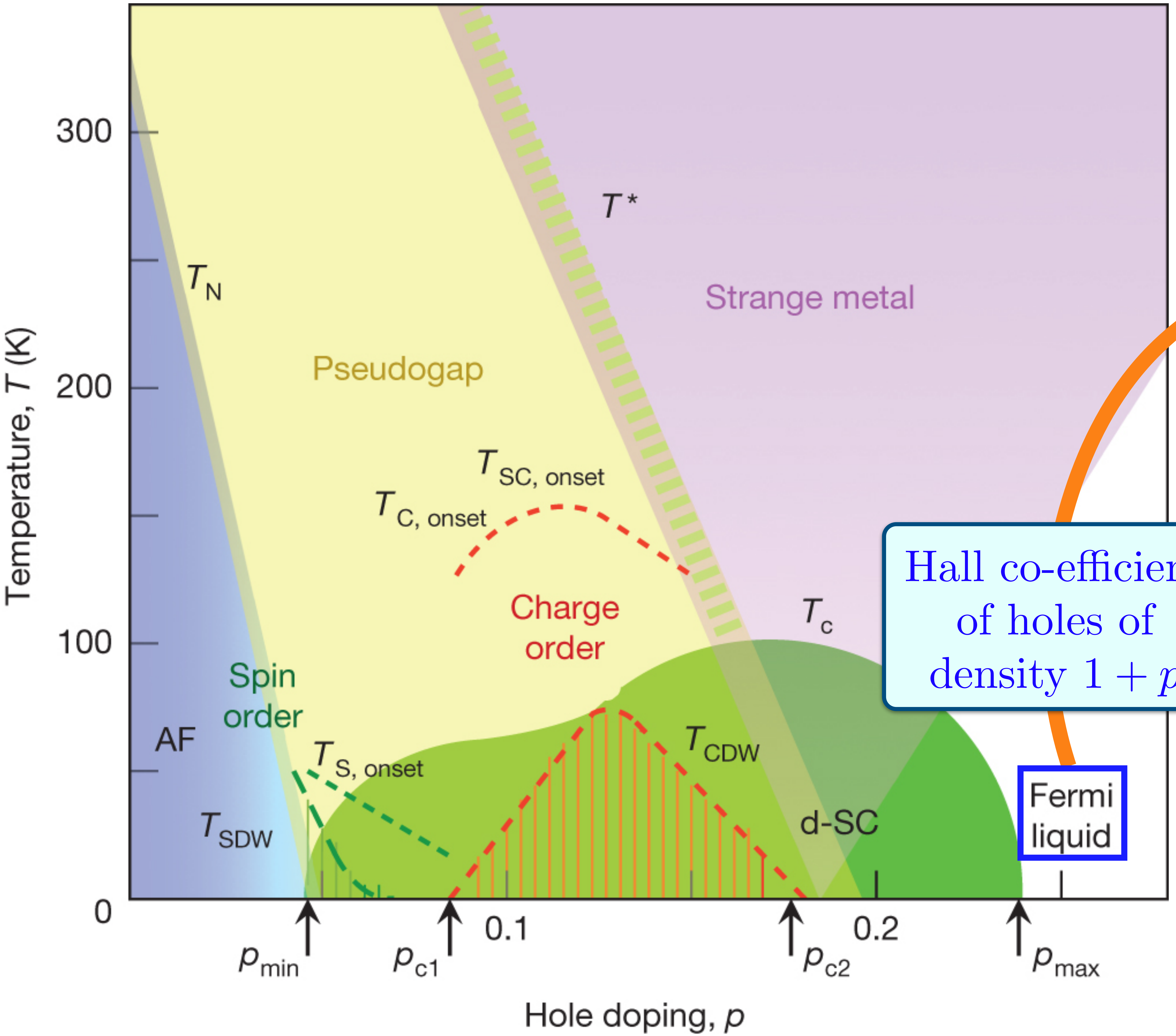
d-SC obtained upon doping AF with density p holes. Hole density relative to the filled band $\rho = 1 + p$. Electron density relative to the empty band $\rho_e = 1 - p$.

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

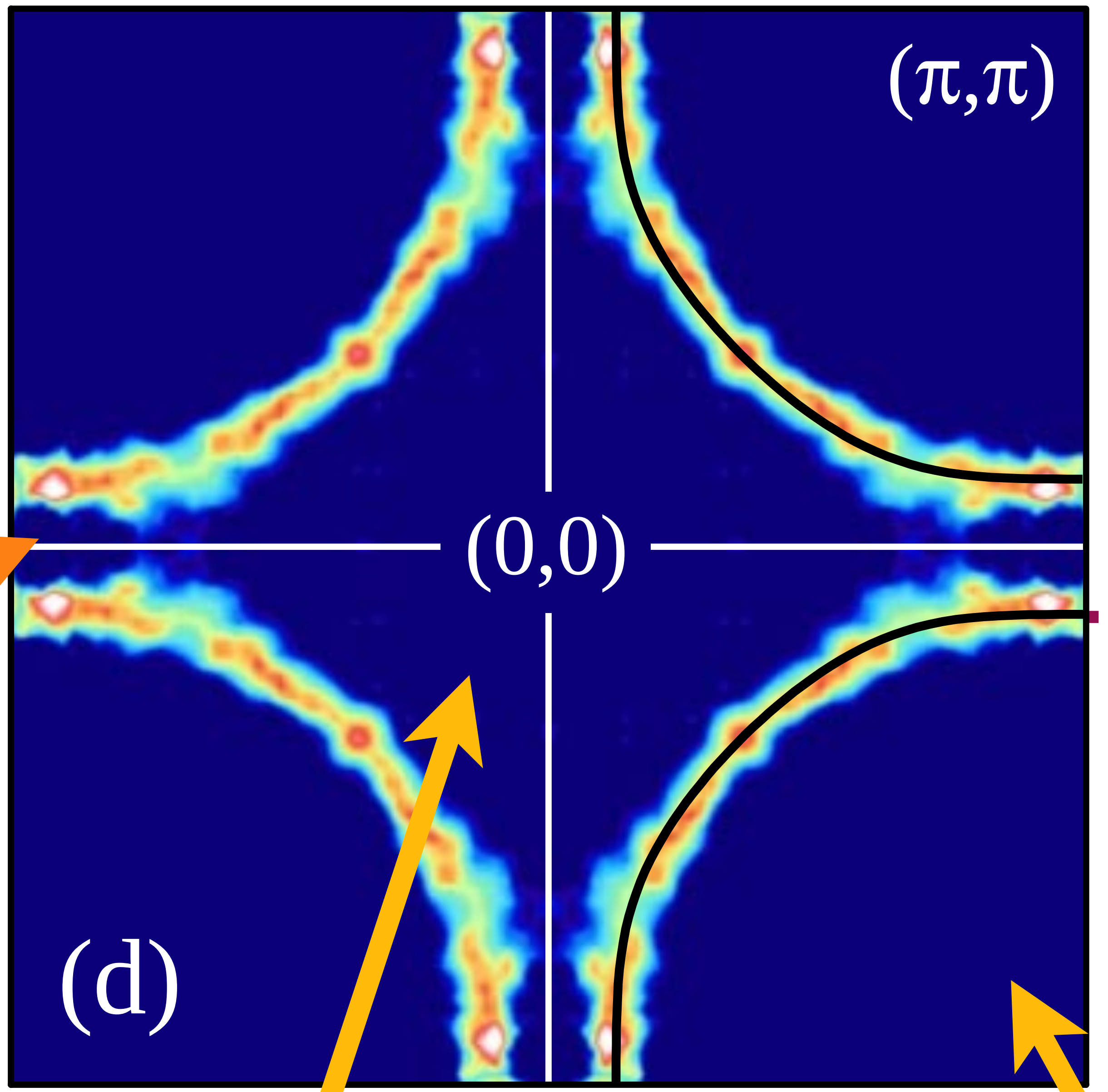


Hall co-efficient of holes of density $1 + p$

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



M. Platé *et al.*, PRL **95**, 077001 (2005)



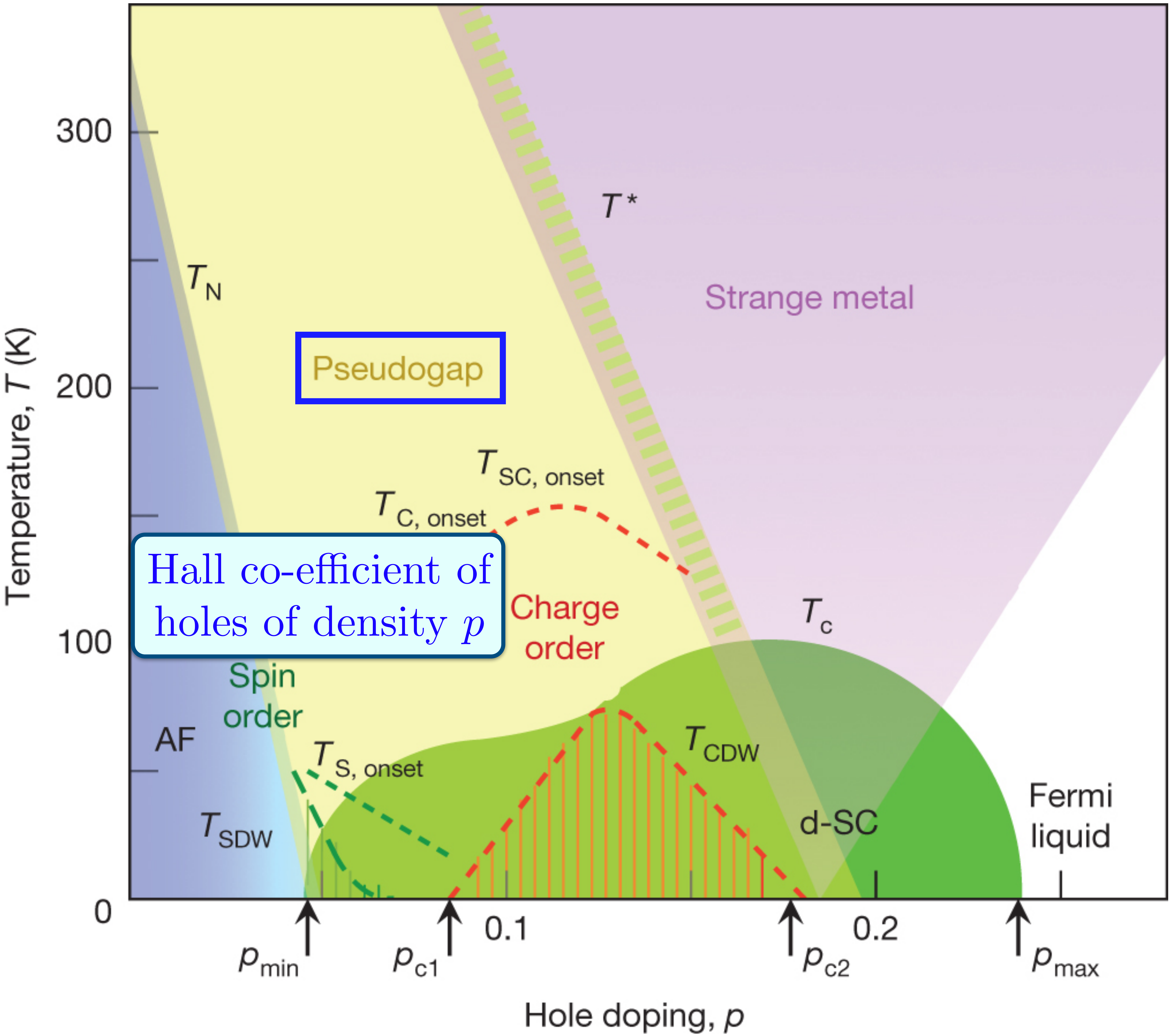
Hall co-efficient of holes of density $1 + p$

1-p electrons

1+p holes

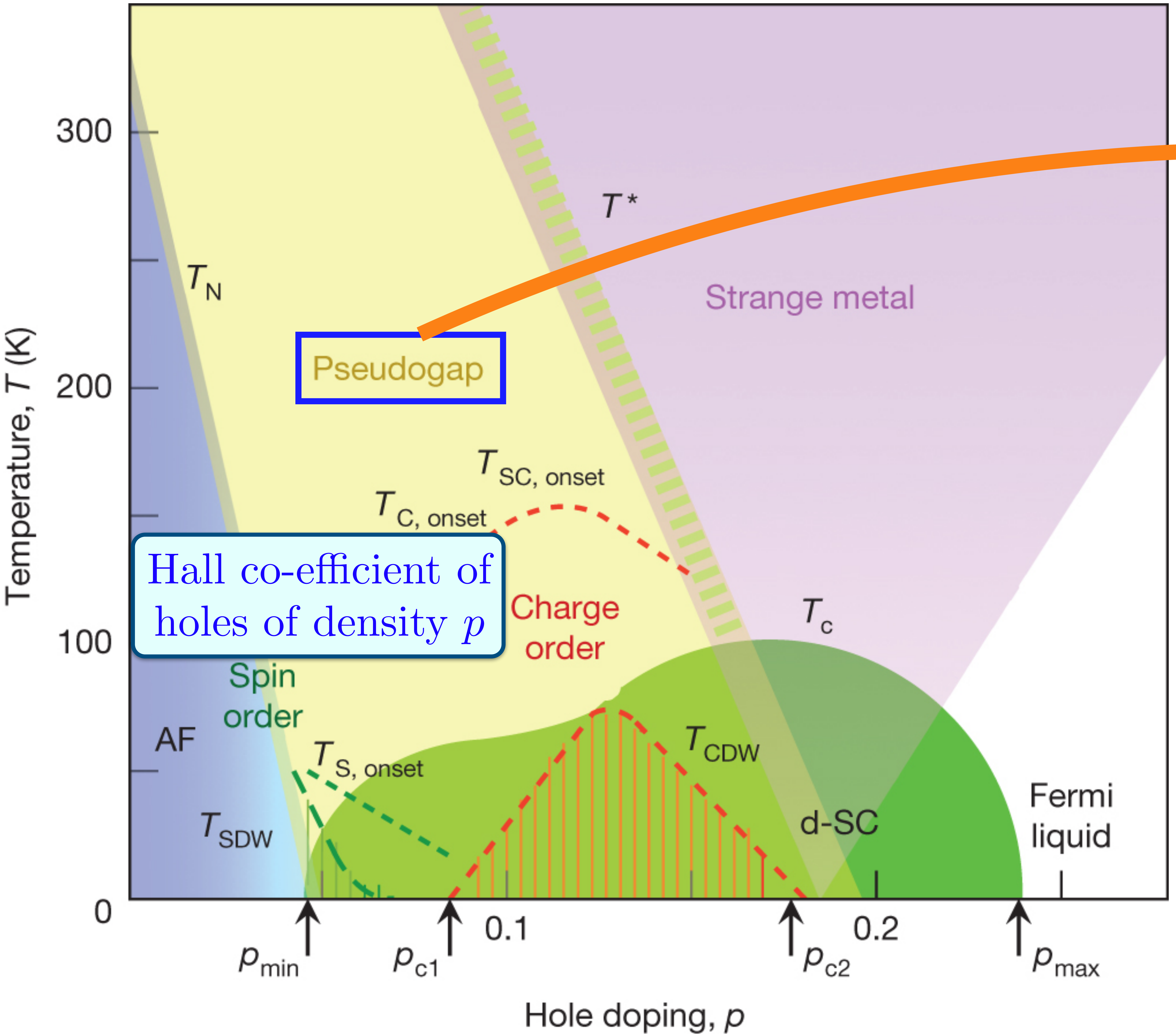
Luttinger, 1960: Area enclosed by the Fermi surface is the same as that for free fermions *with the same symmetry.*

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

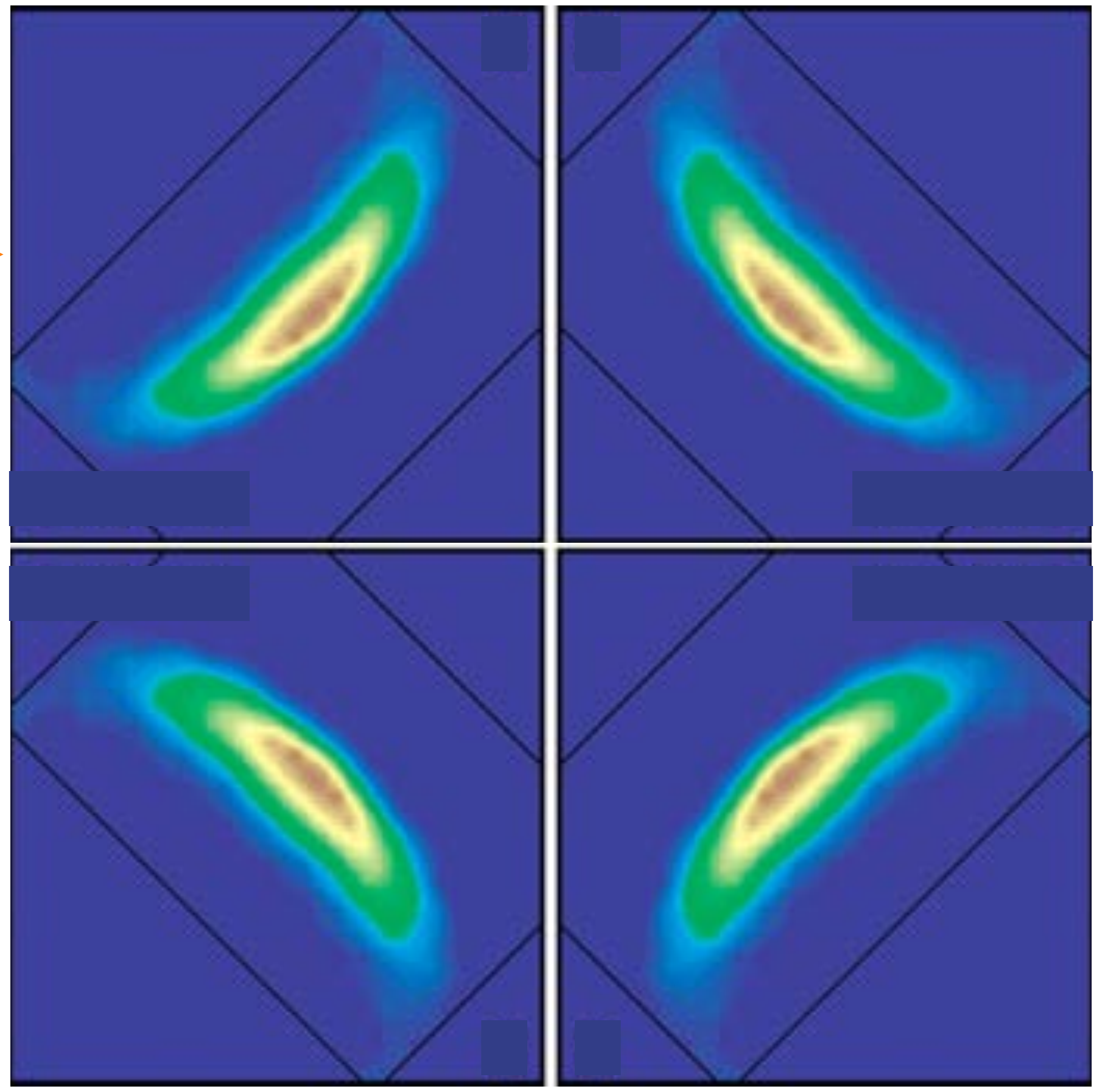


But there is no antiferromagnetic order to justify carrier density p

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

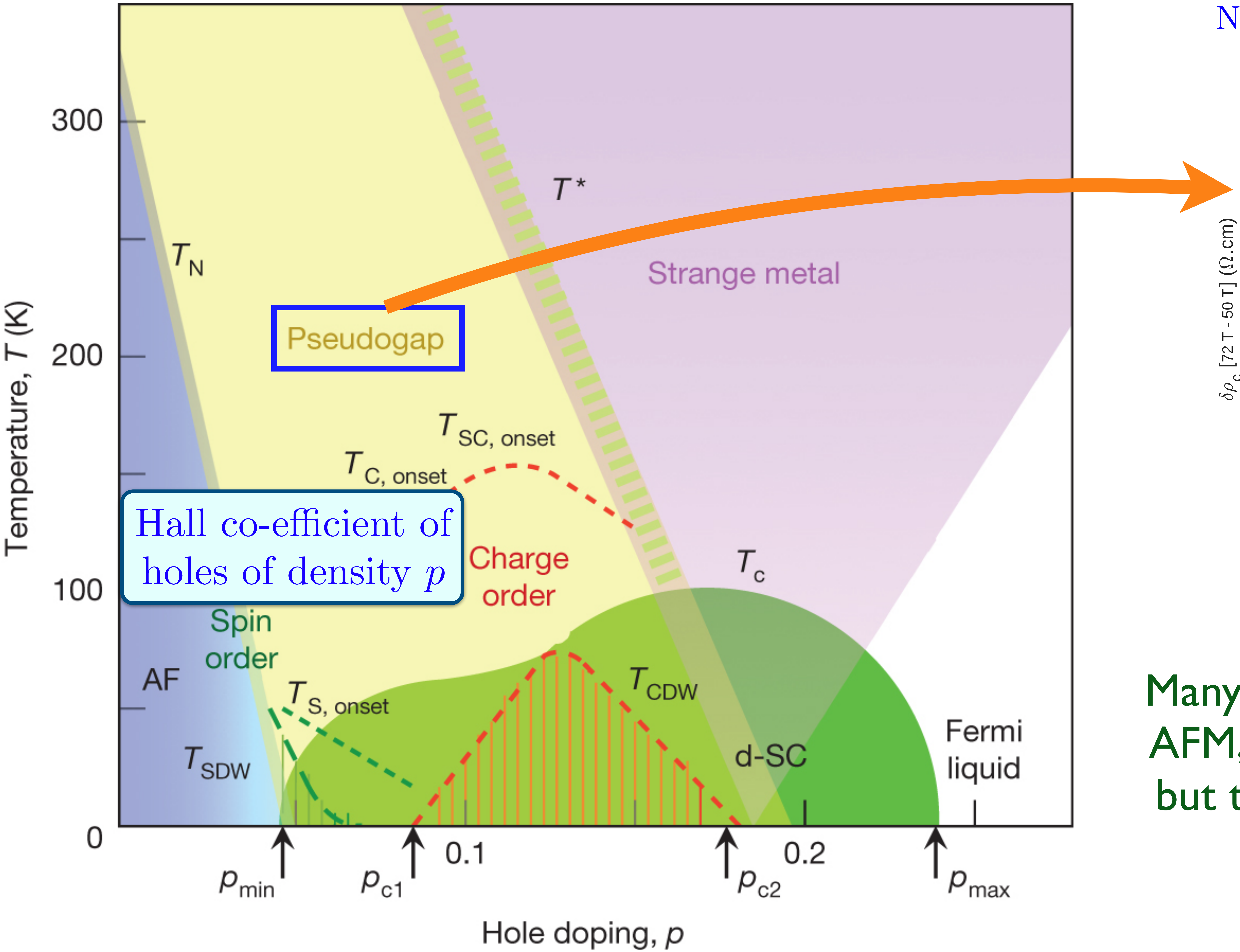


Kyle M. Shen *et al.*, Science **307**, 901 (2005)

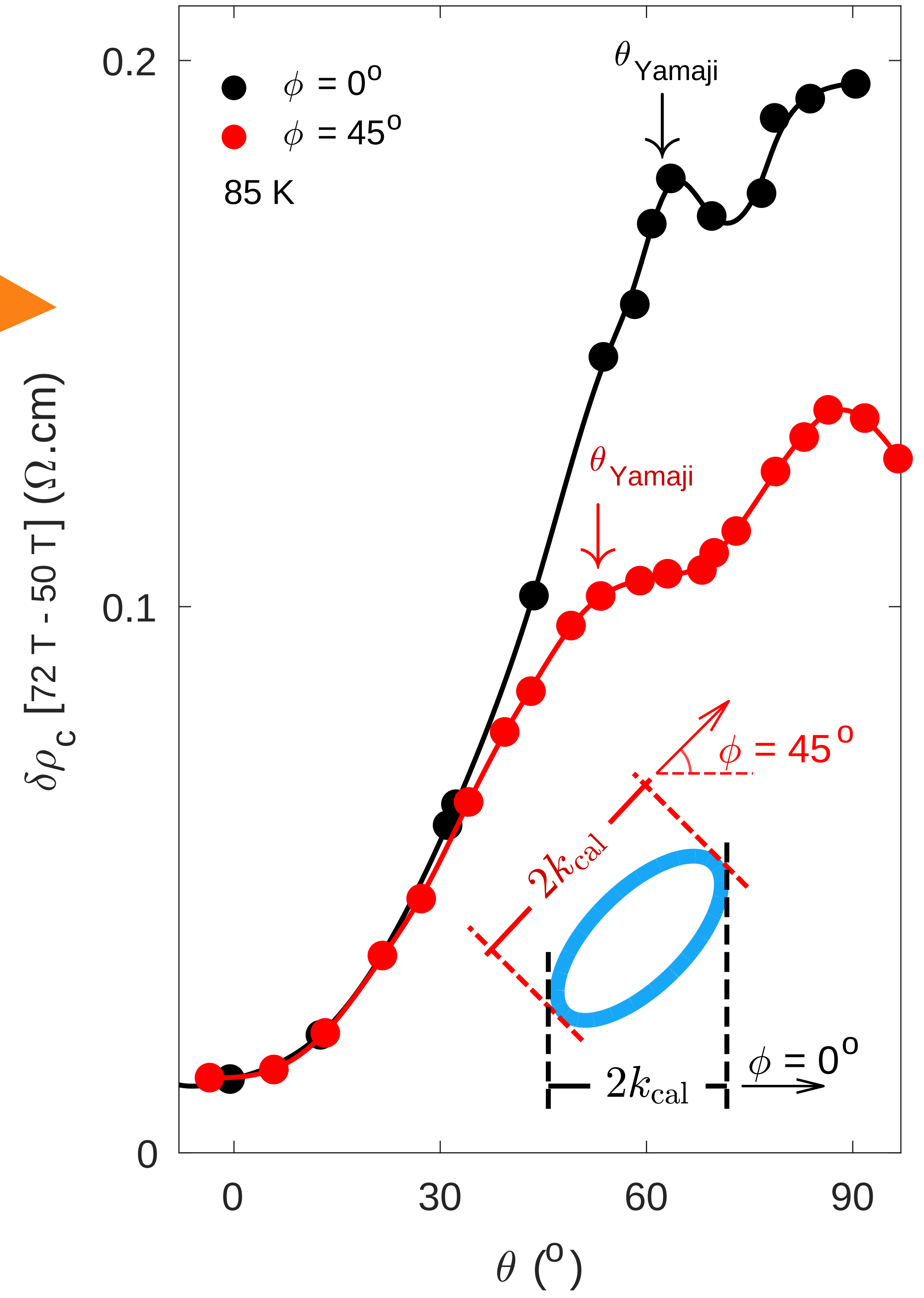


Many theories with fluctuating AFM, d-SC and charge orders:

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

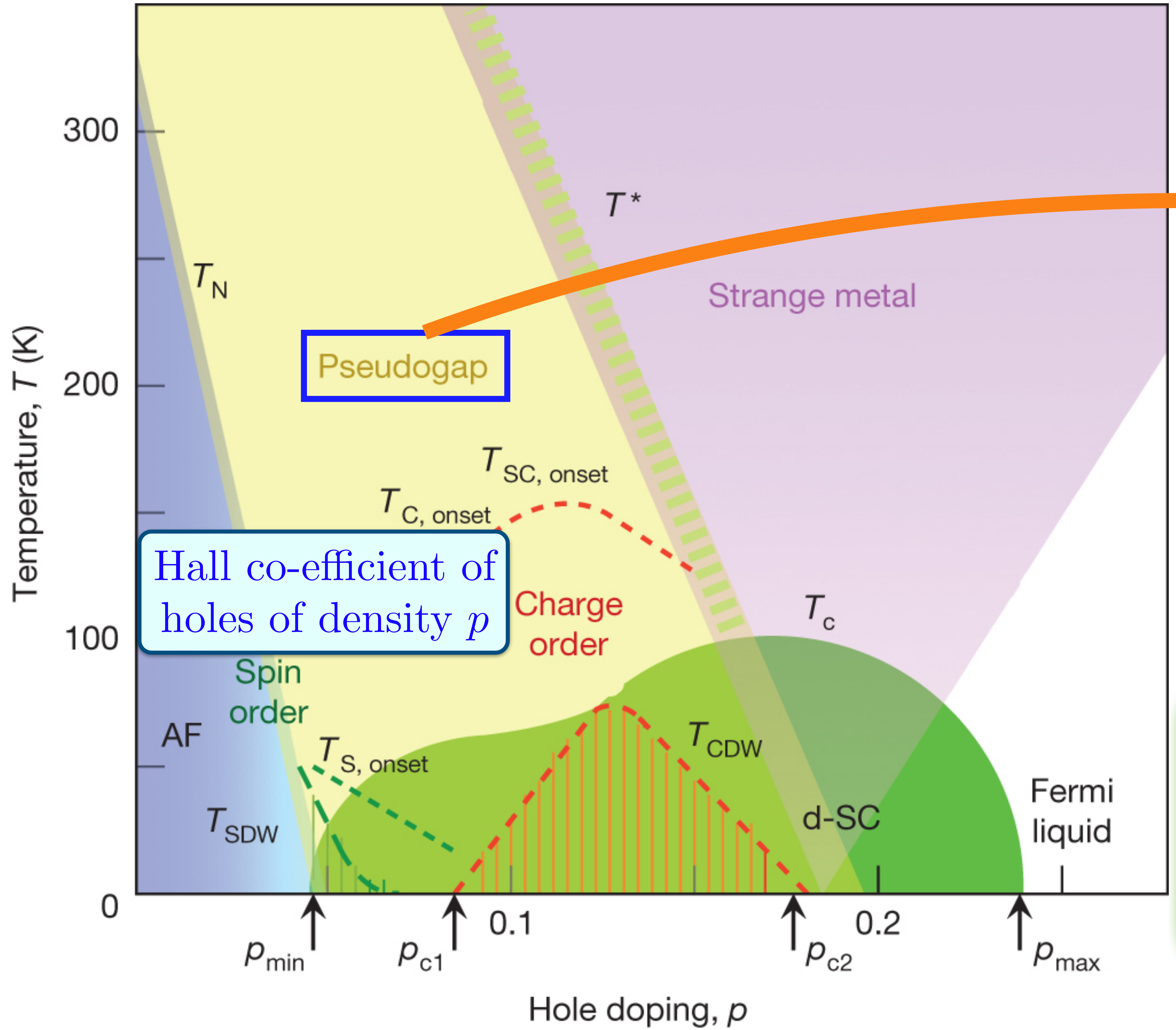


Mun K. Chan *et al.*, Nature Physics **21**, 1753 (2025)



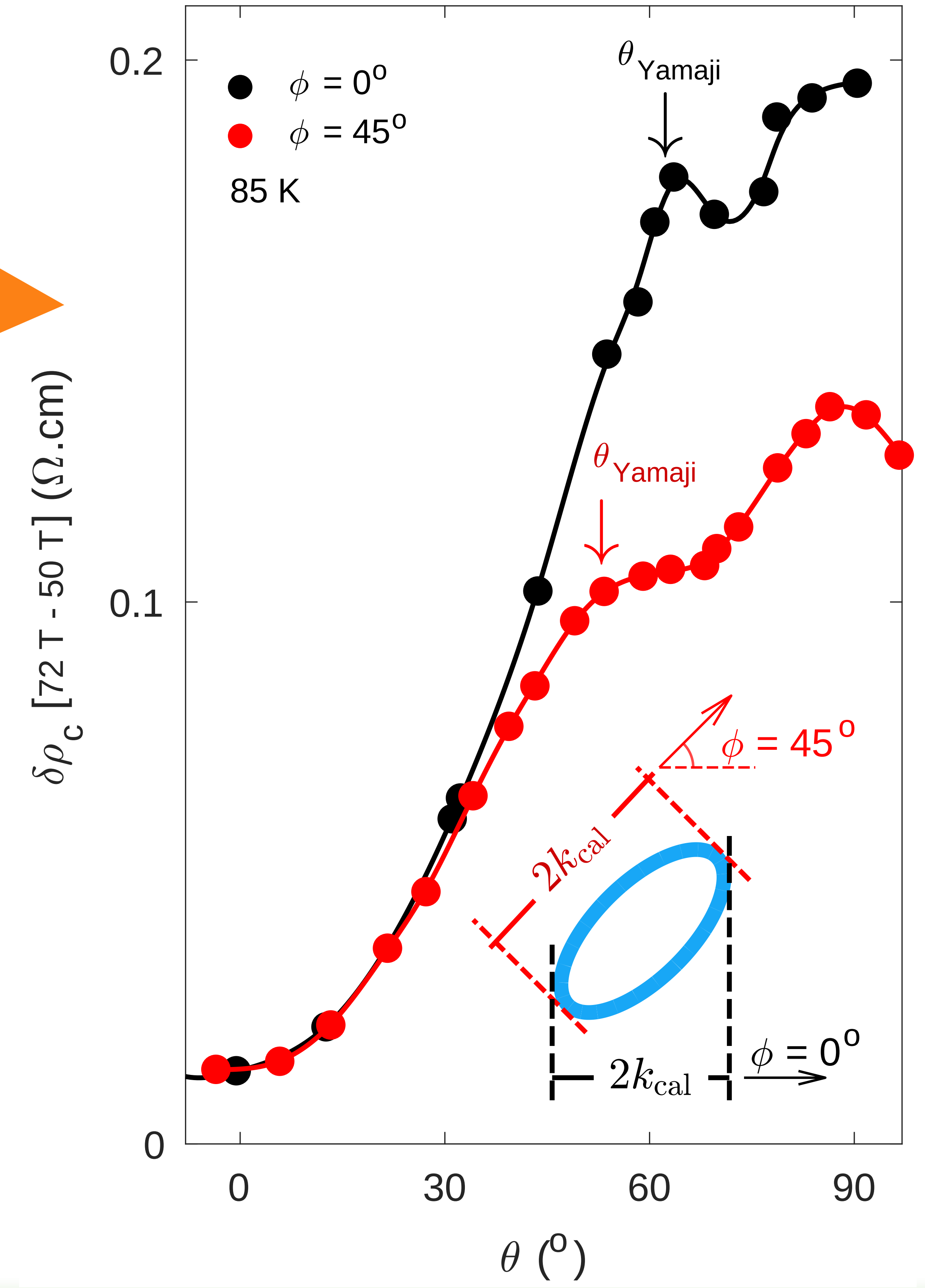
Many theories with fluctuating AFM, d-SC and charge orders: but they have difficulties with the Yamaji effect

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Hall co-efficient of holes of density p

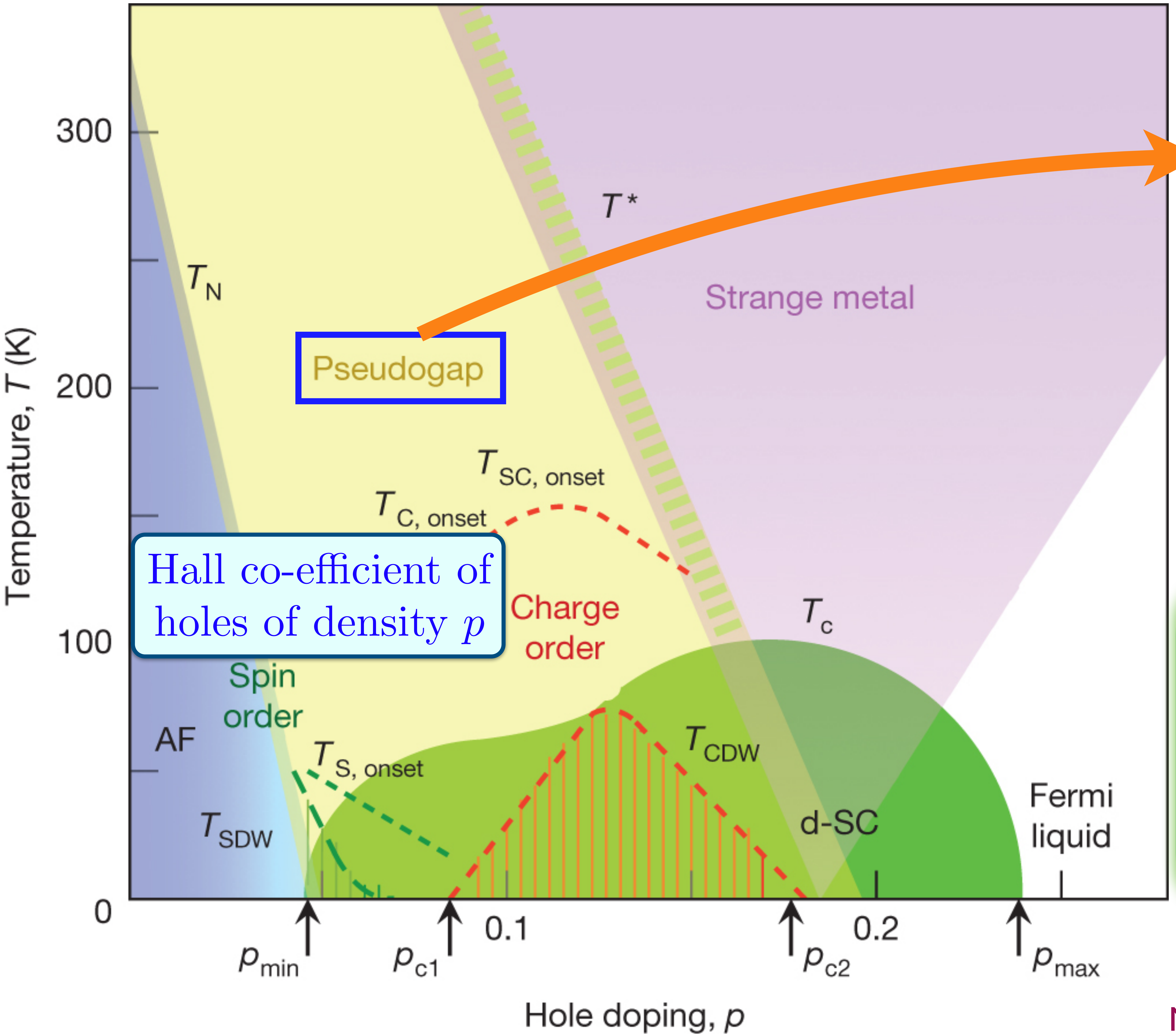
Mun K. Chan *et al.*, Nature Physics **21**, 1753 (2025)



Will explain with a Fractionalized Fermi Liquid (FL*), a novel quantum metal with *no broken symmetry*: evades the Luttinger constraint, but satisfies the Oshikawa anomaly

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003)

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



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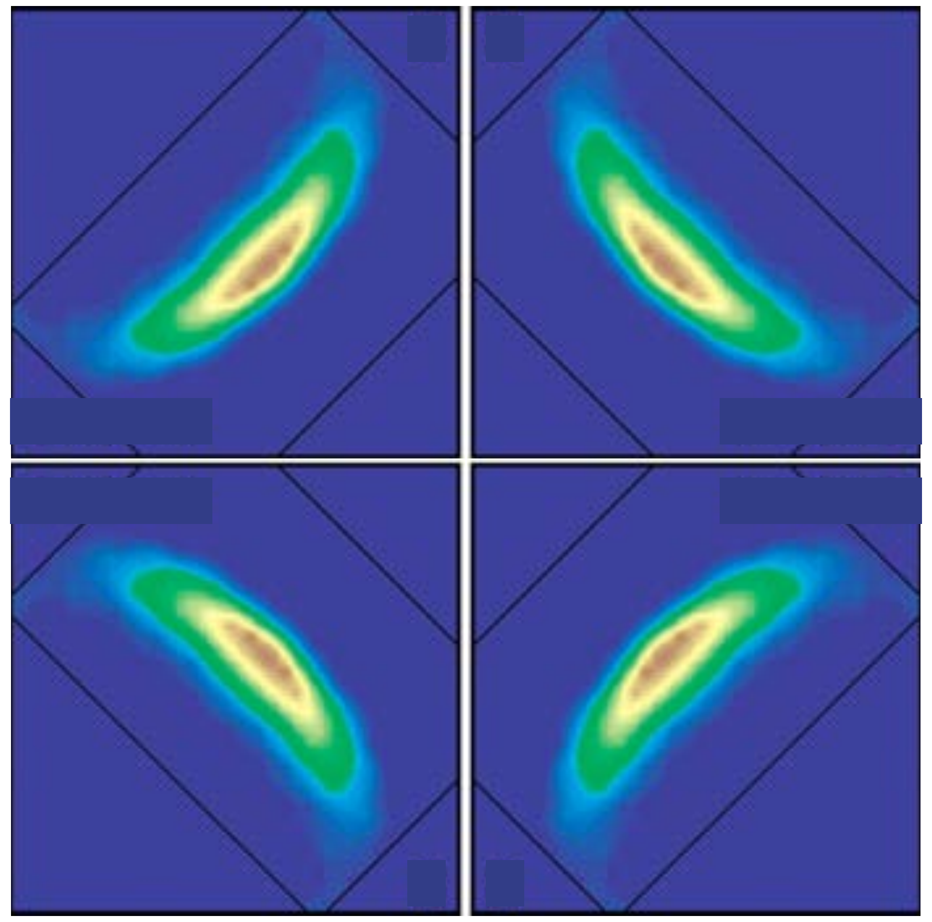
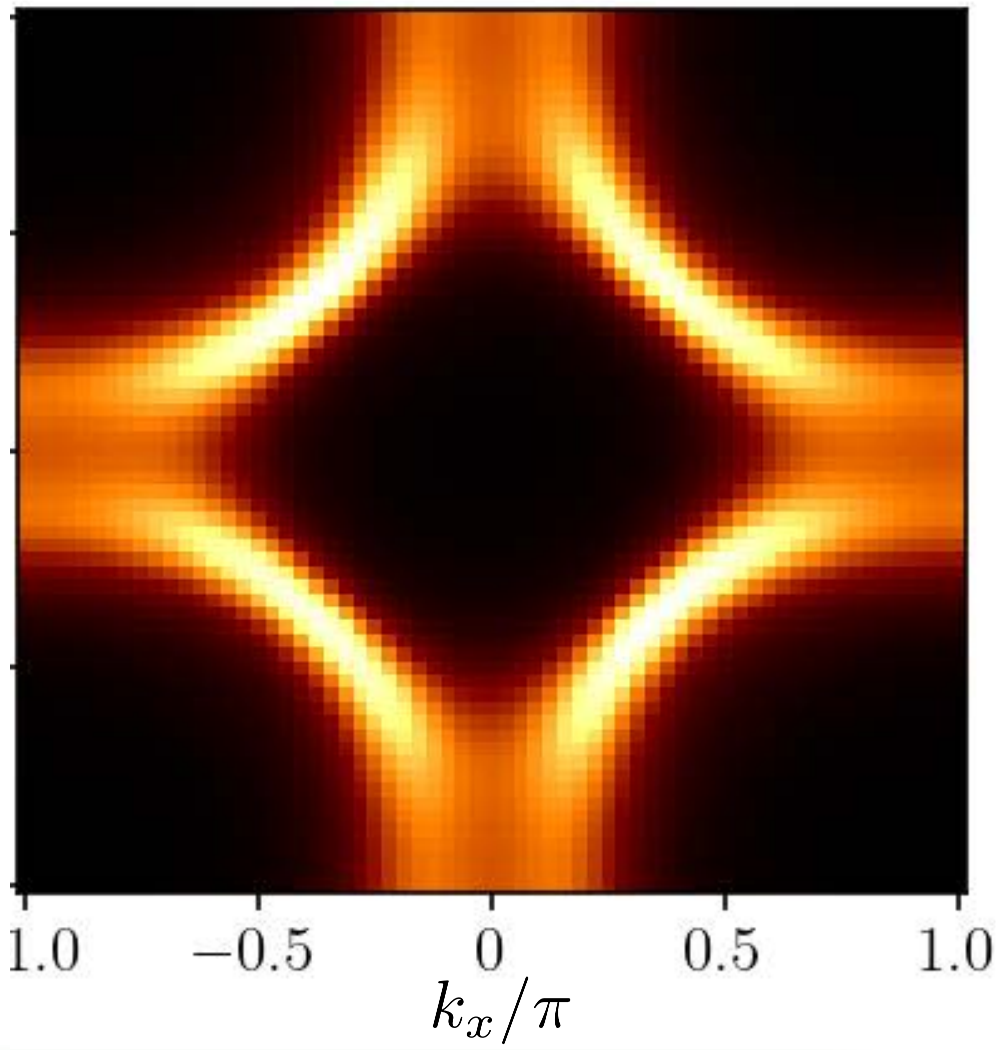


H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336



Fluctuating orders appear as composites of a more fundamental fractionalized order parameter, *B*, which carries an emergent SU(2) gauge charge

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, S. S., PNAS **120,** e2302701120 (2023);

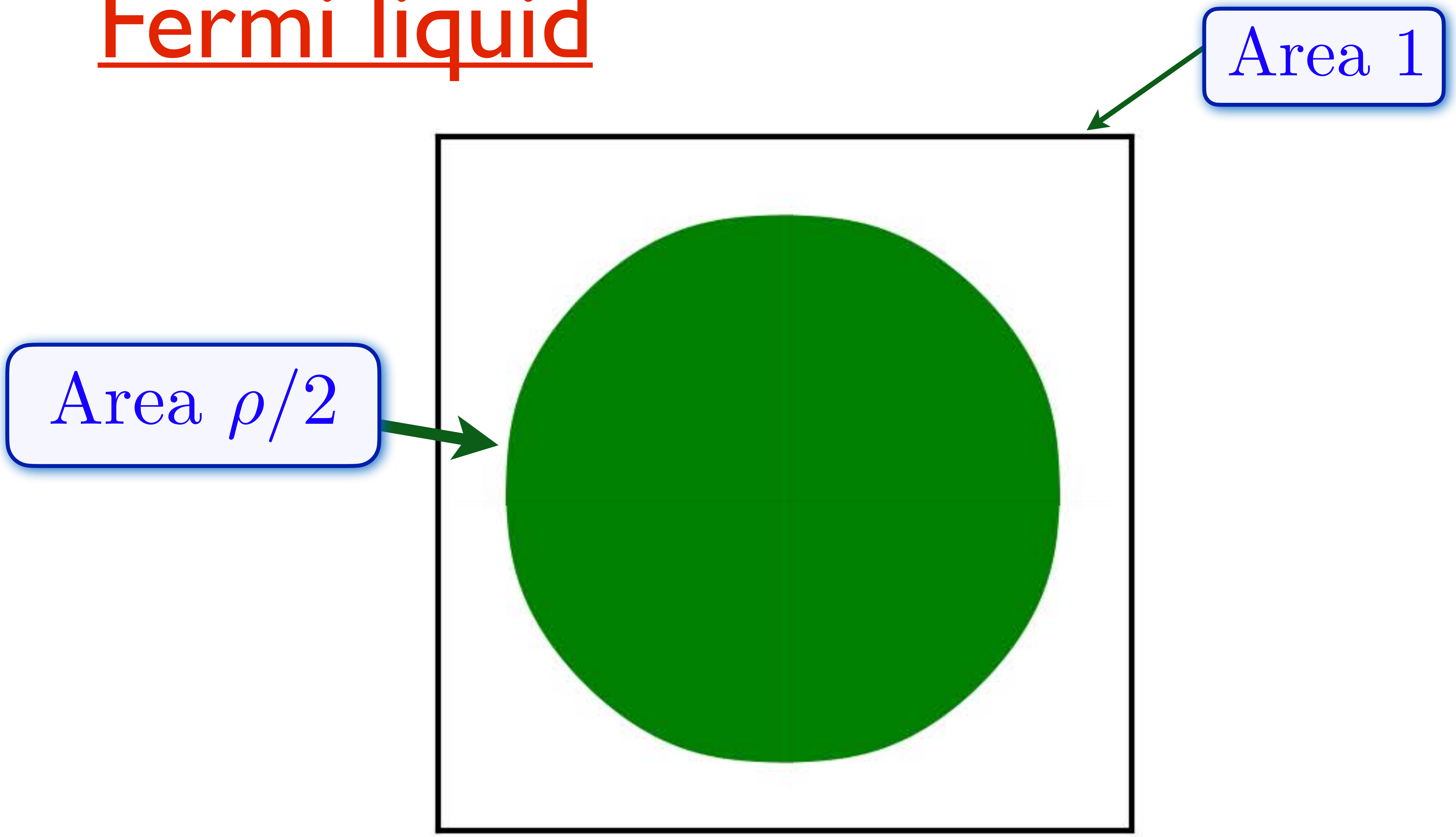
Fractionalized Fermi liquids (FL*)

Fermi liquid

Spin-1/2 holes of density

$\rho = 1 + p$

Positive Hall coefficient of carrier density ρ

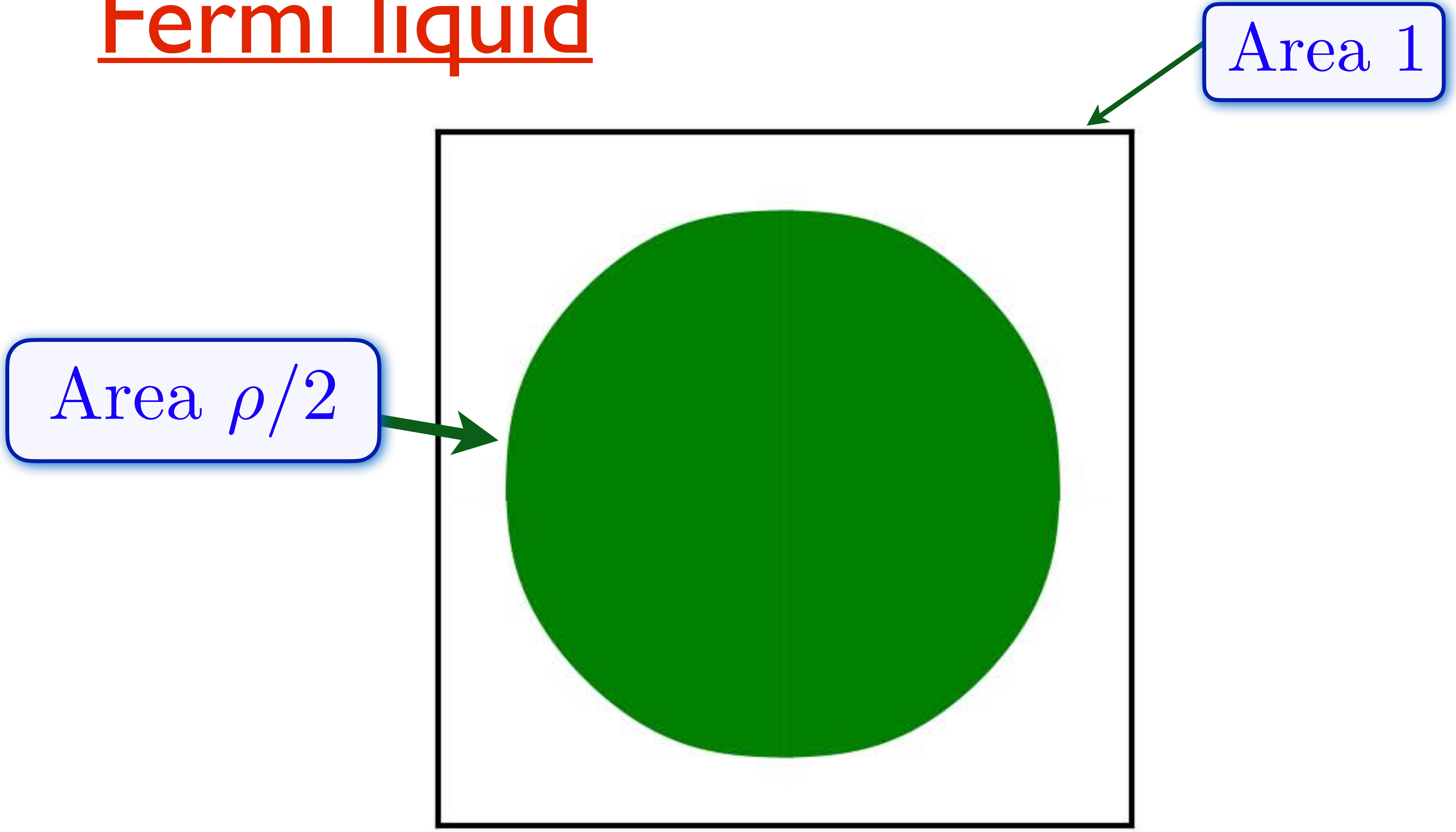


Luttinger, 1960: Area enclosed by the Fermi surface is the same as that for free fermions *with the same symmetry.*

Fermi liquid

Spin-1/2 holes of density
$$\rho = 1 + p$$

Positive Hall coefficient
of carrier density ρ



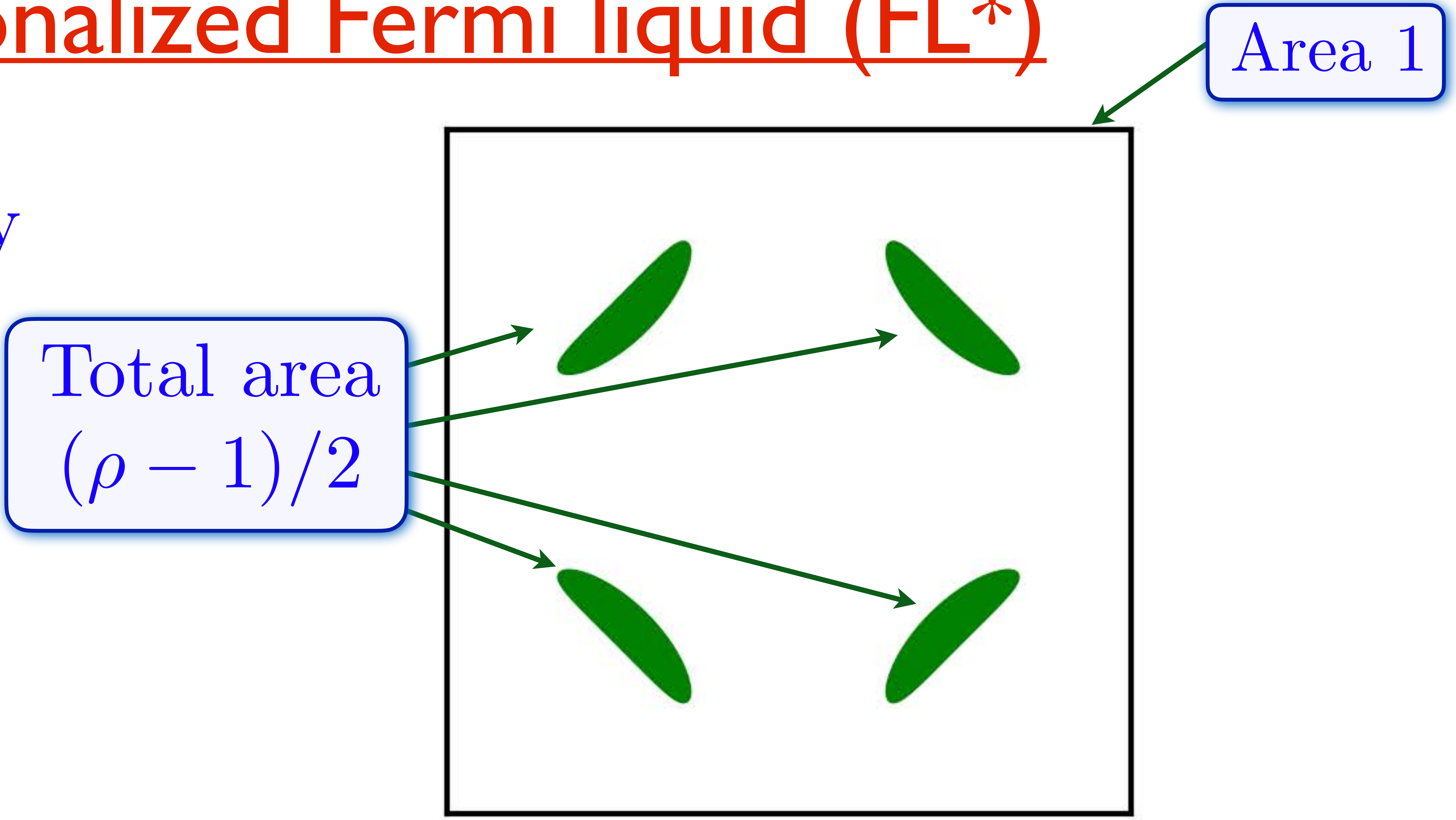
Luttinger, 1960: Area enclosed by the Fermi surface is the same as that for free fermions *with the same symmetry.*

Oshikawa, 2000: Area constrained by an anomaly-argument of global U(1) and translations

Fractionalized Fermi liquid (FL*)

Spin-1/2 holes of density
$$\rho = 1 + p$$

Positive Hall coefficient of carrier density $\rho - 1$



T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

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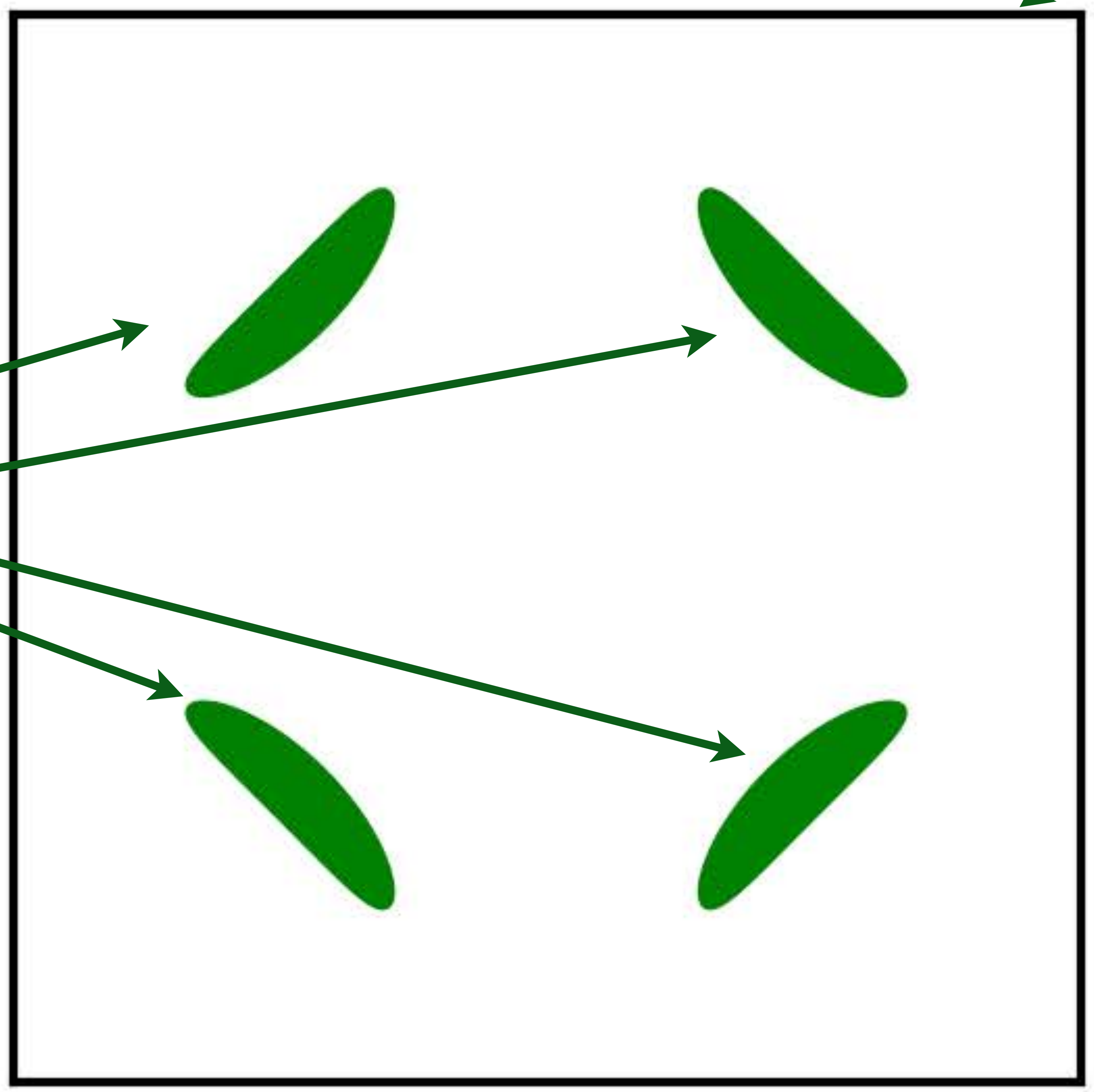

Oshikawa anomaly-argument is satisfied by
the sum of spin liquid (1) and
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T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

Fractionalized Fermi liquid (FL*)

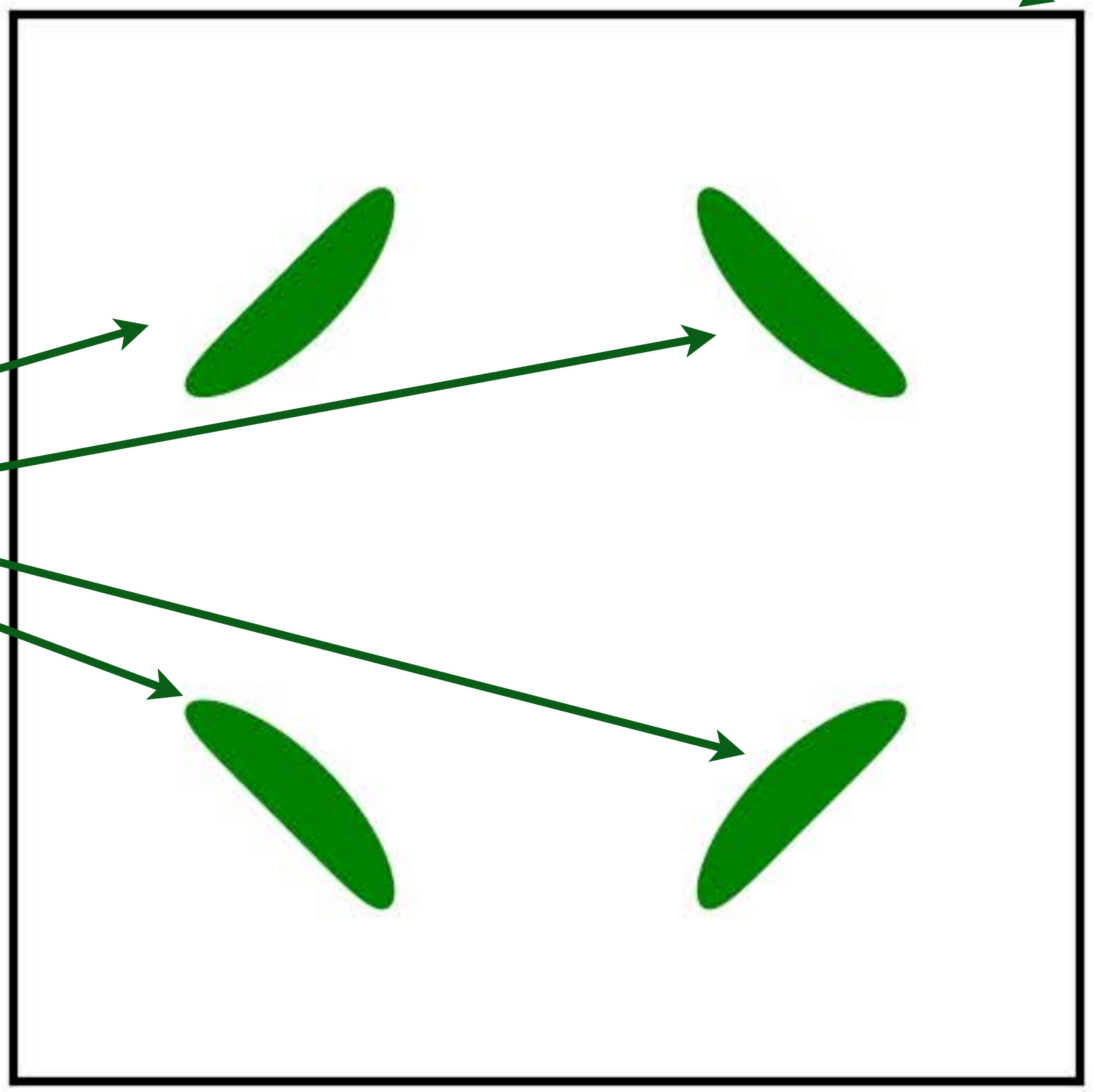
Spin-1/2 holes of density

$$\rho = 1 + p$$

Positive Hall coefficient of carrier density $\rho - 1$

Total area $(\rho - 1)/2$

Area 1



The density deficit (1) in the area is **quantized** by rigid structure of the spin liquid.

Oshikawa anomaly-argument is satisfied by the sum of spin liquid (1) and Fermi surface anomalies $(\rho - 1)$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

Fractionalized Fermi liquid (FL*)

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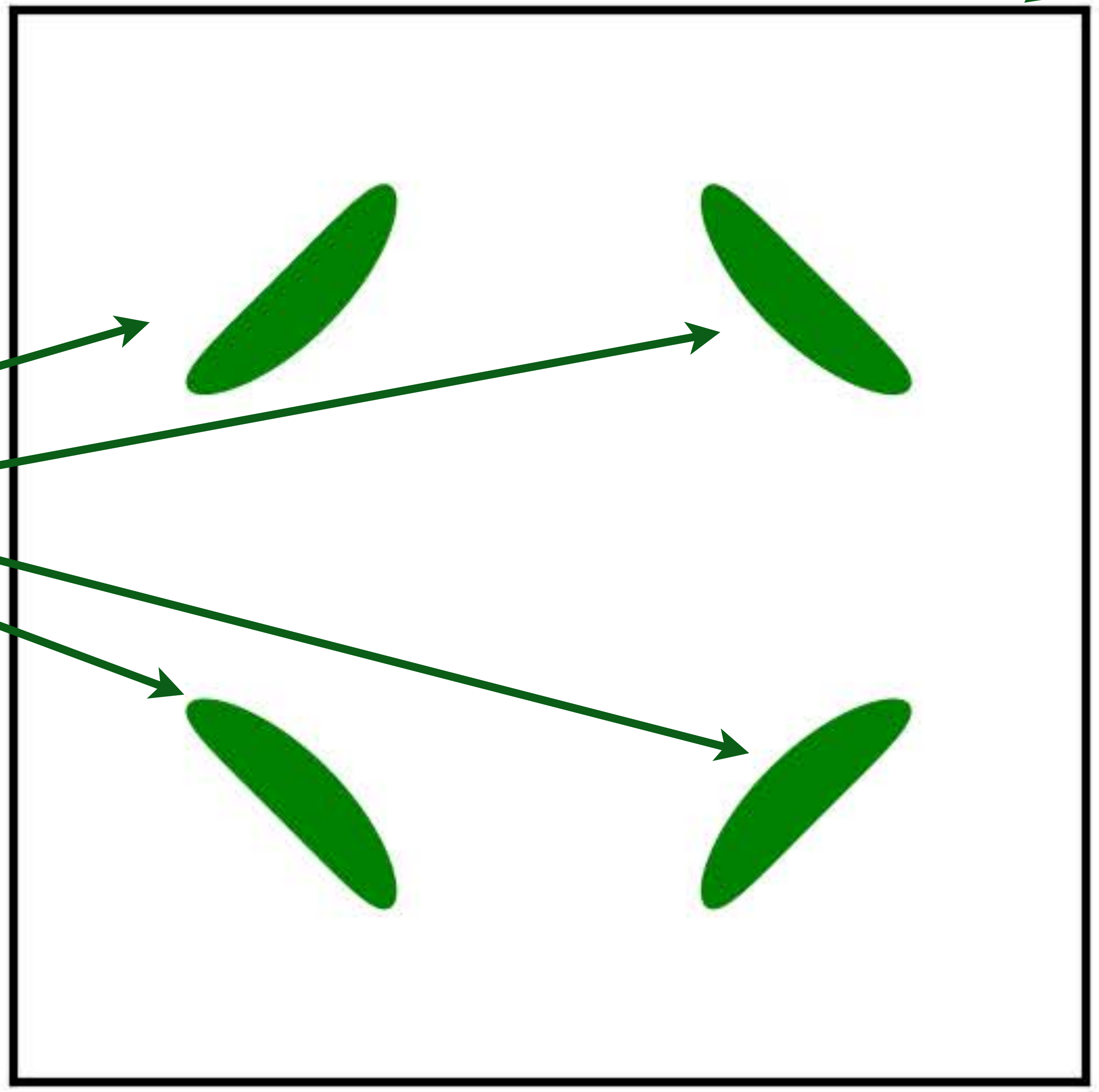
Positive Hall coefficient
of carrier density $\rho - 1$

Total area
$(\rho - 1)/2$

Area 1



Measuring non-Luttinger Fermi surface area is direct evidence for spin liquid quantum entanglement.

Oshikawa anomaly-argument is satisfied by
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T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

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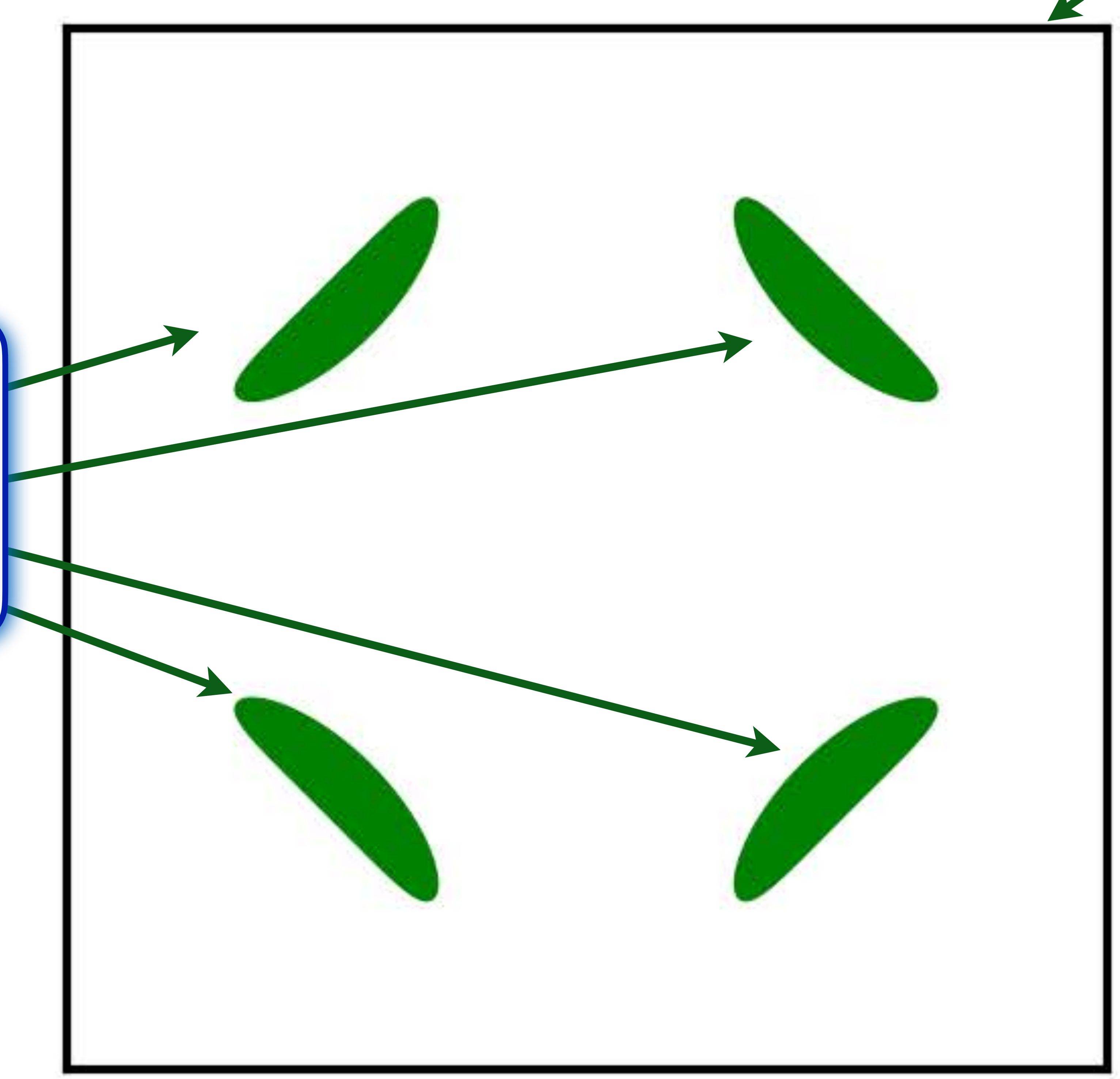

Area of
each
hole pocket
$= p/8$

Oshikawa anomaly-argument is satisfied by
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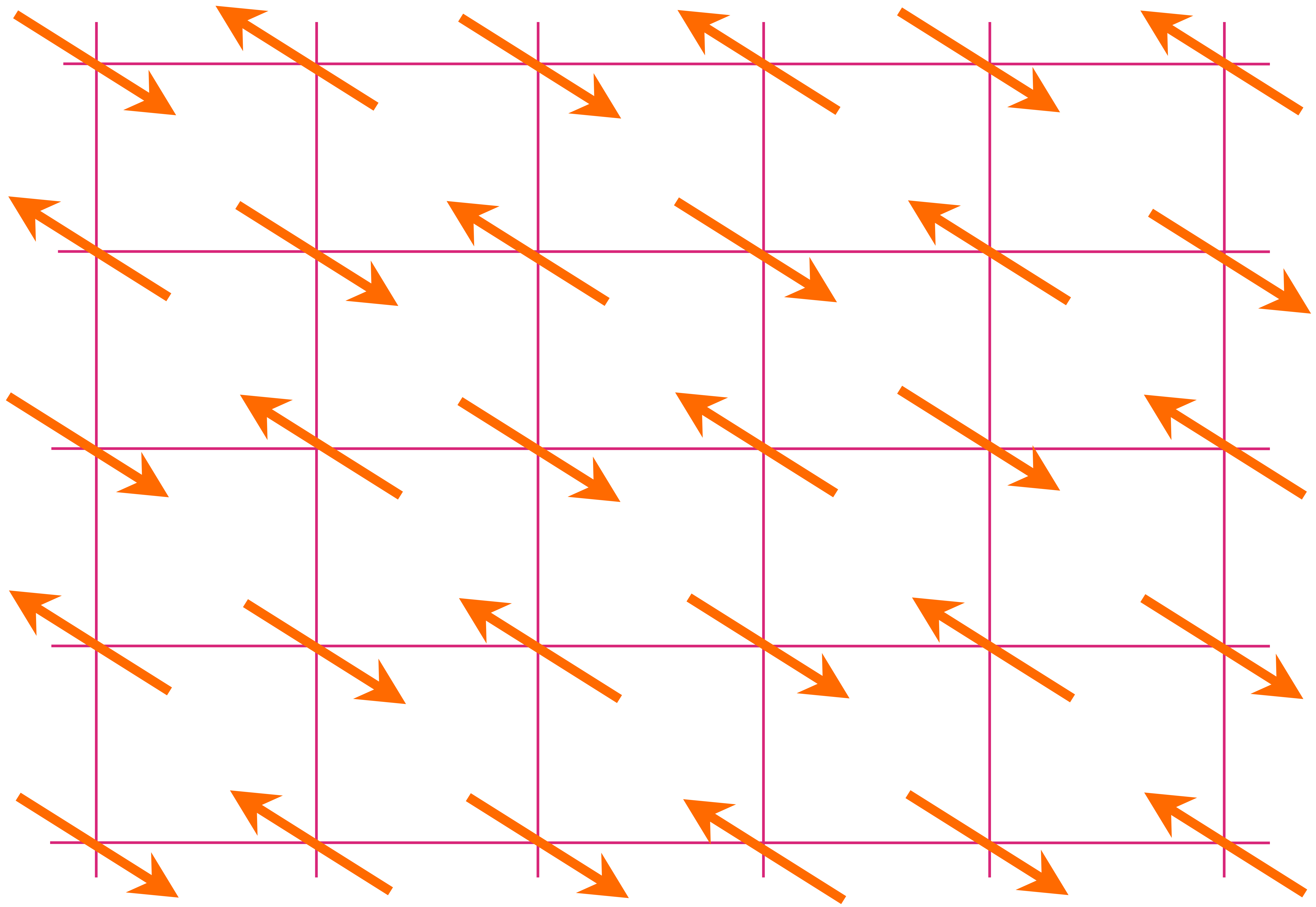
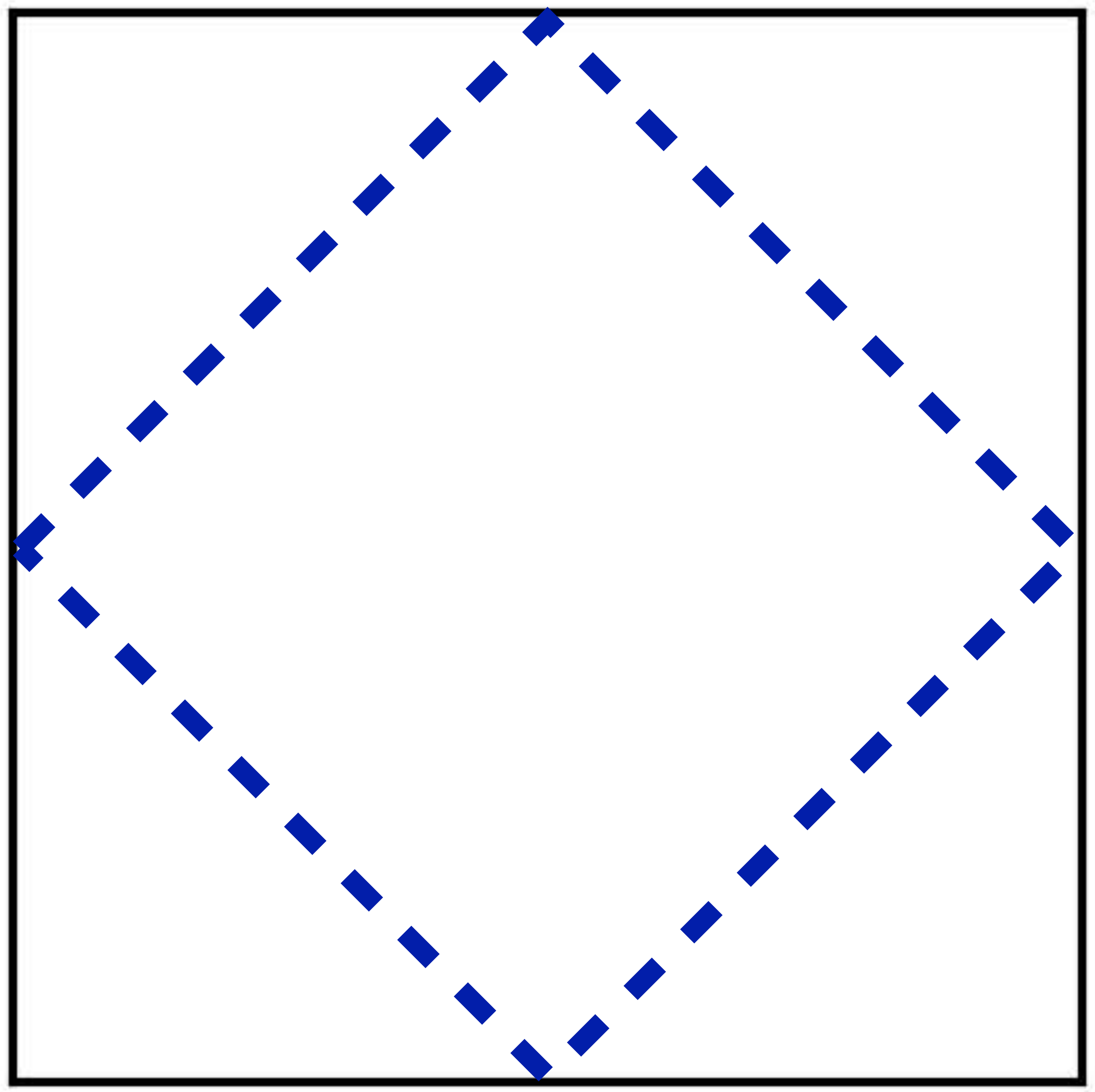
T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

Insulating antiferromagnet



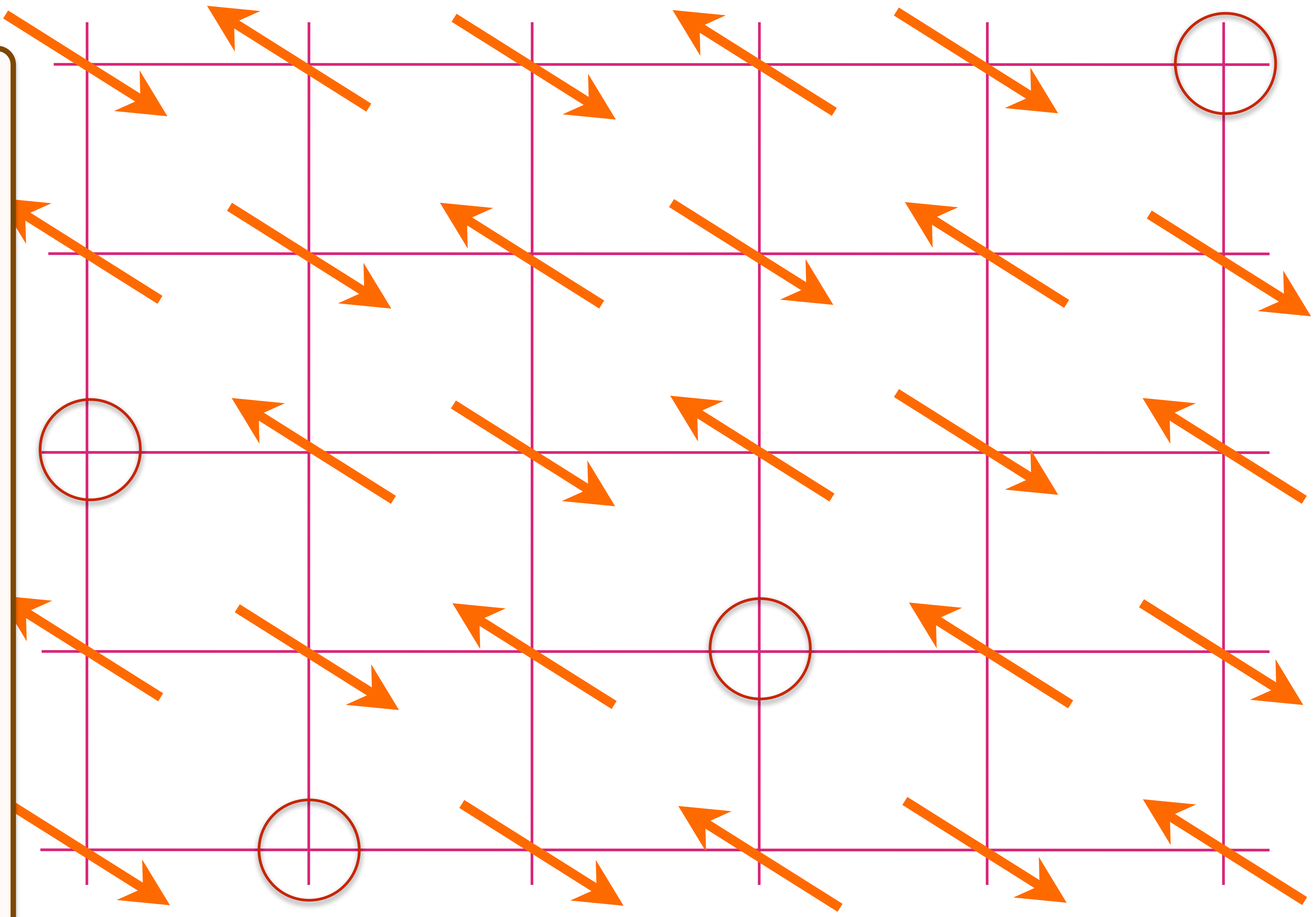
Reduced Brillouin
Zone.
Broken symmetry



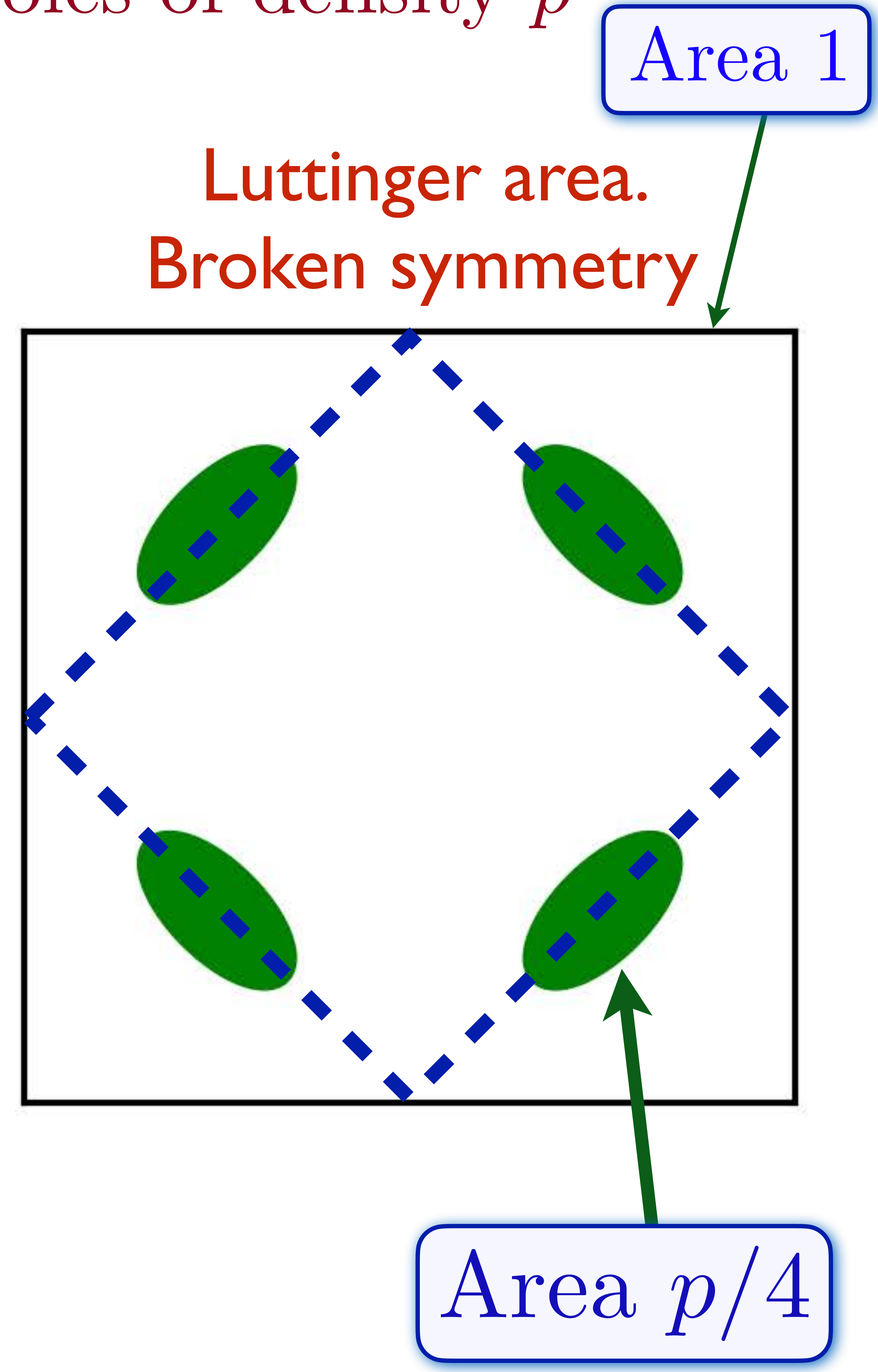
Doping an insulating antiferromagnet with holes of density p

AF metal

Fermi liquid with density p of spin 1/2, charge $+e$ holes. Coherent inter-layer transport requires inter-layer spin correlations.



Luttinger area. Broken symmetry

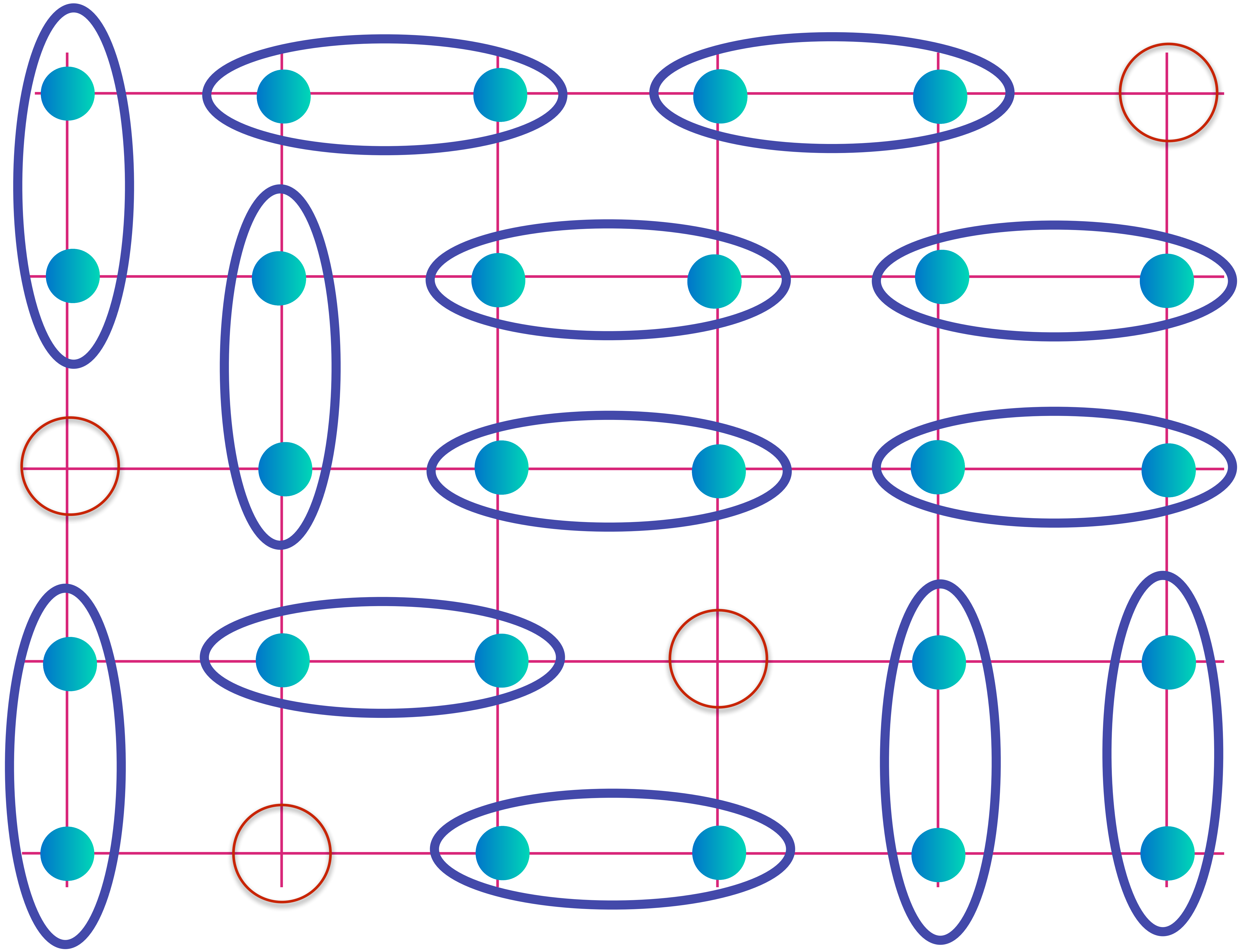


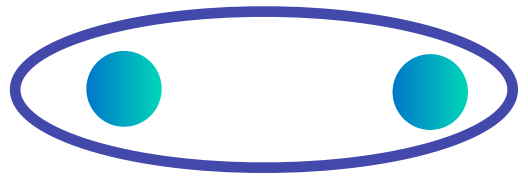
Doping an insulating antiferromagnet with holes of density p

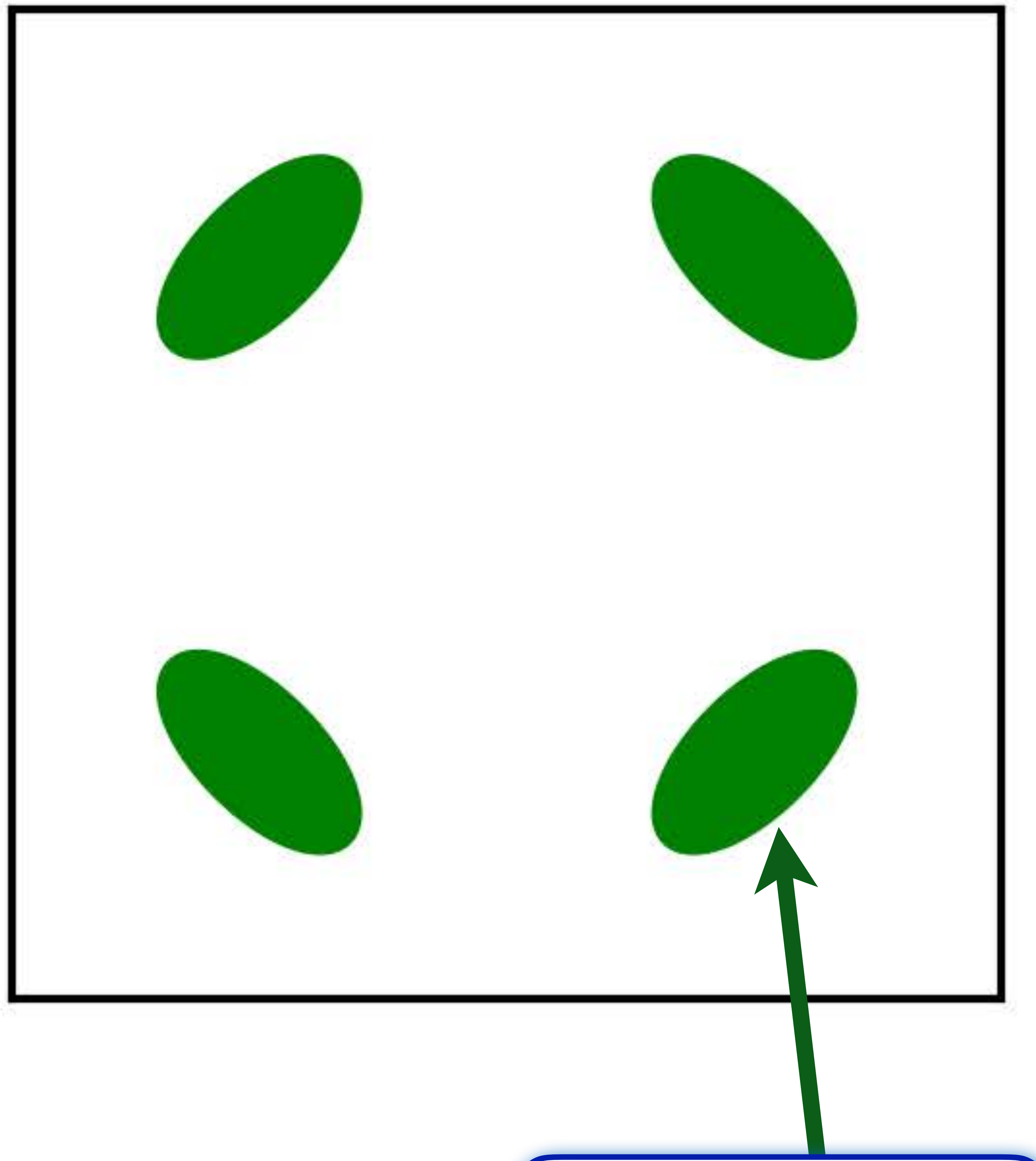
Holon metal

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Spin liquid with density p of spinless, charge $+e$ holons.
No coherent inter-layer transport.



 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$



Area $p/4$

R. K. Kaul, Y. B. Kim, S. S., T. Senthil, Nature Physics **4**, 28 (2008); S.S. M.A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

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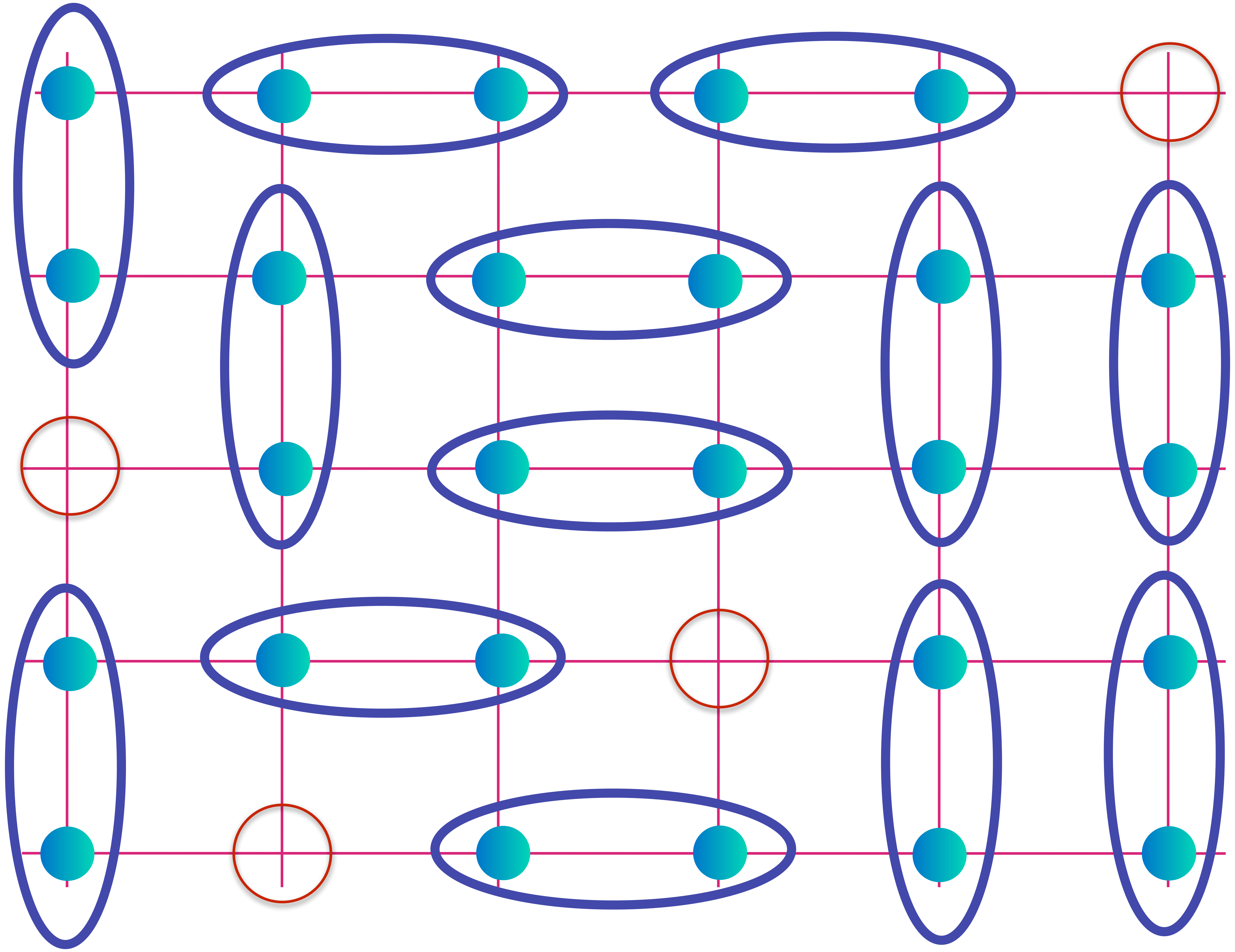
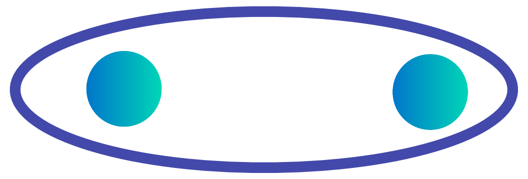
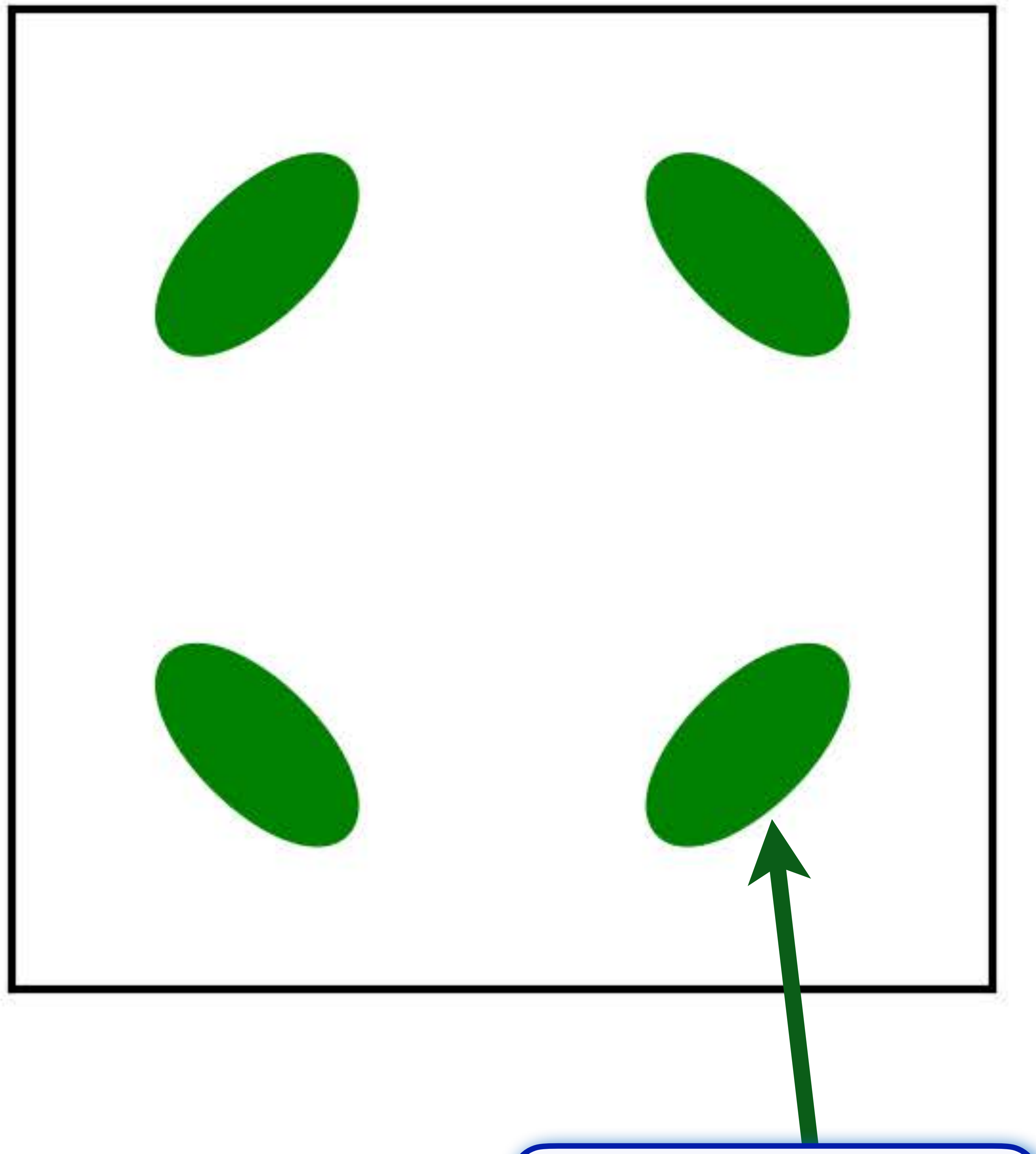


 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



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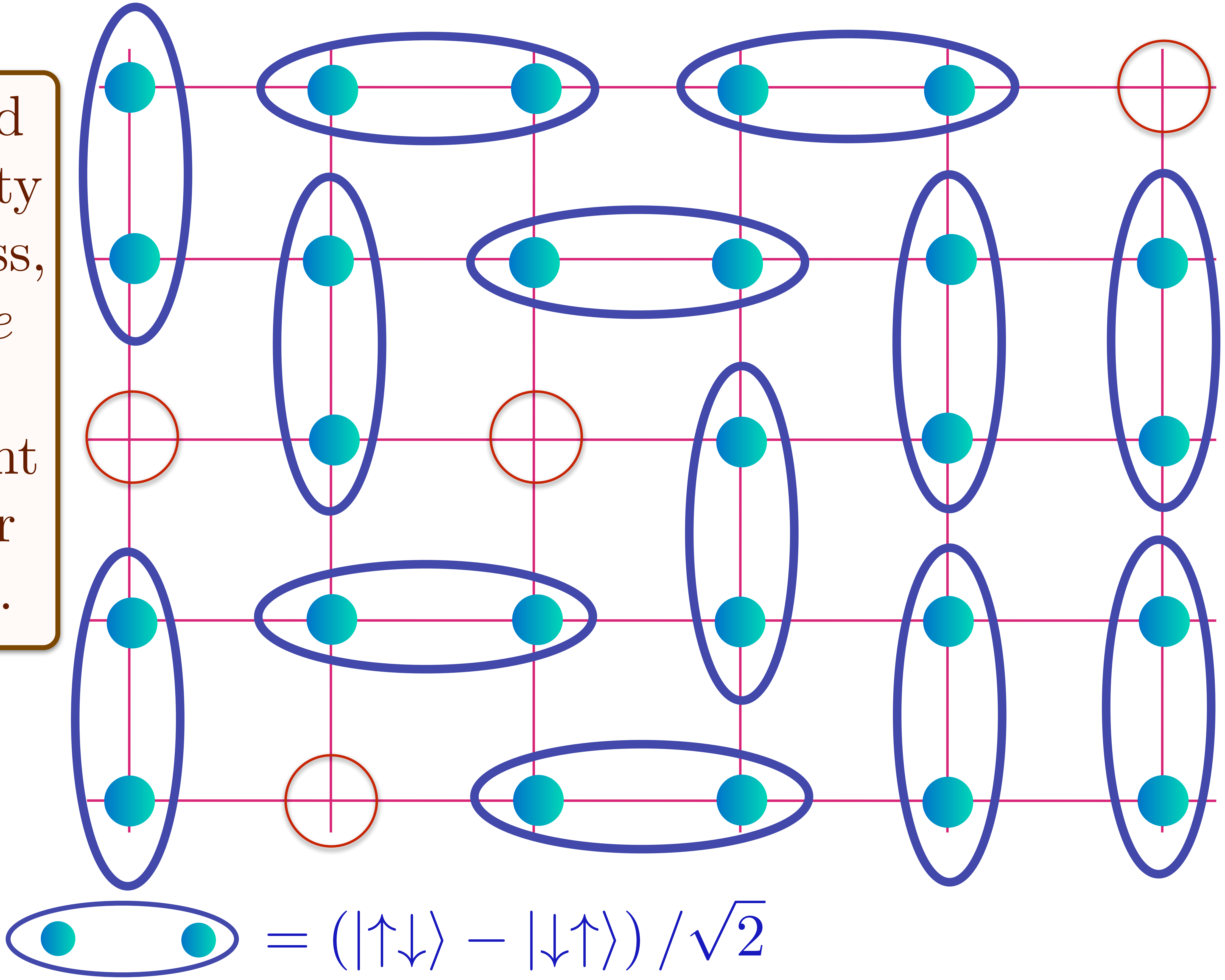

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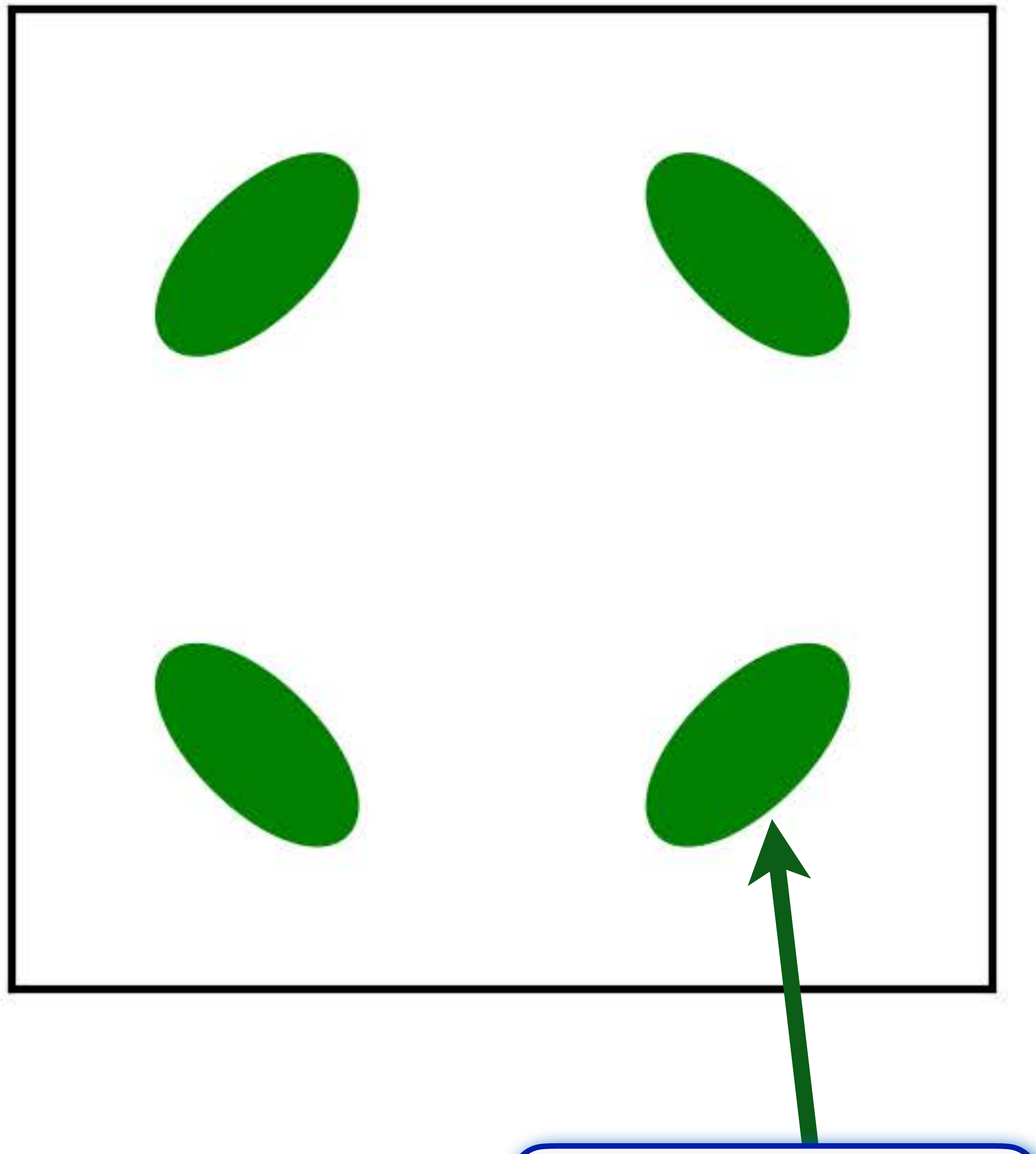

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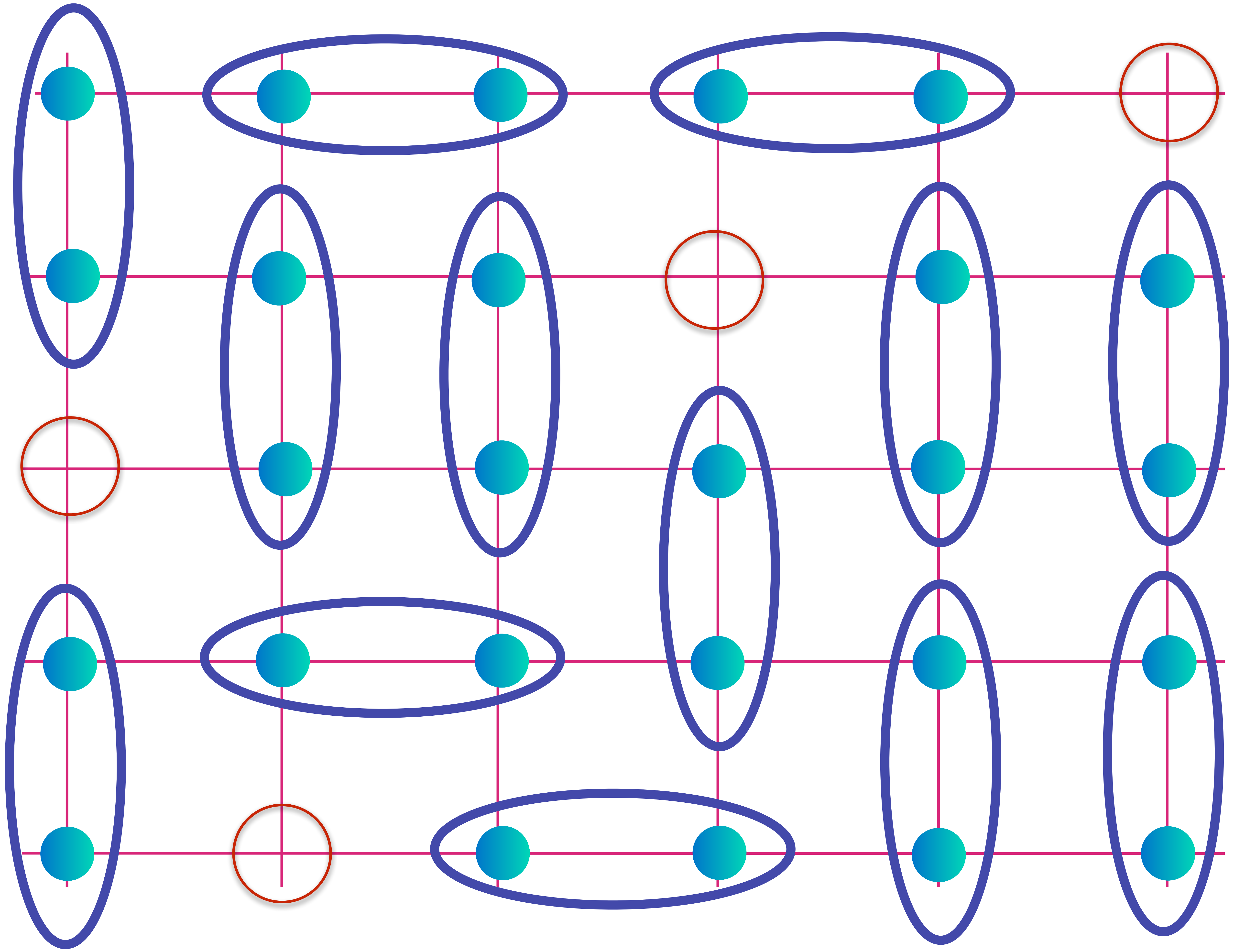

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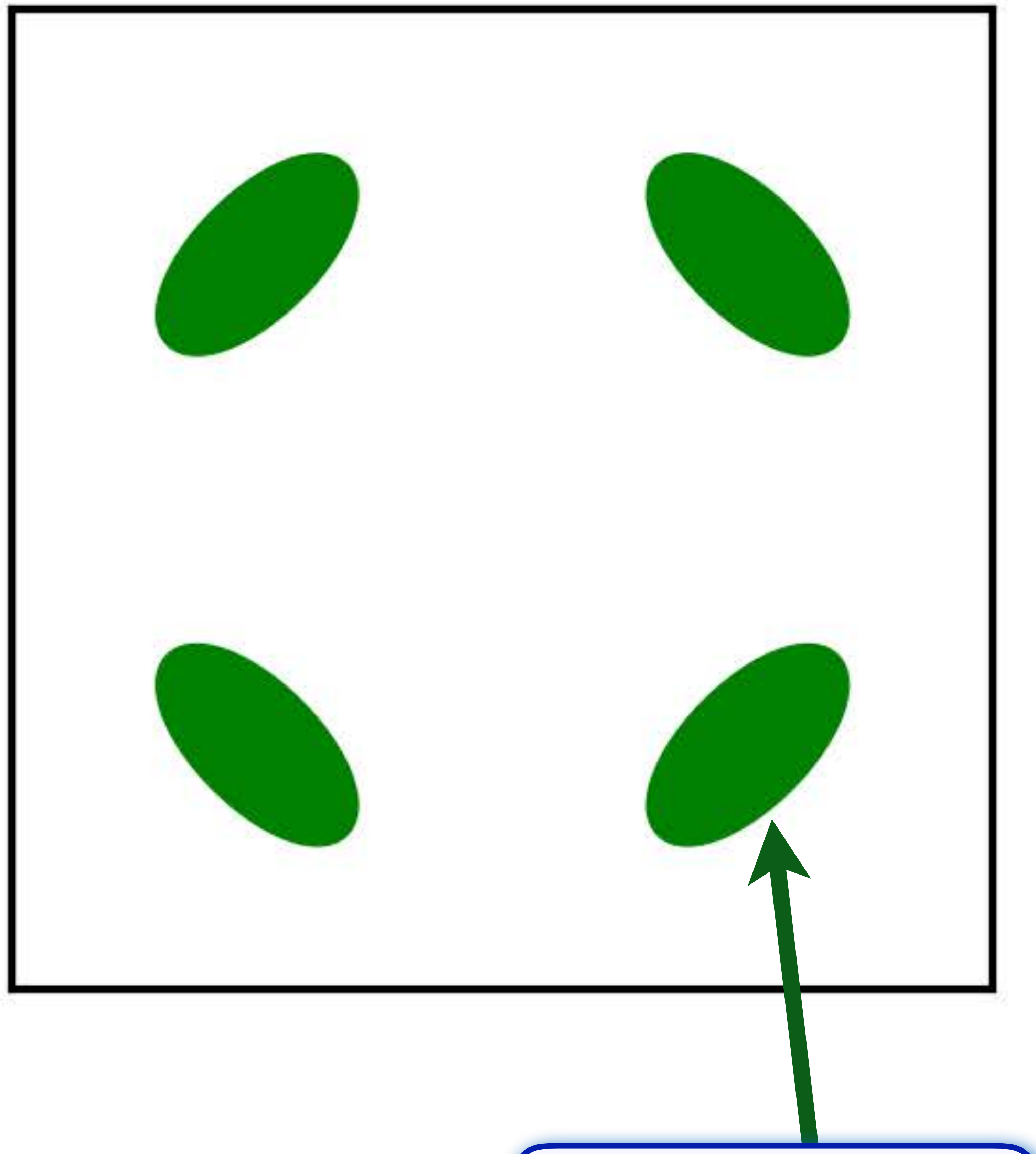

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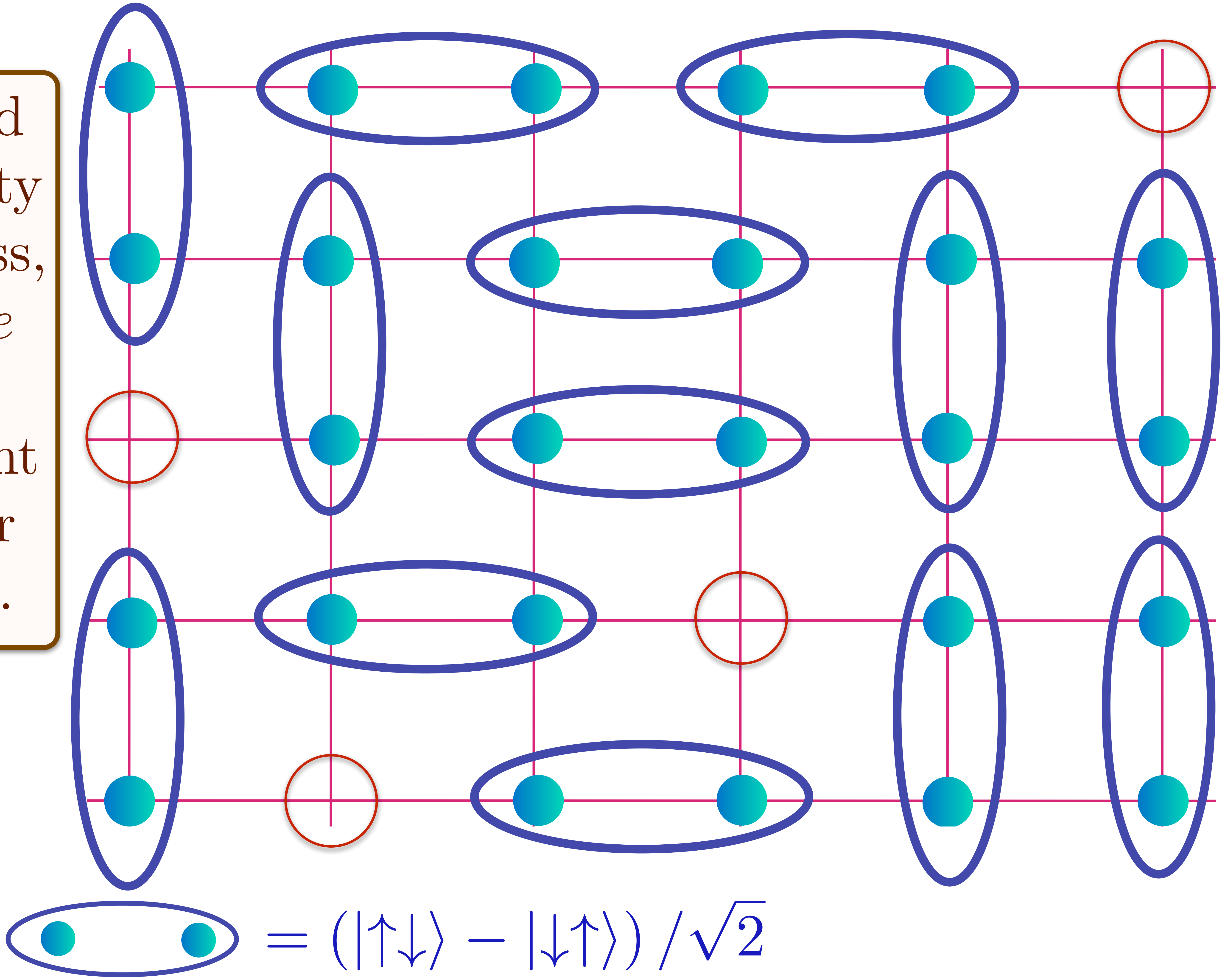
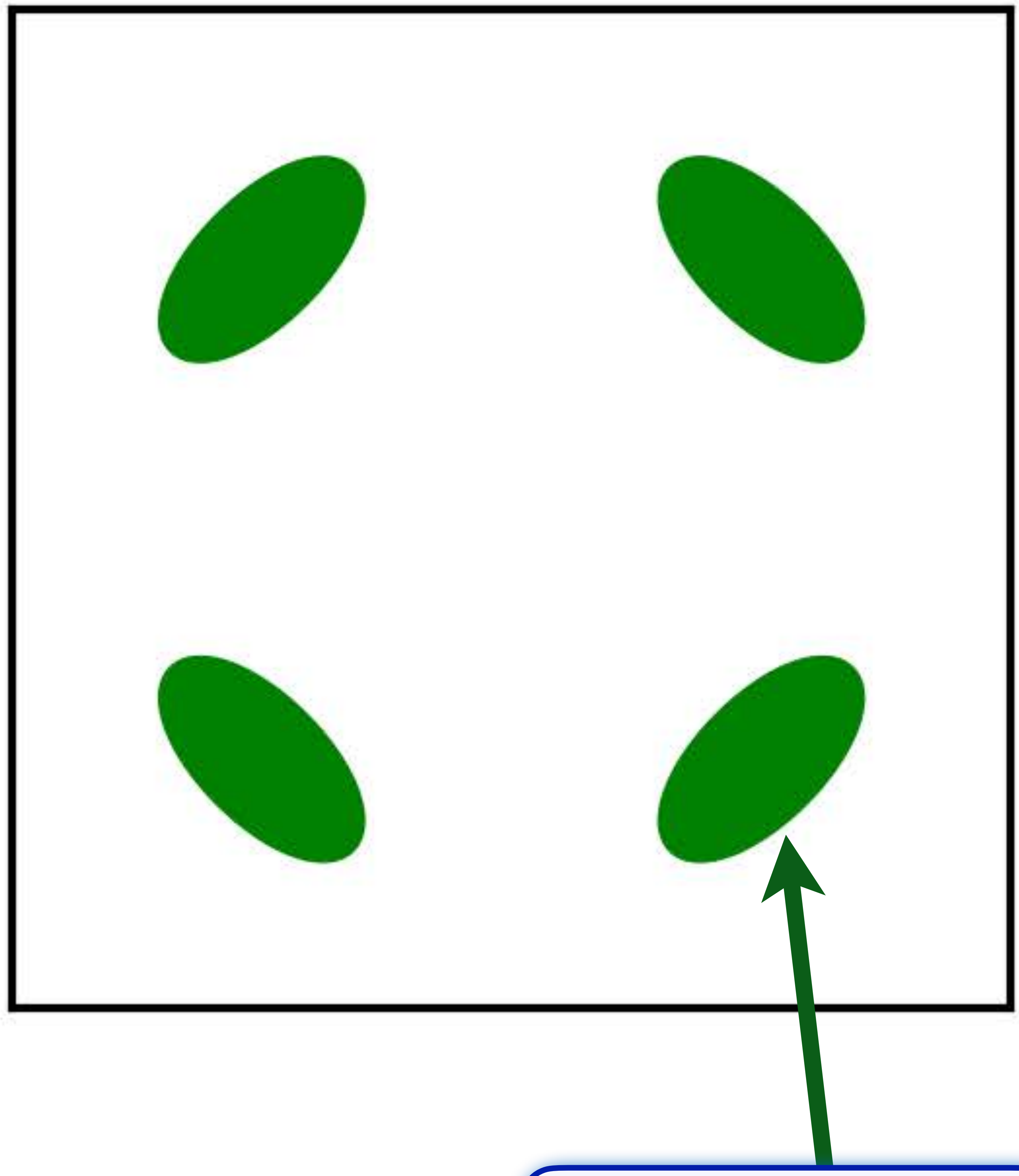


$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$



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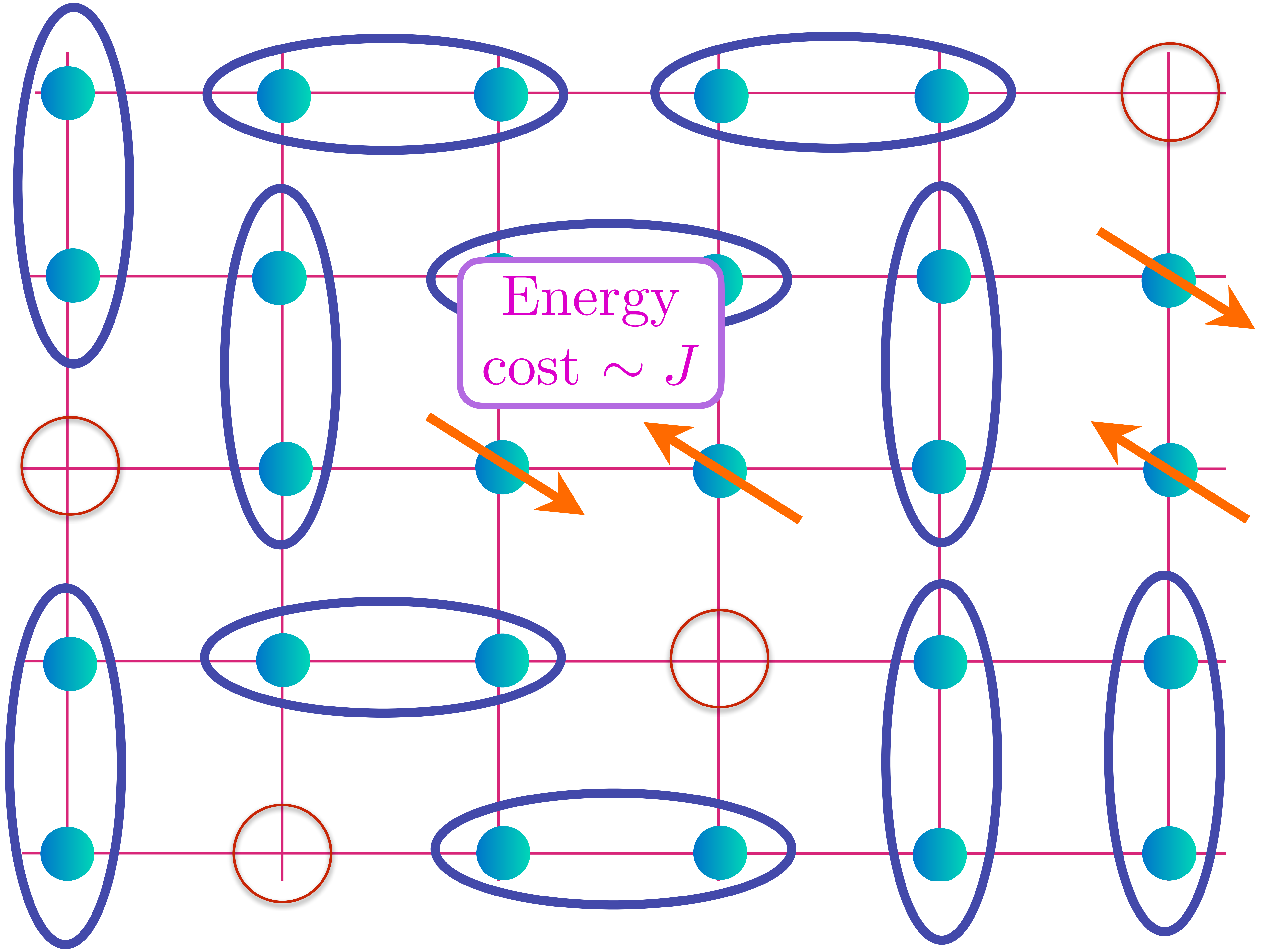
Doping an insulating antiferromagnet with holes of density p

Holon metal excited states

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Charge 0, spin-1/2 spinons



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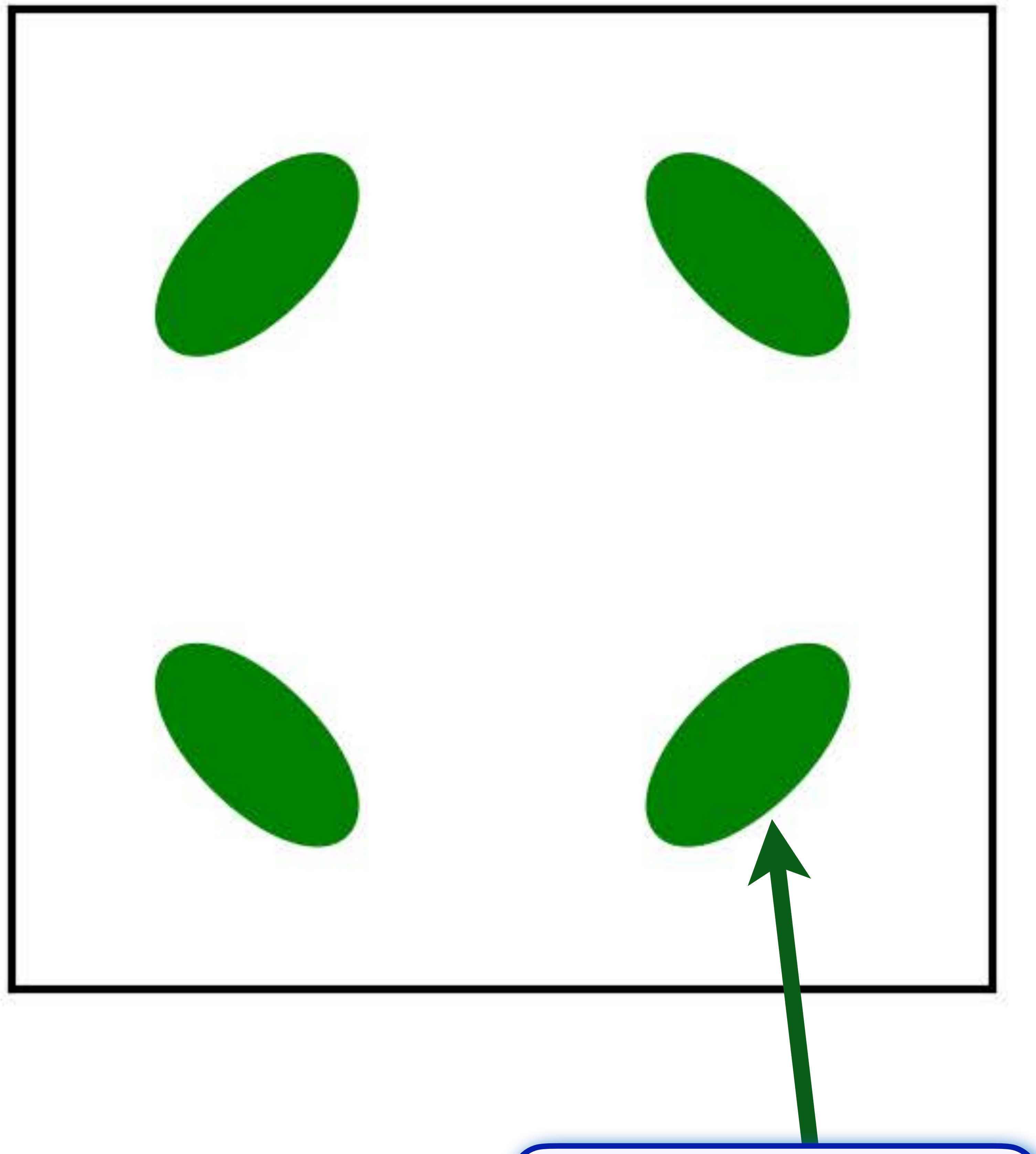

Doping an insulating antiferromagnet with holes of density p

Holon metal excited states

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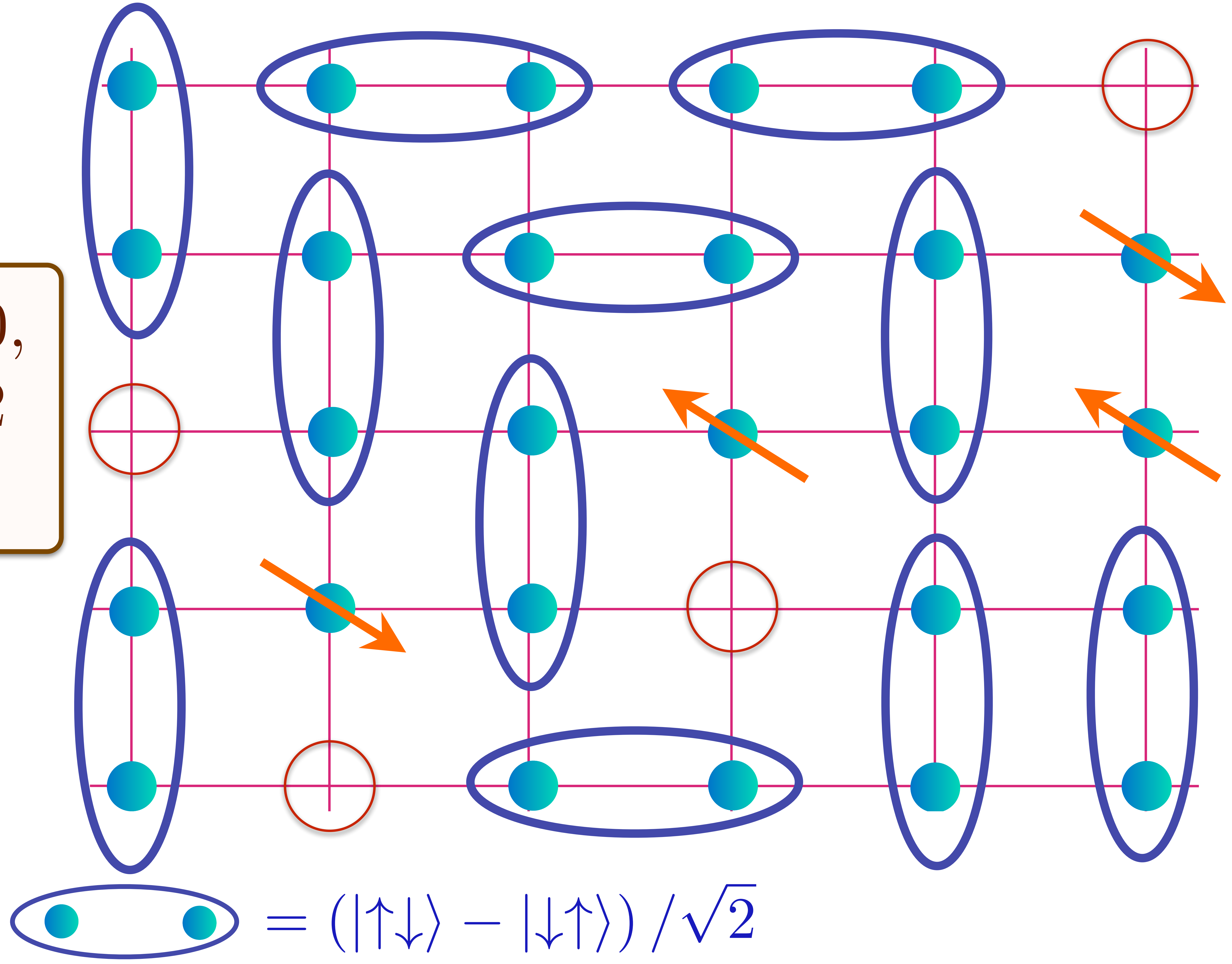

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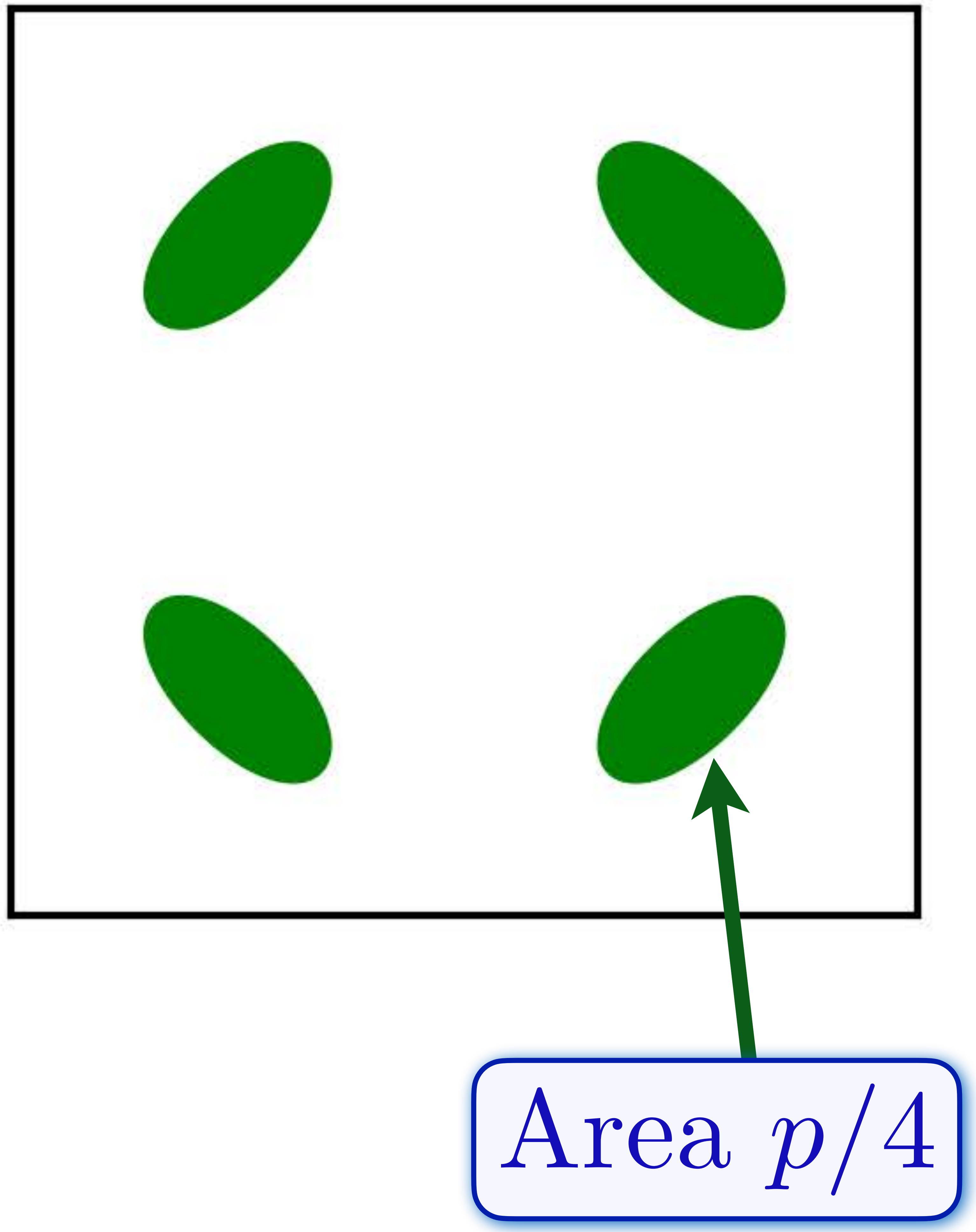

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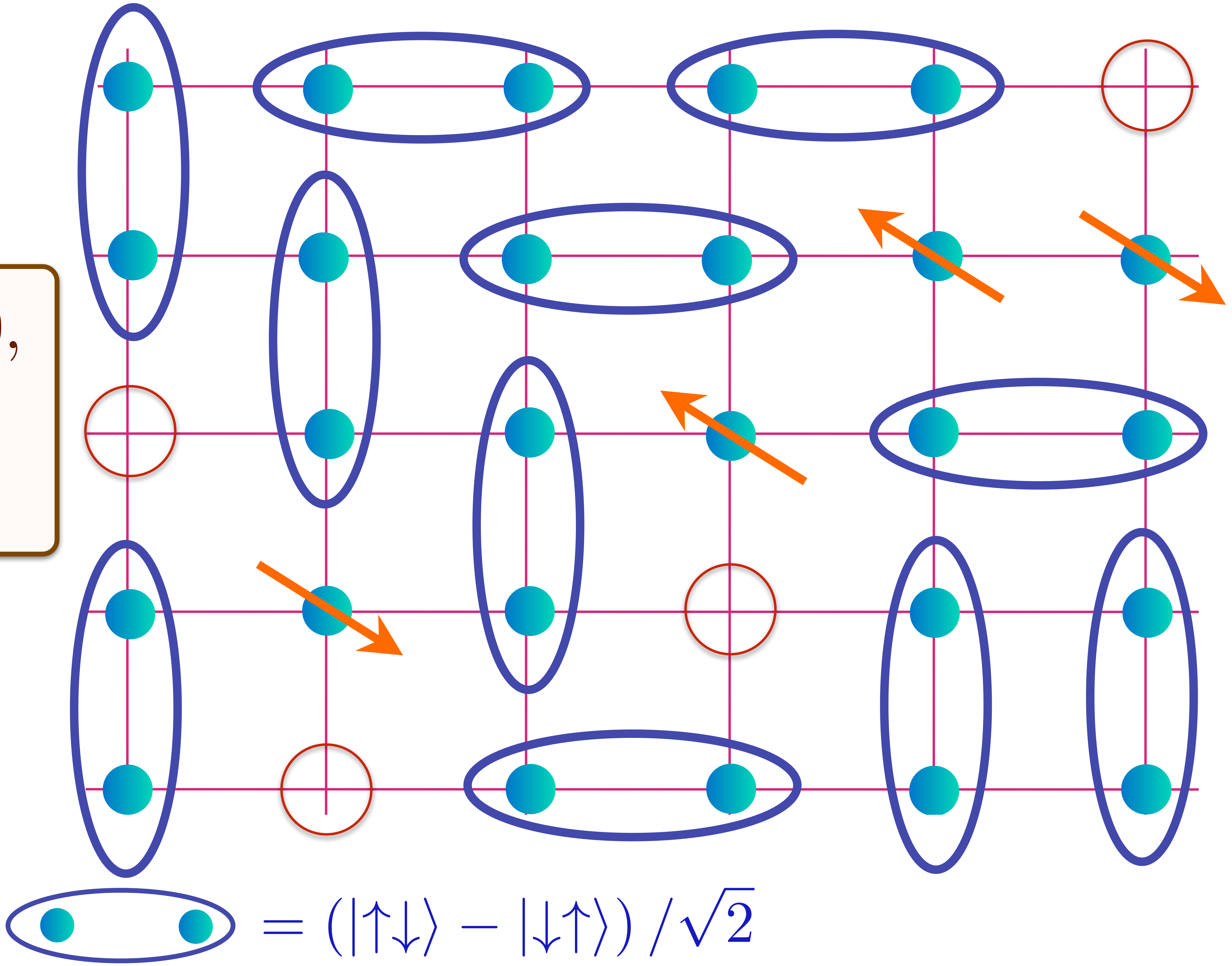

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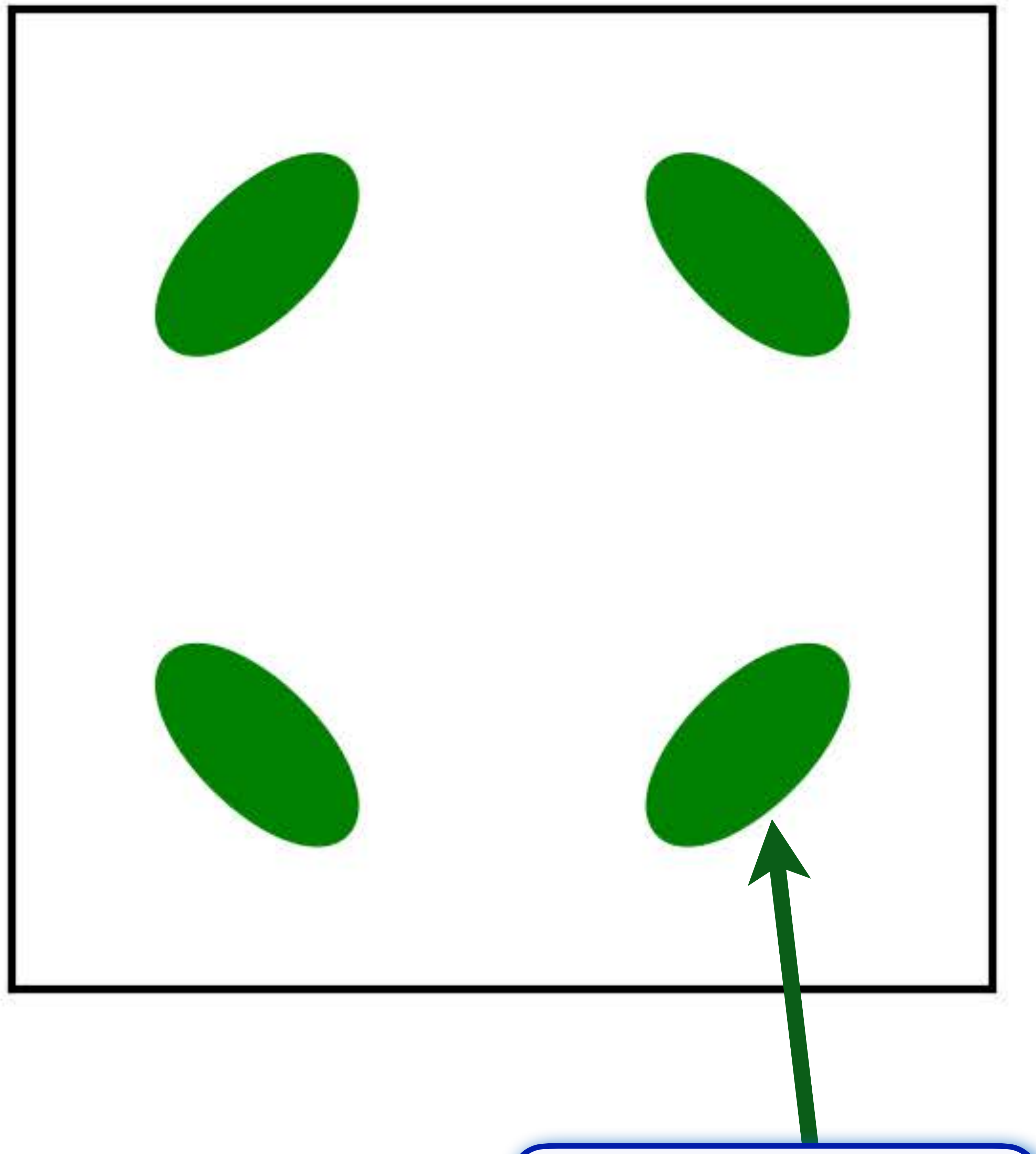

Area $p/4$

Doping an insulating antiferromagnet with holes of density p

Holon metal excited states

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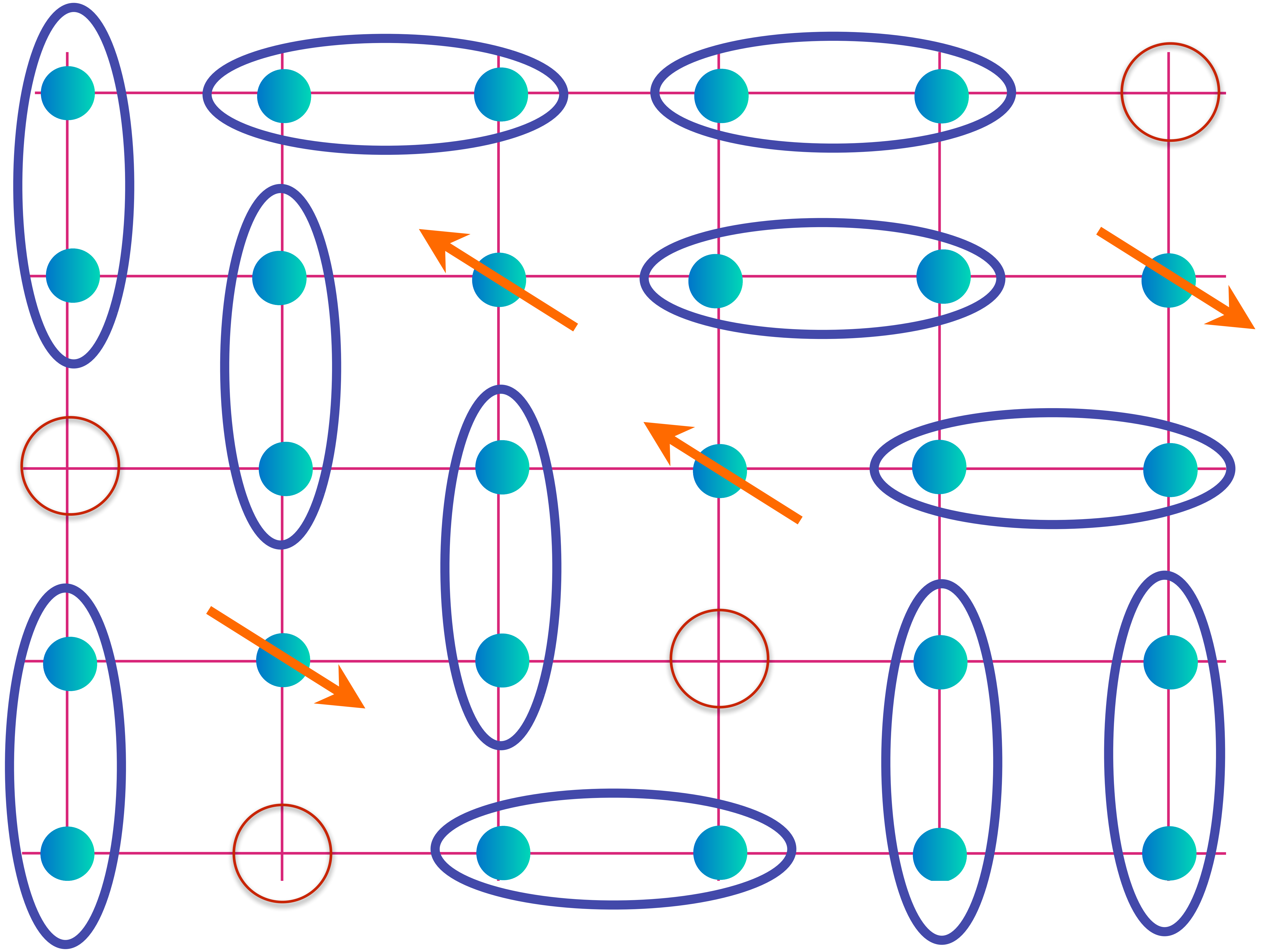
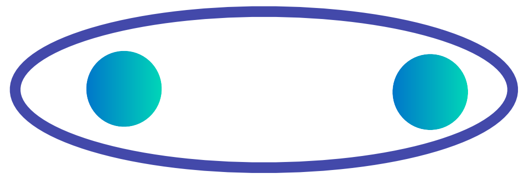
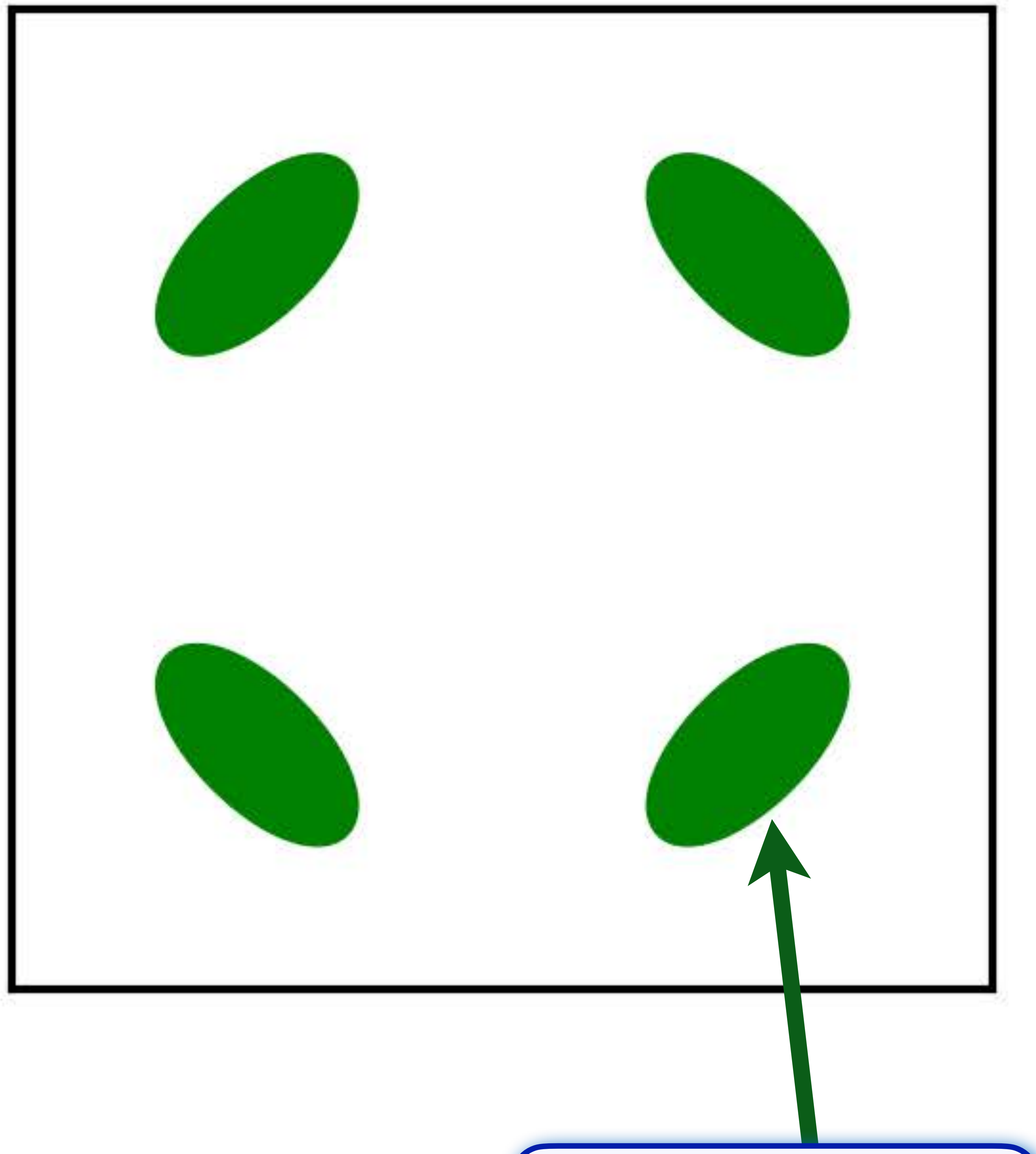


 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$



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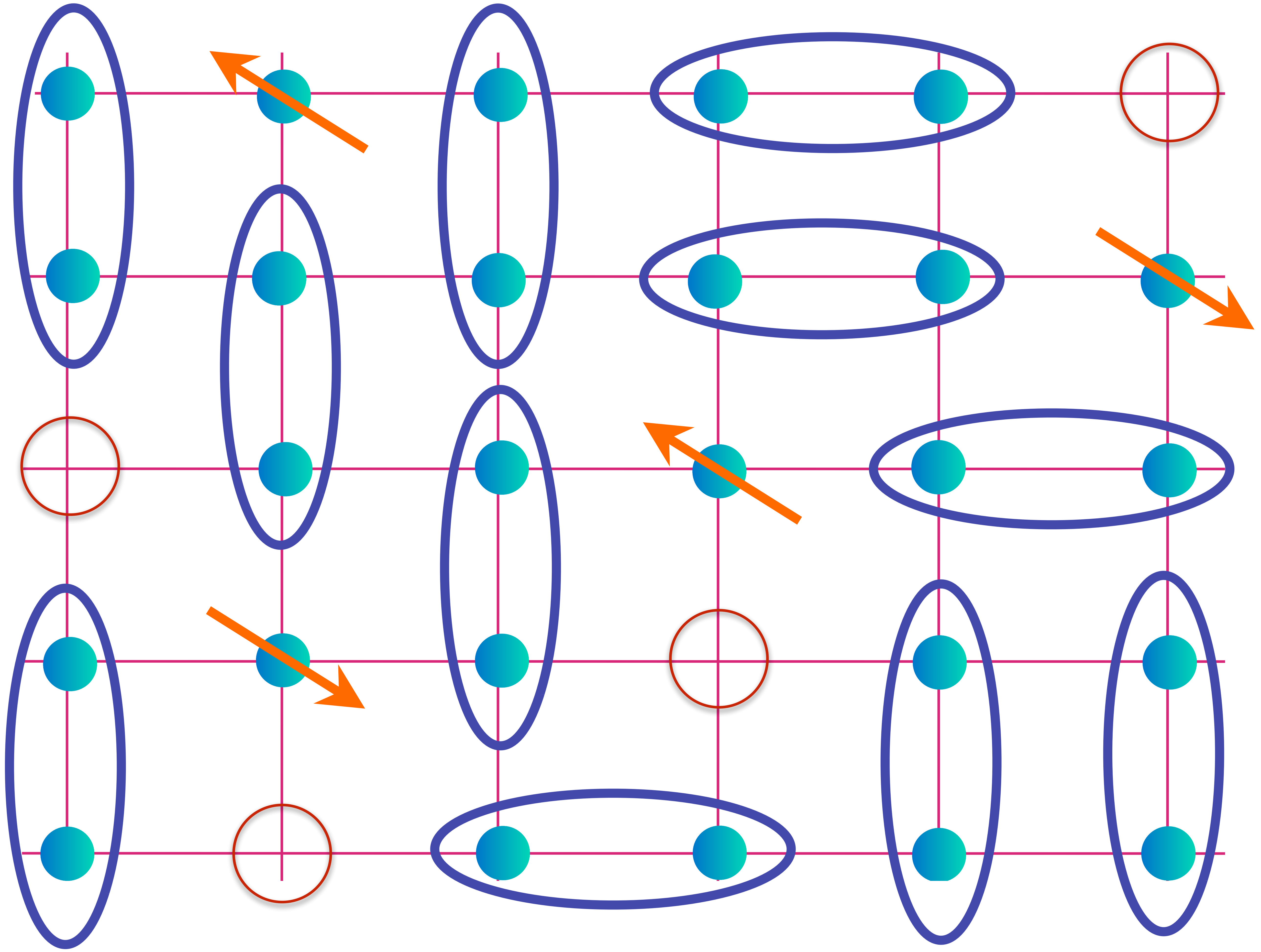

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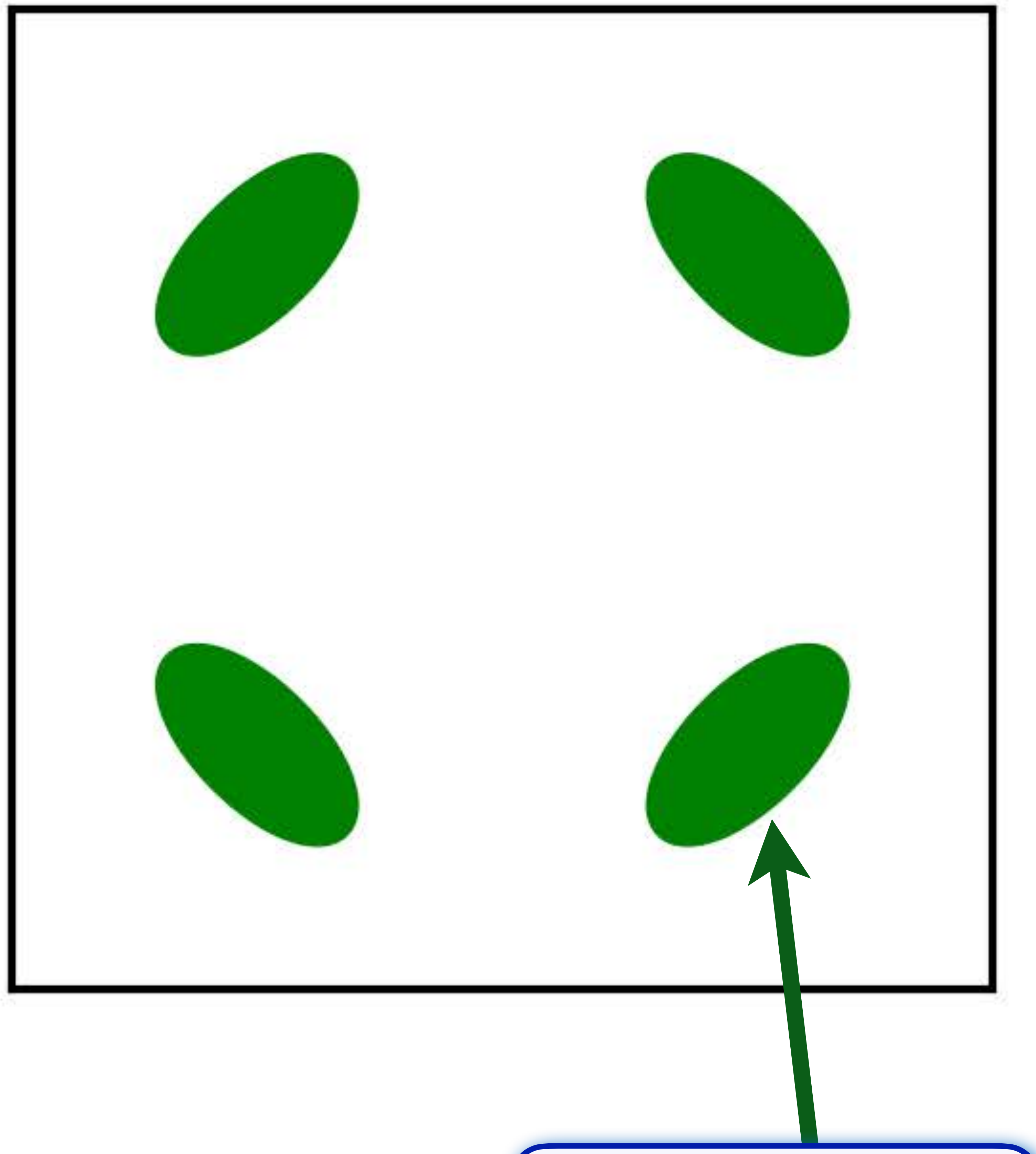

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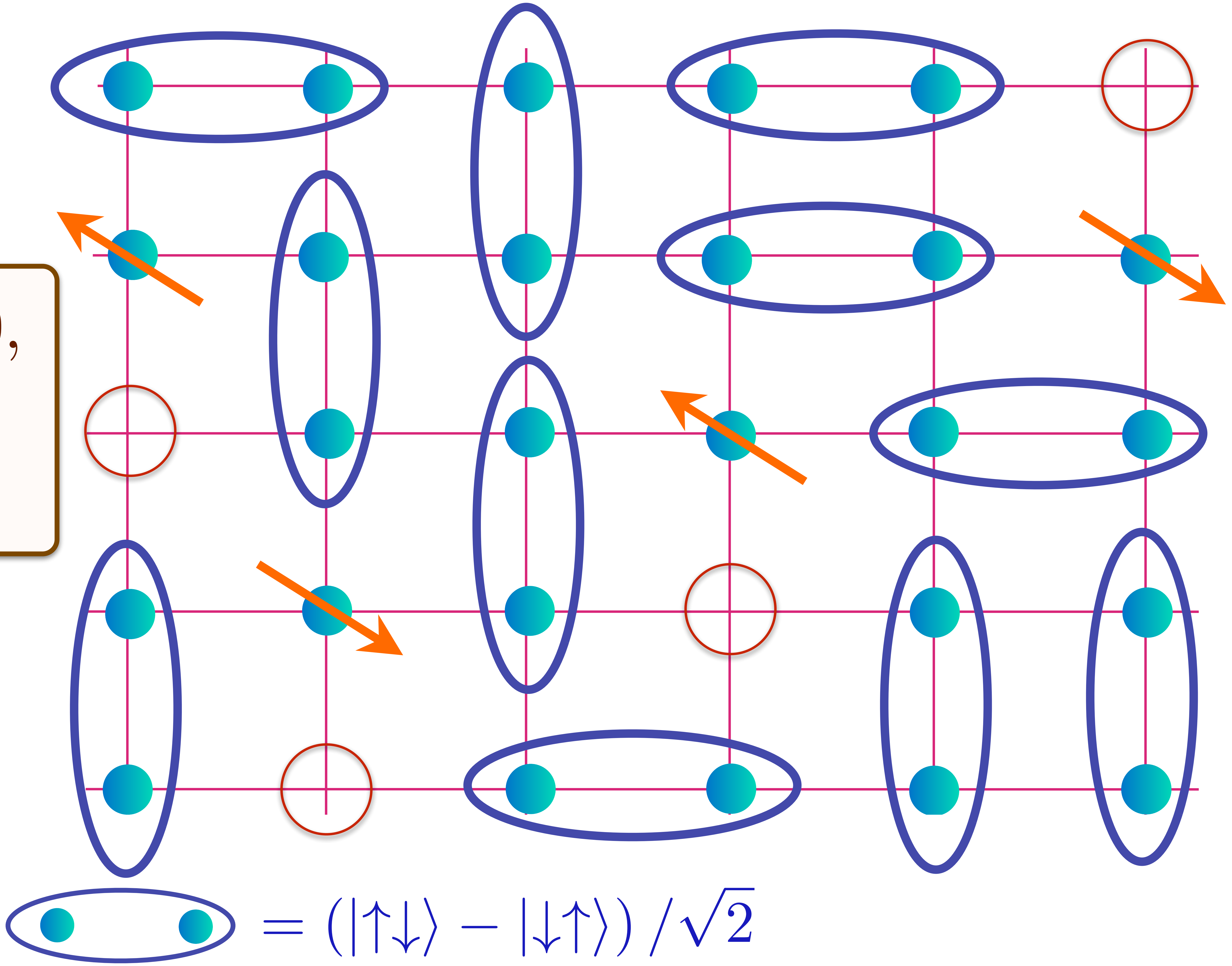

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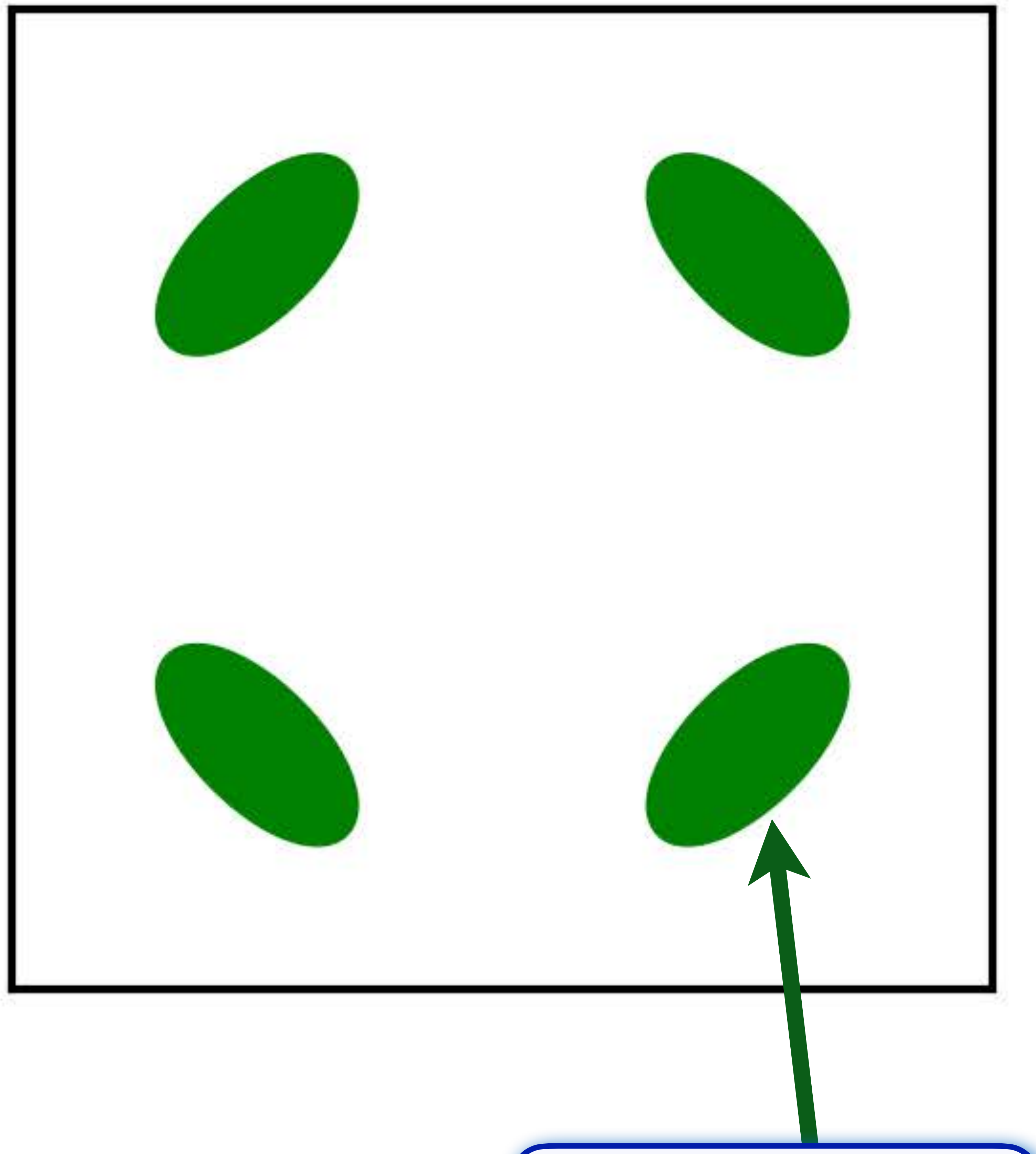

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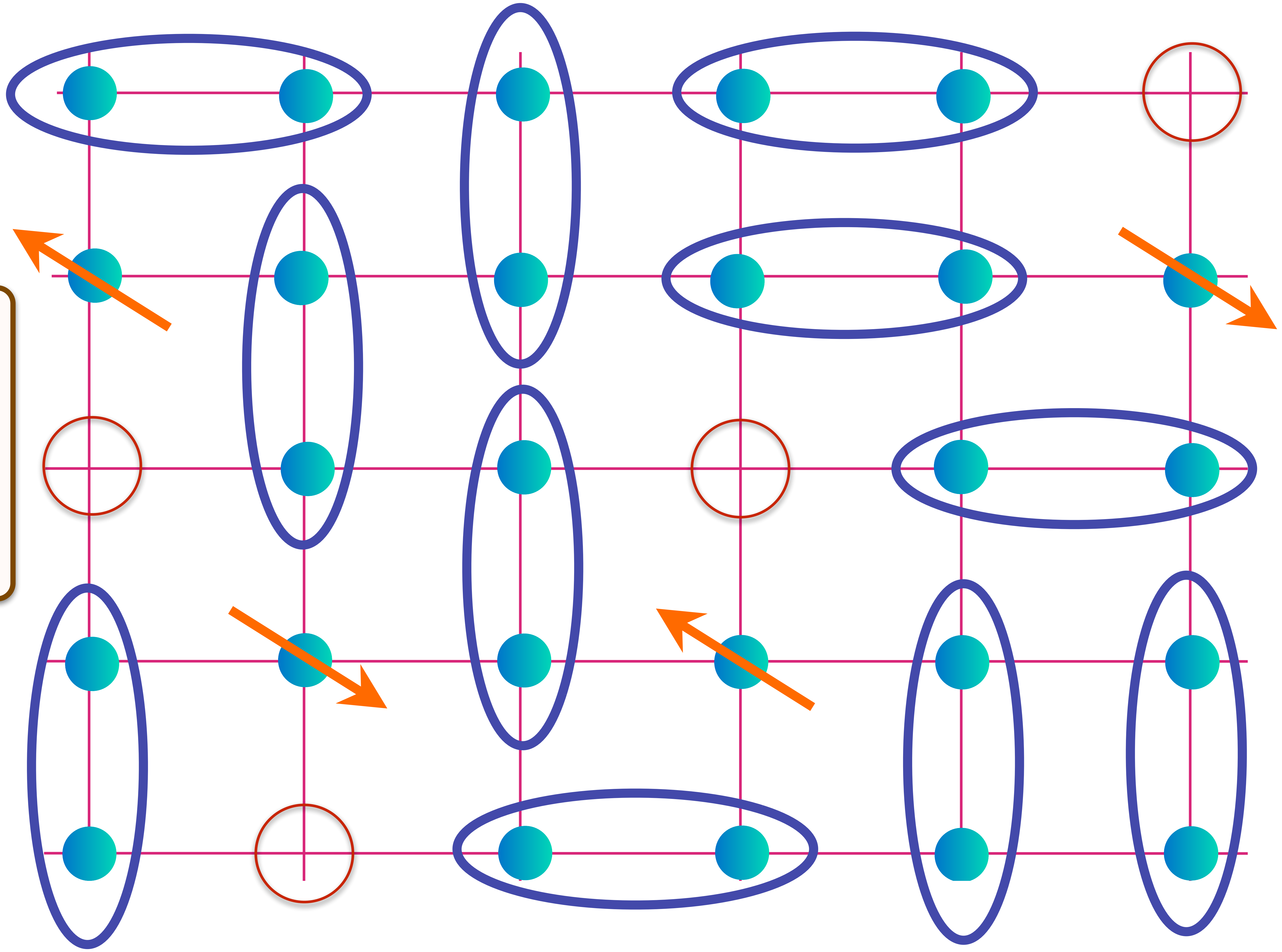

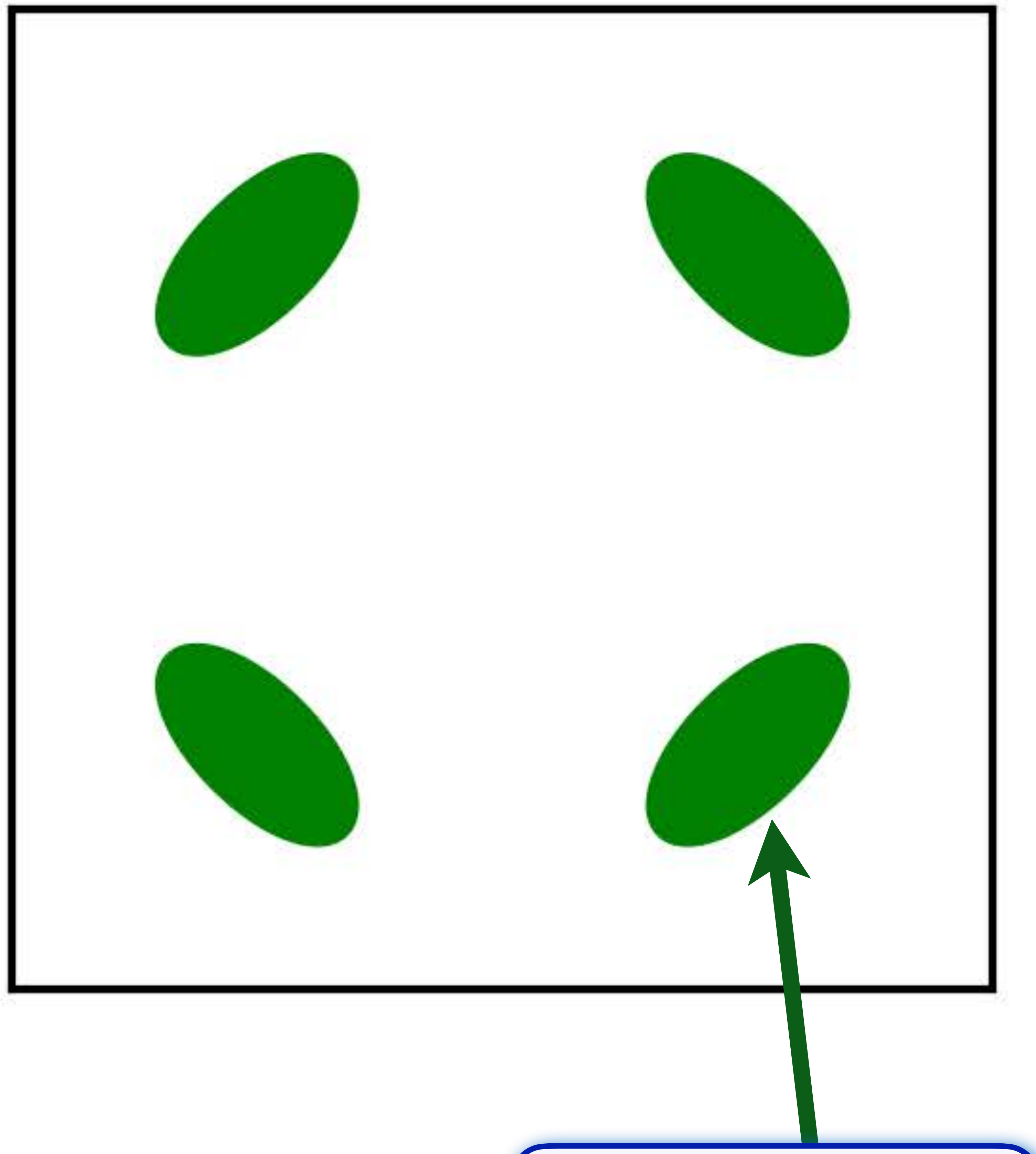

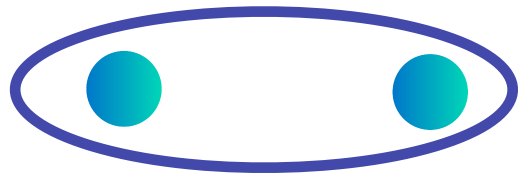 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

Area $p/4$

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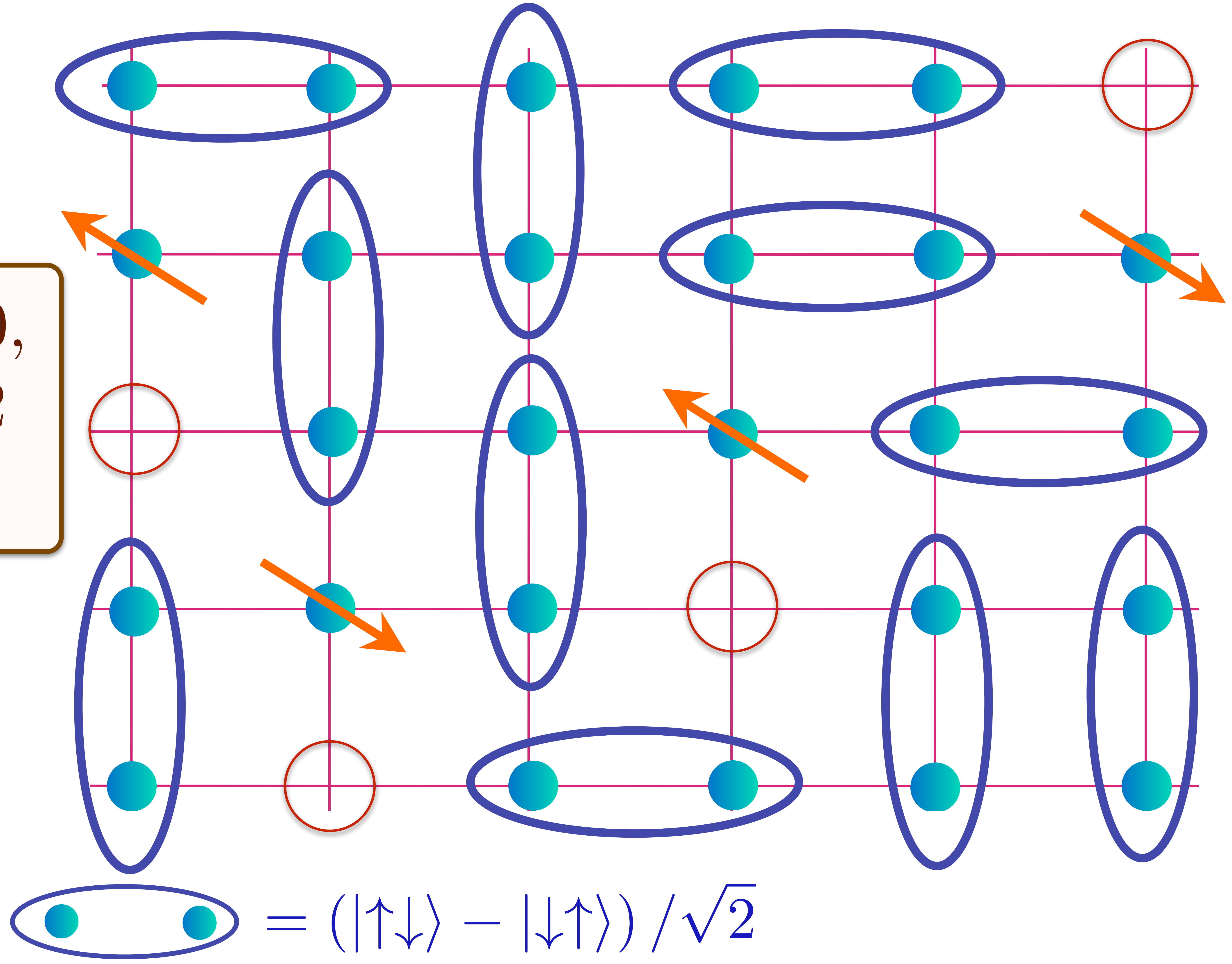

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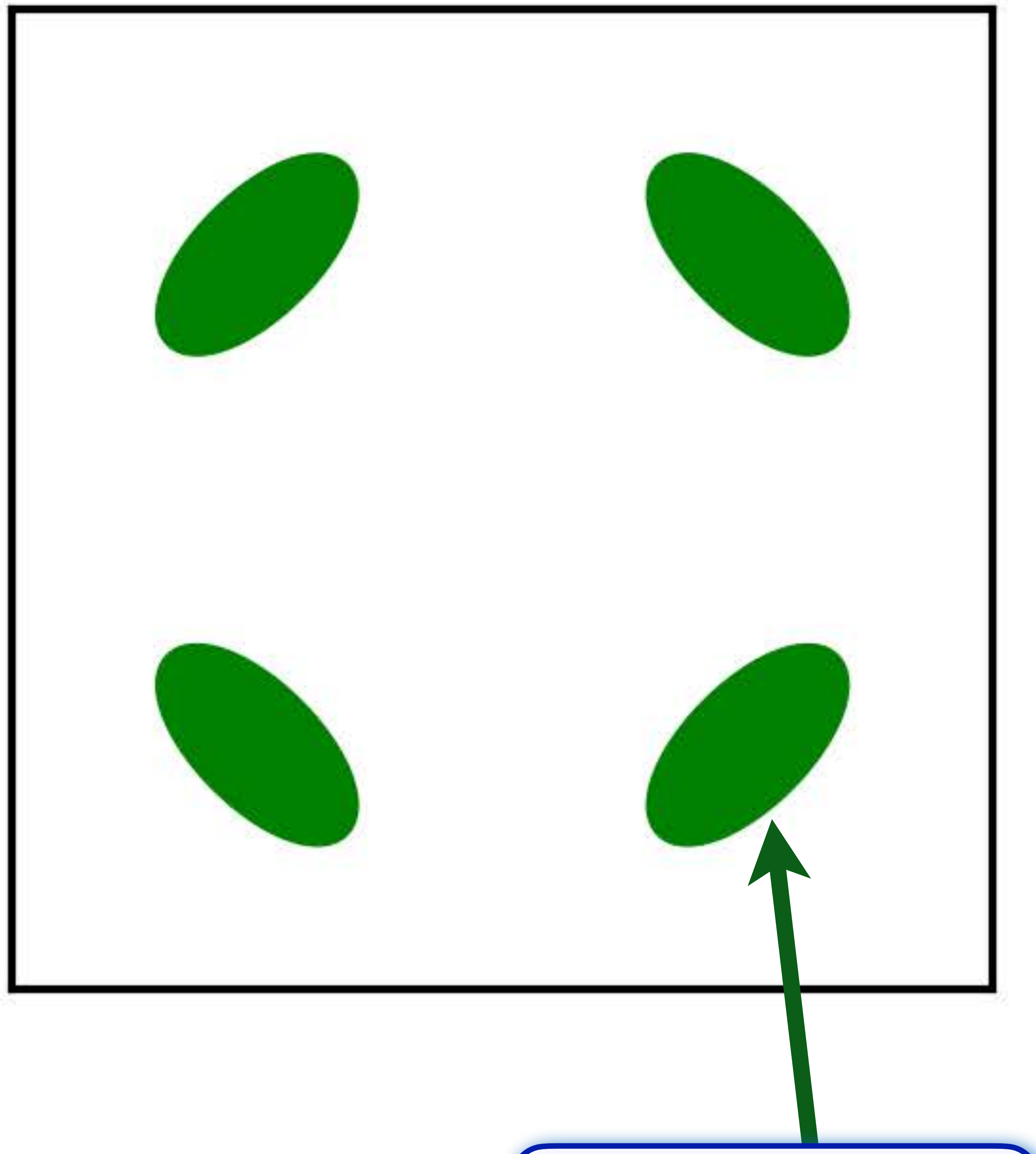

Area $p/4$

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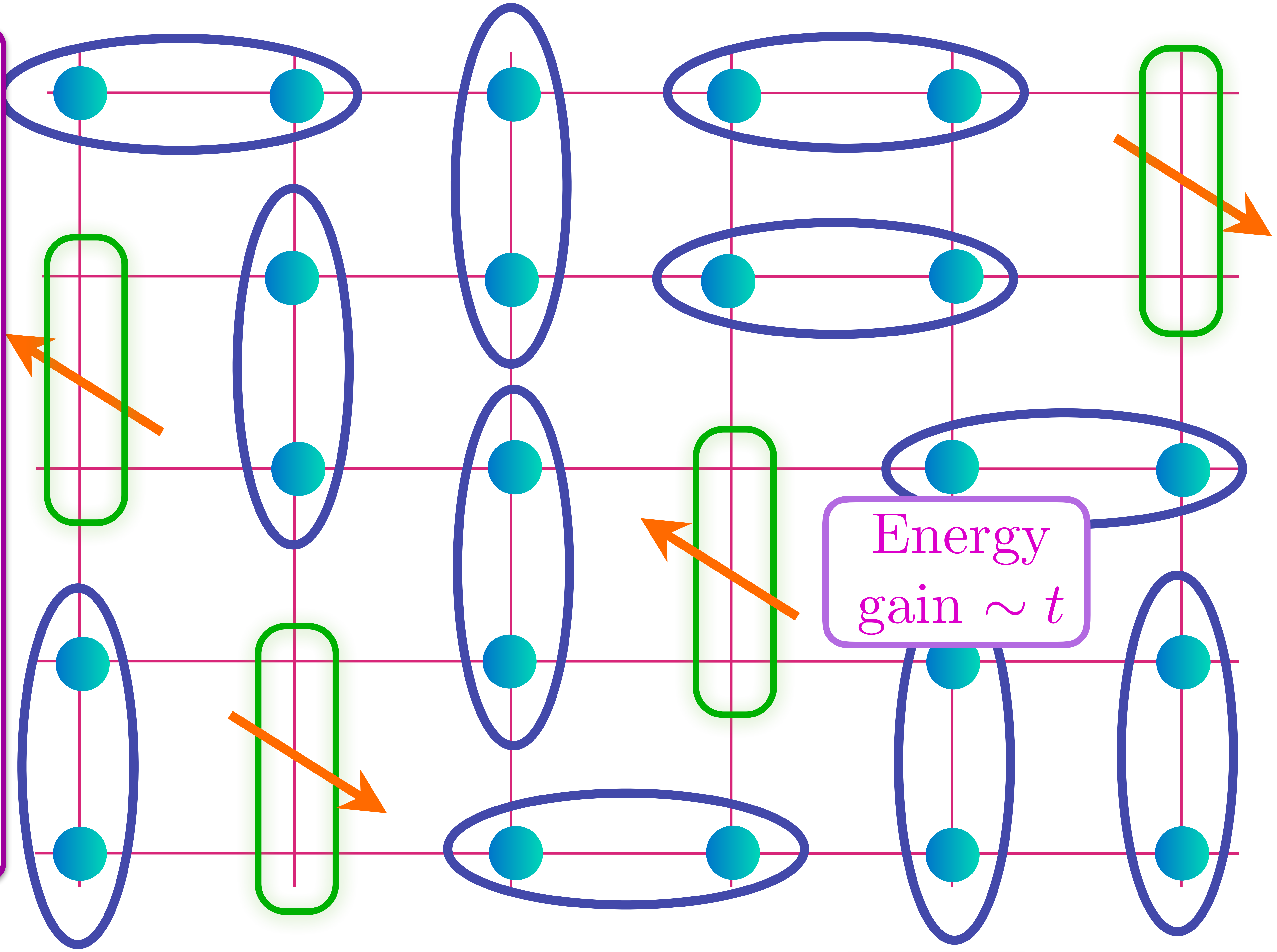
FL*

Metal with density p of spin-1/2, charge $+e$ 'holes' (or 'magnetic polarons') with coherent inter-layer transport.



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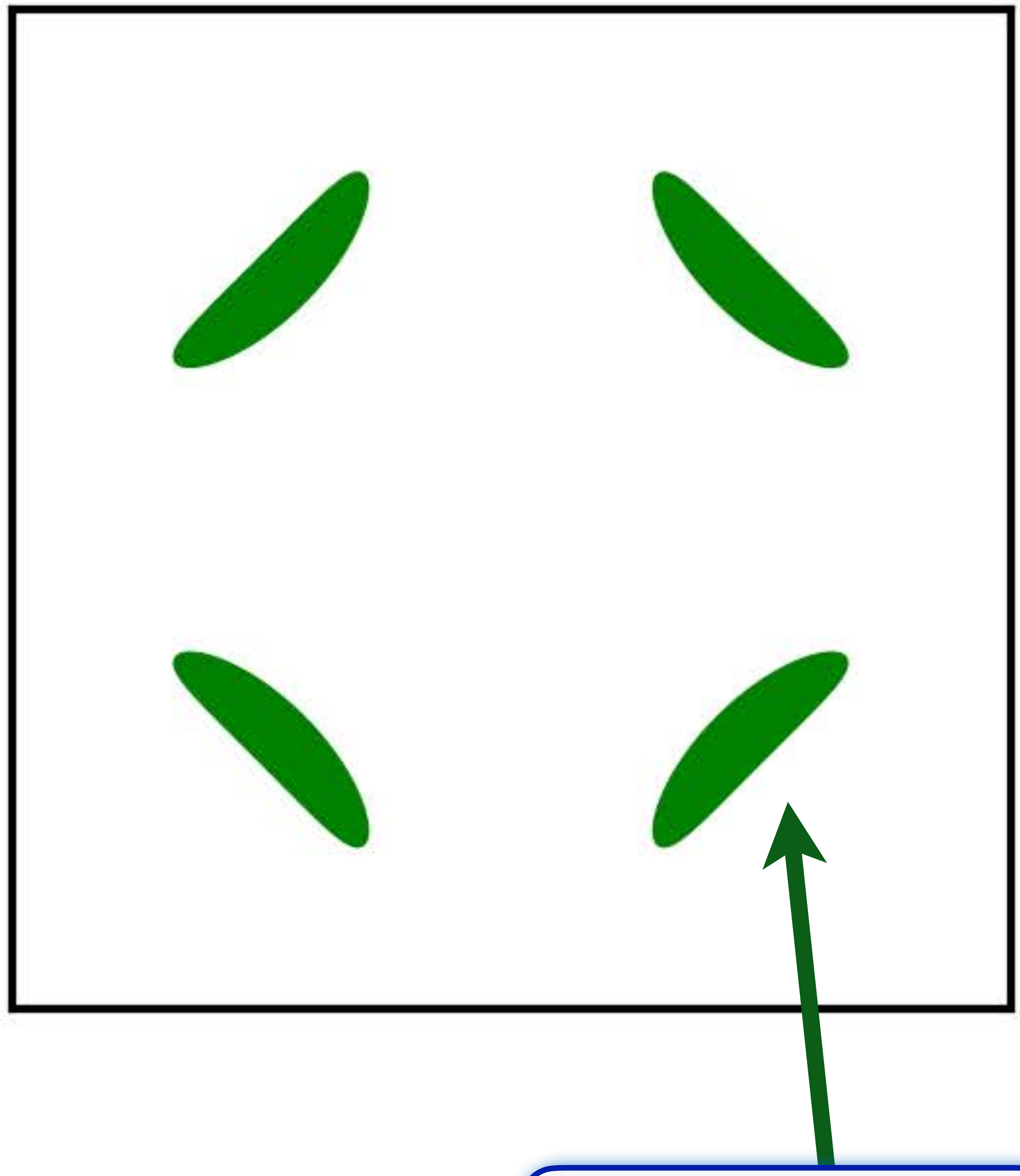
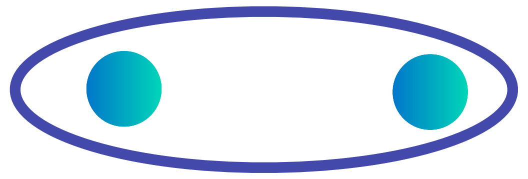
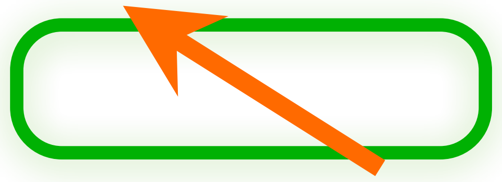


Area $p/8$

 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

 $= (|\uparrow \circ\rangle + |\circ \uparrow\rangle)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

Doping an insulating antiferromagnet with holes of density p

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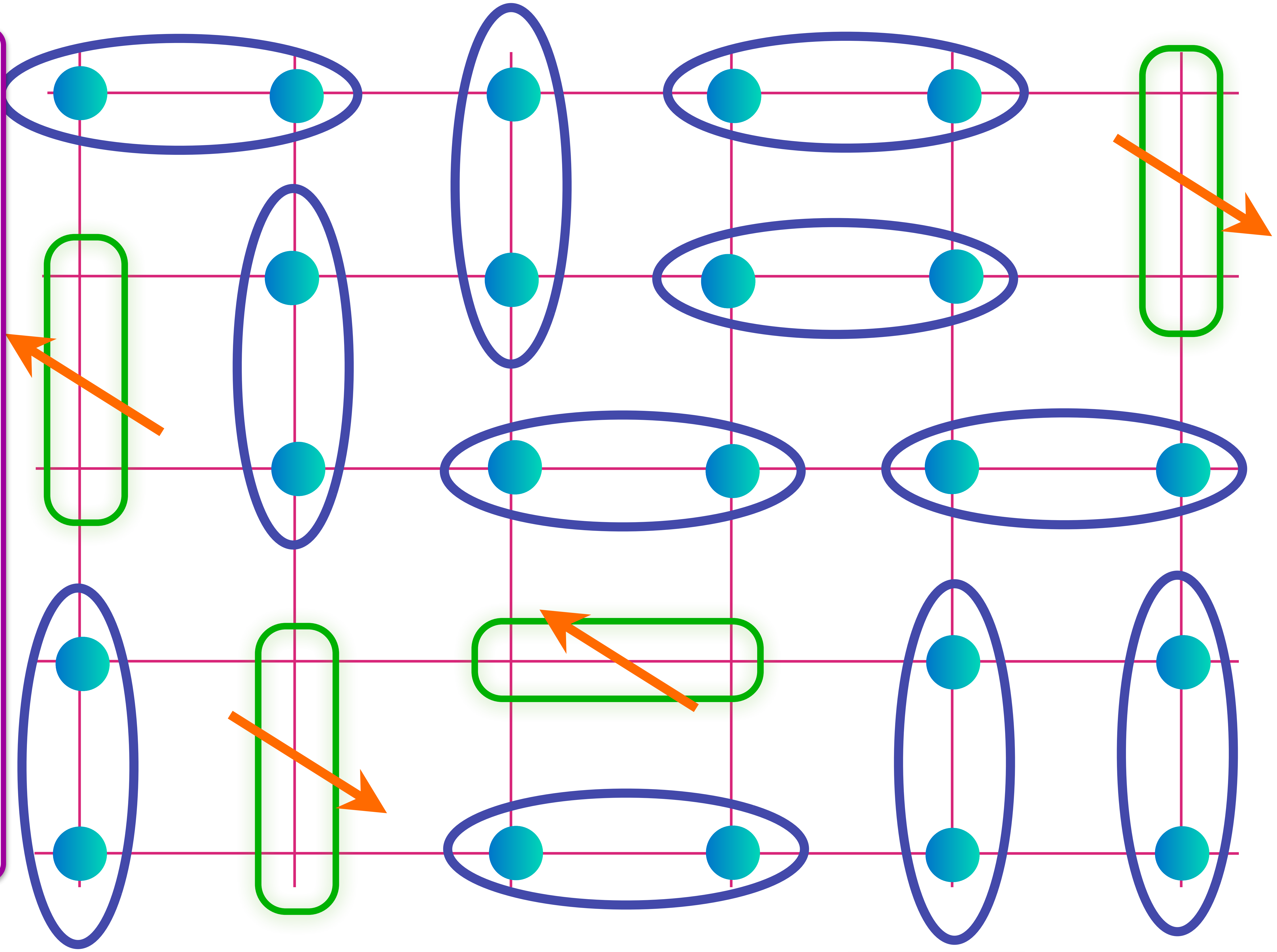

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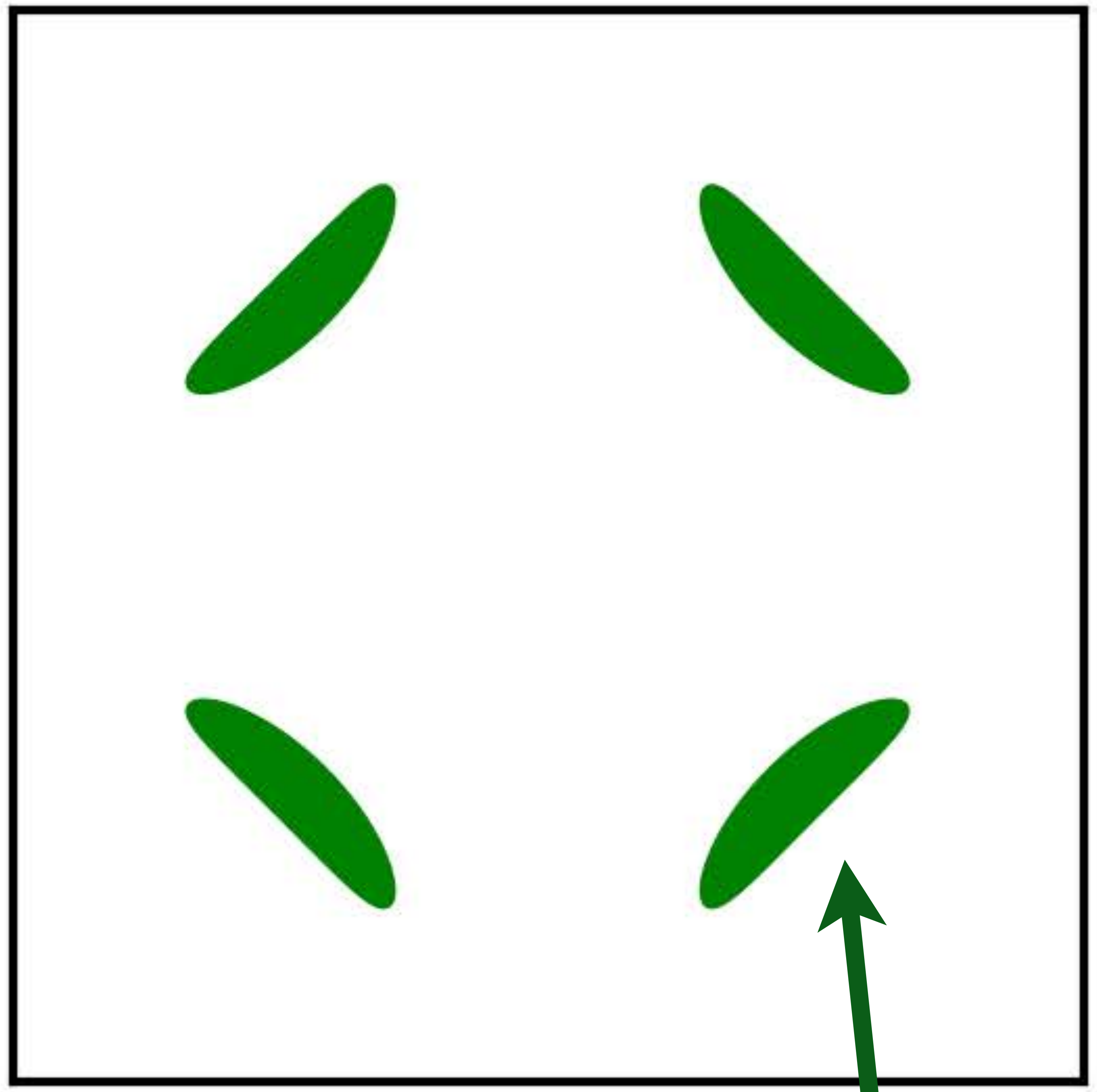

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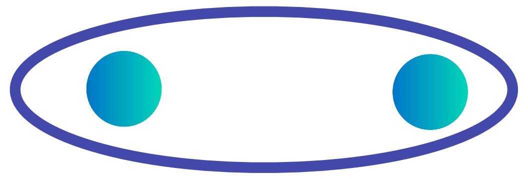
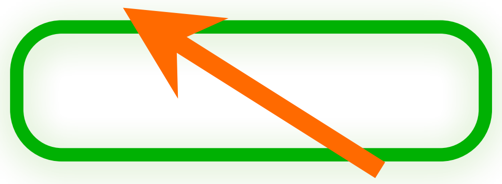
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$  $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

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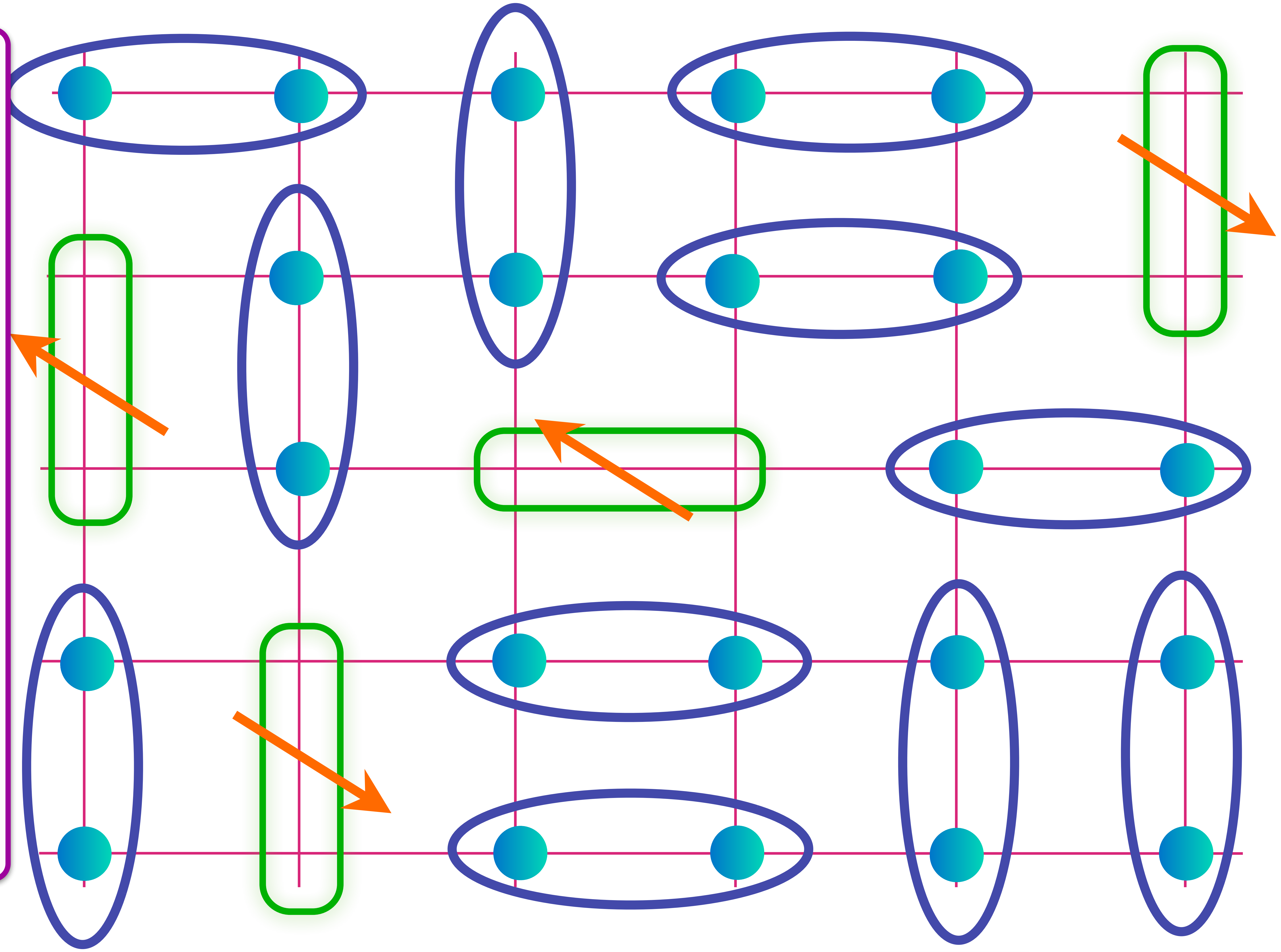
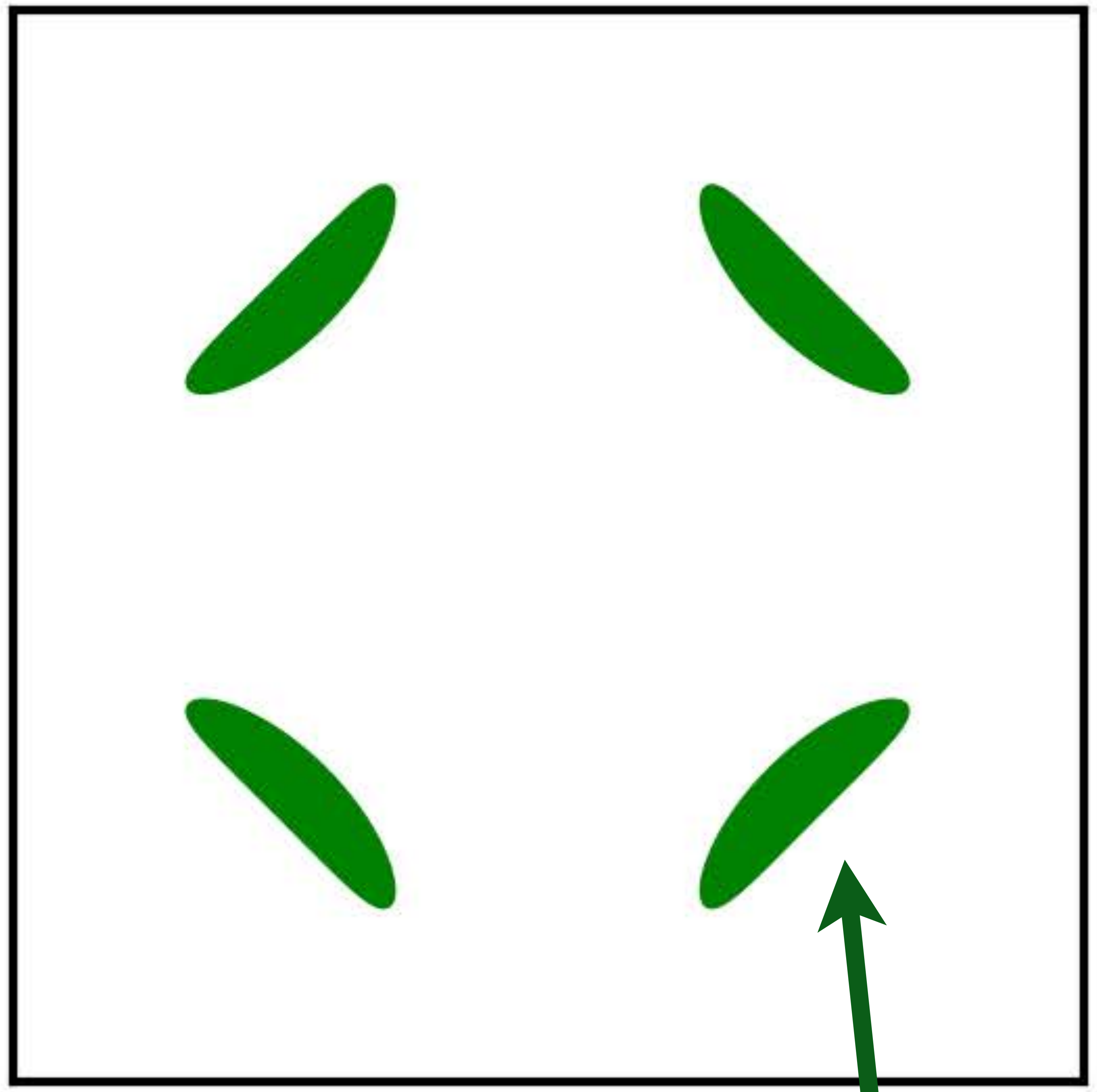
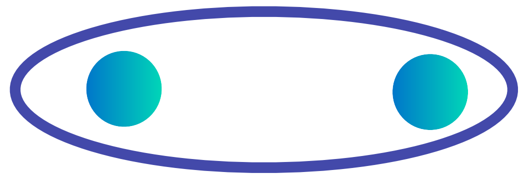
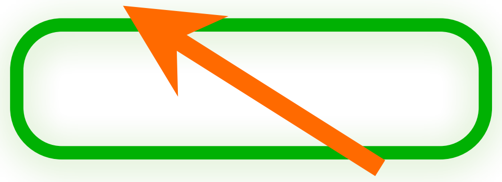


Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)



Area $p/8$

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

Doping an insulating antiferromagnet with holes of density p

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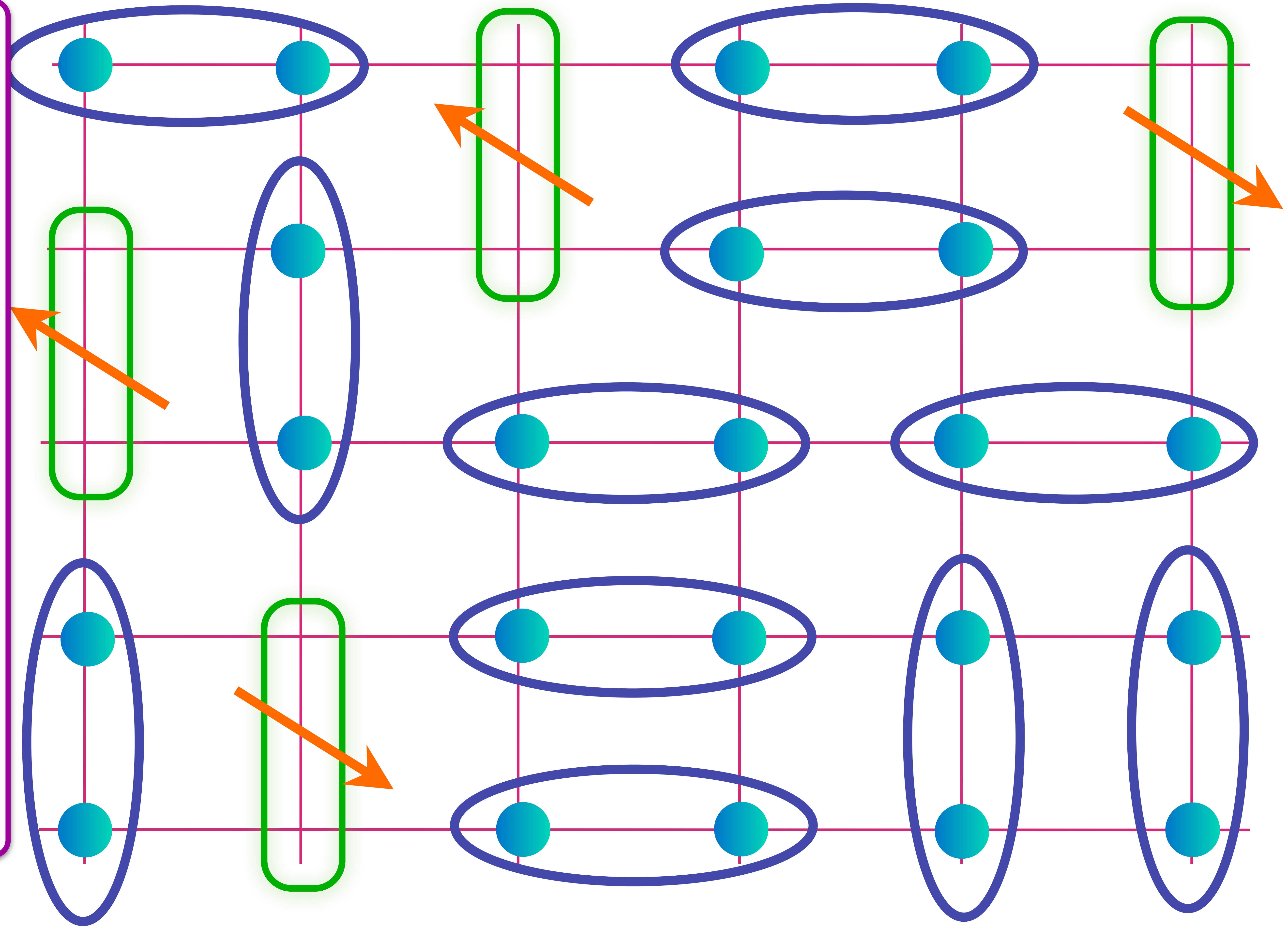

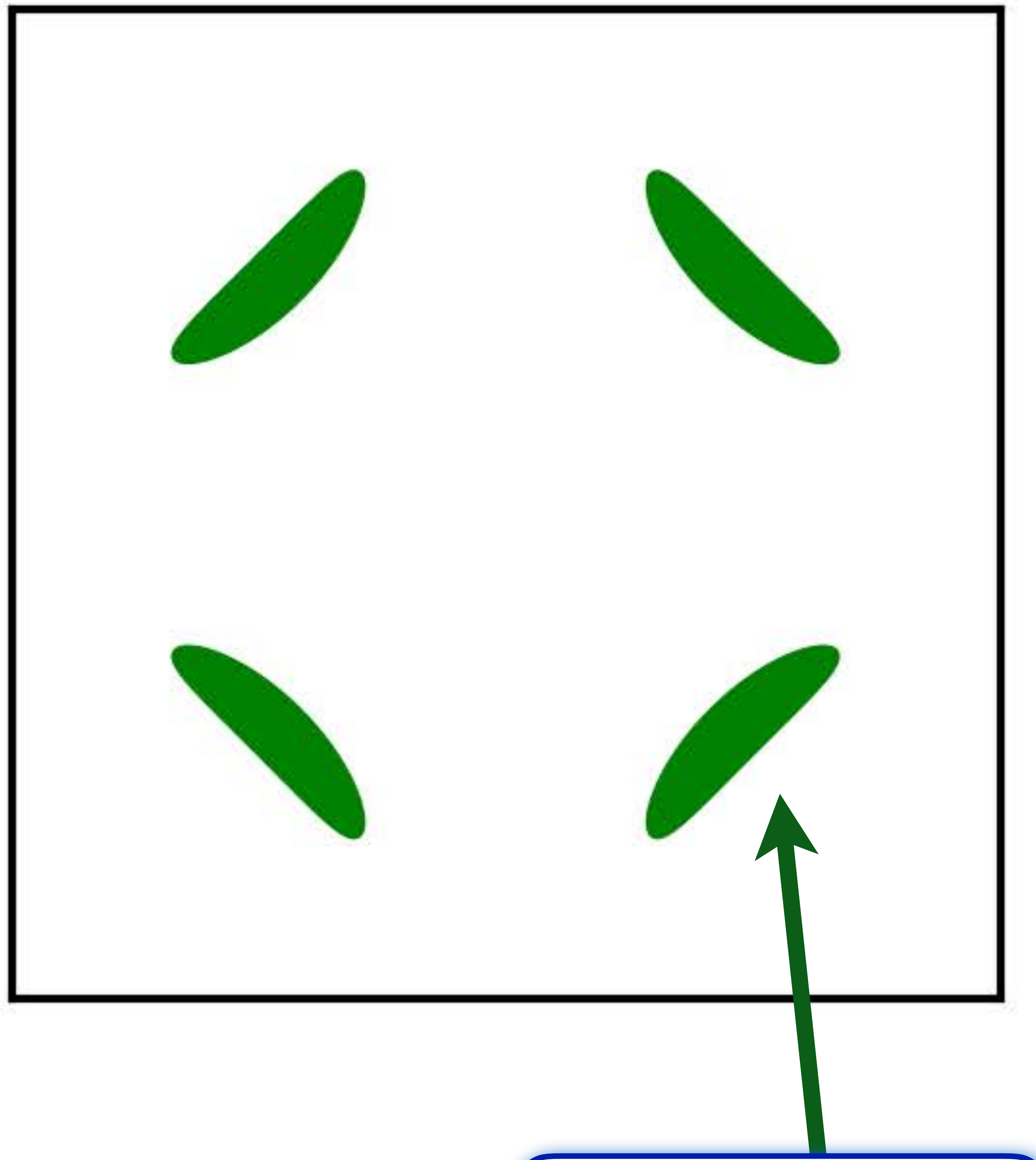

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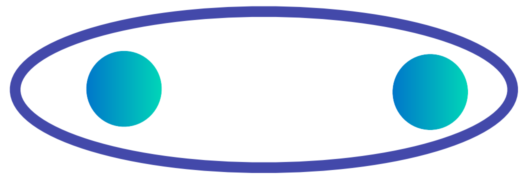
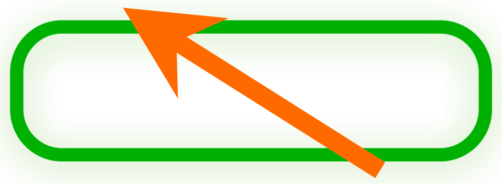
 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

 $= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

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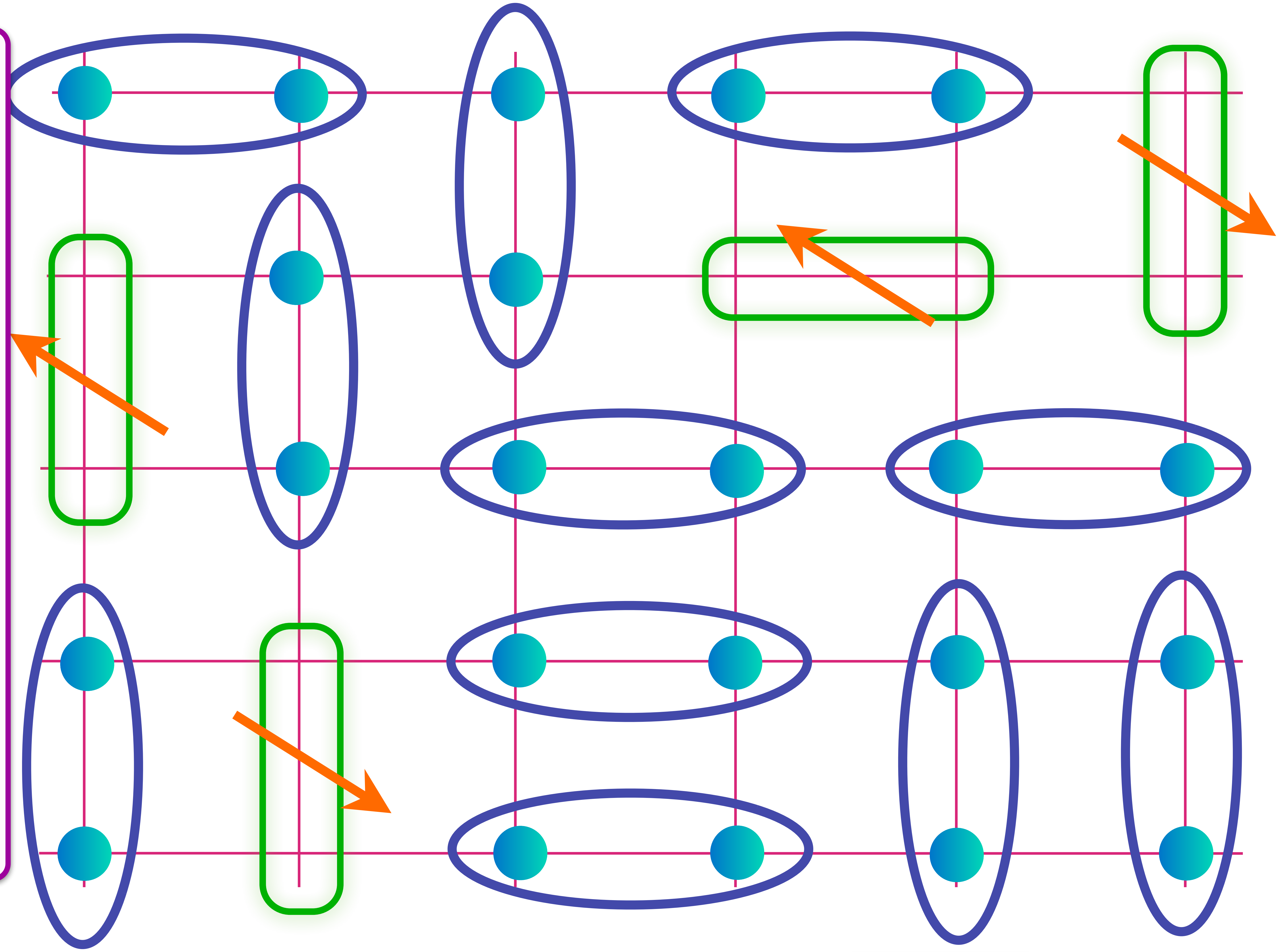

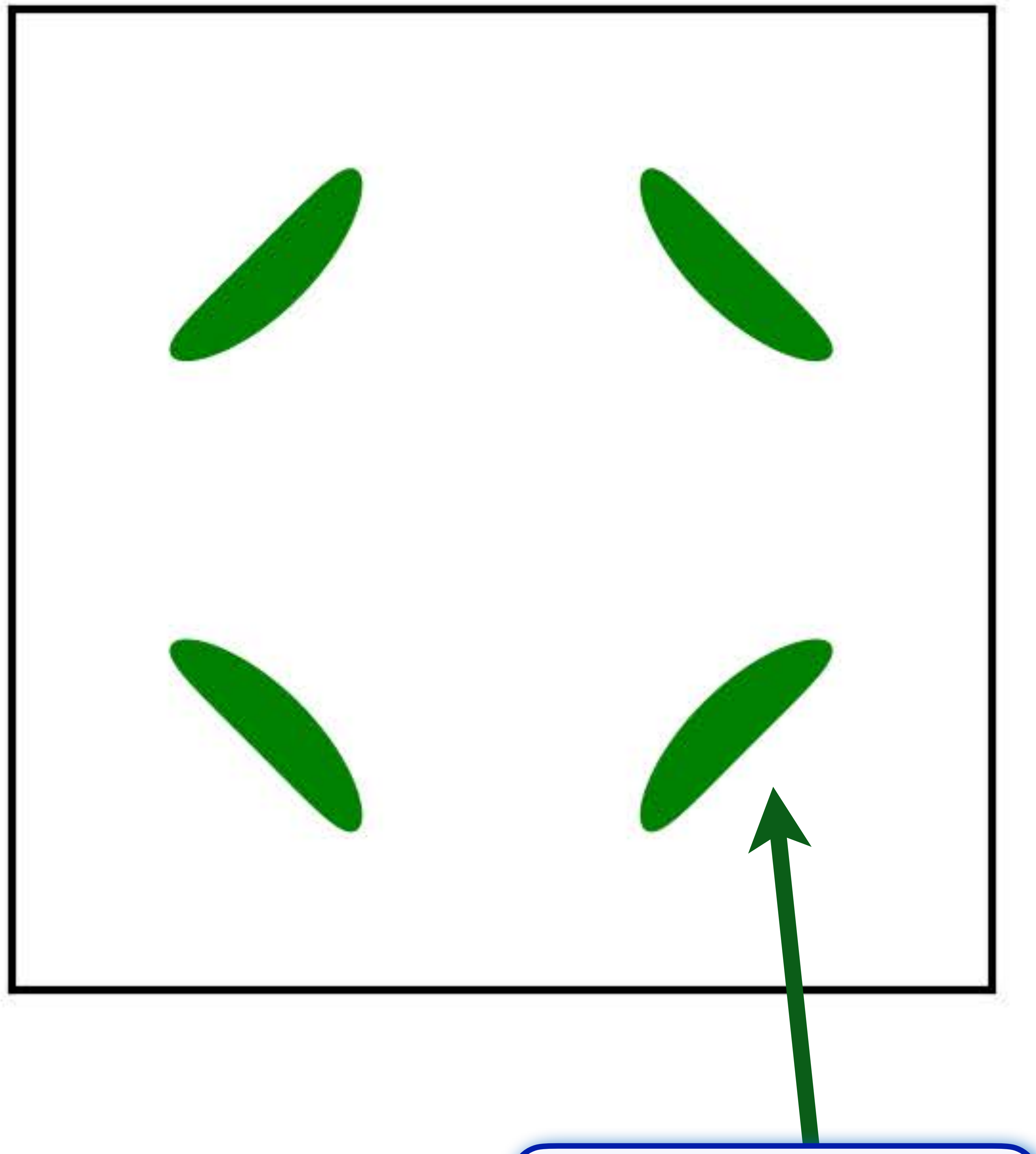

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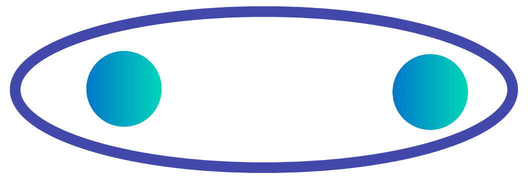
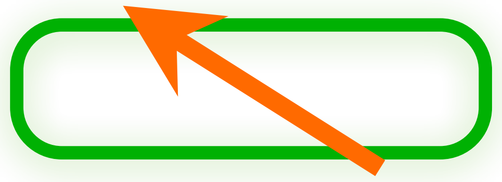
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$  $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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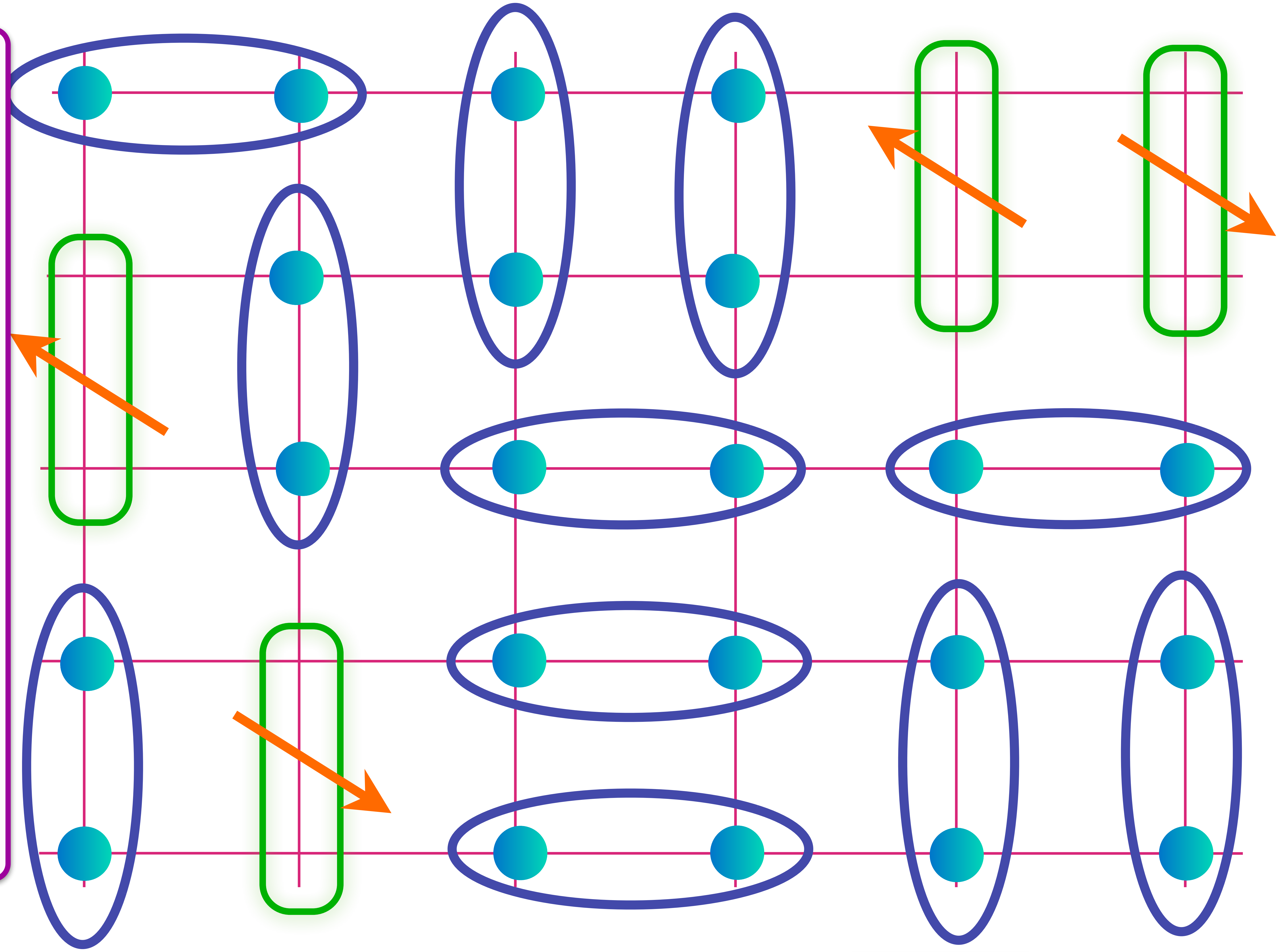

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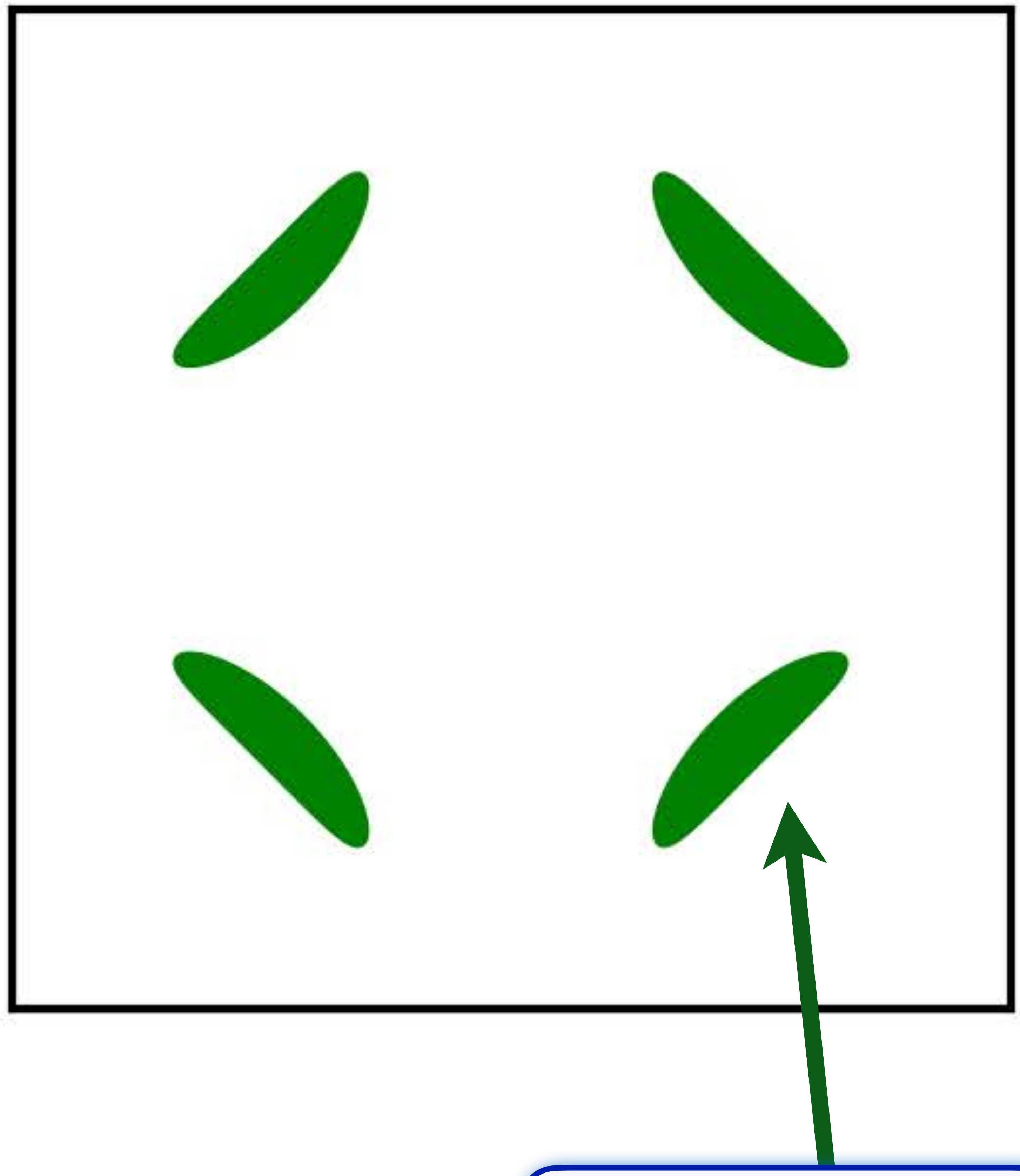

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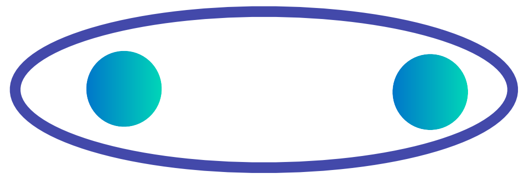
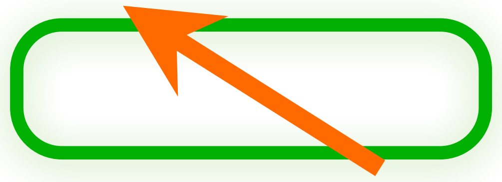
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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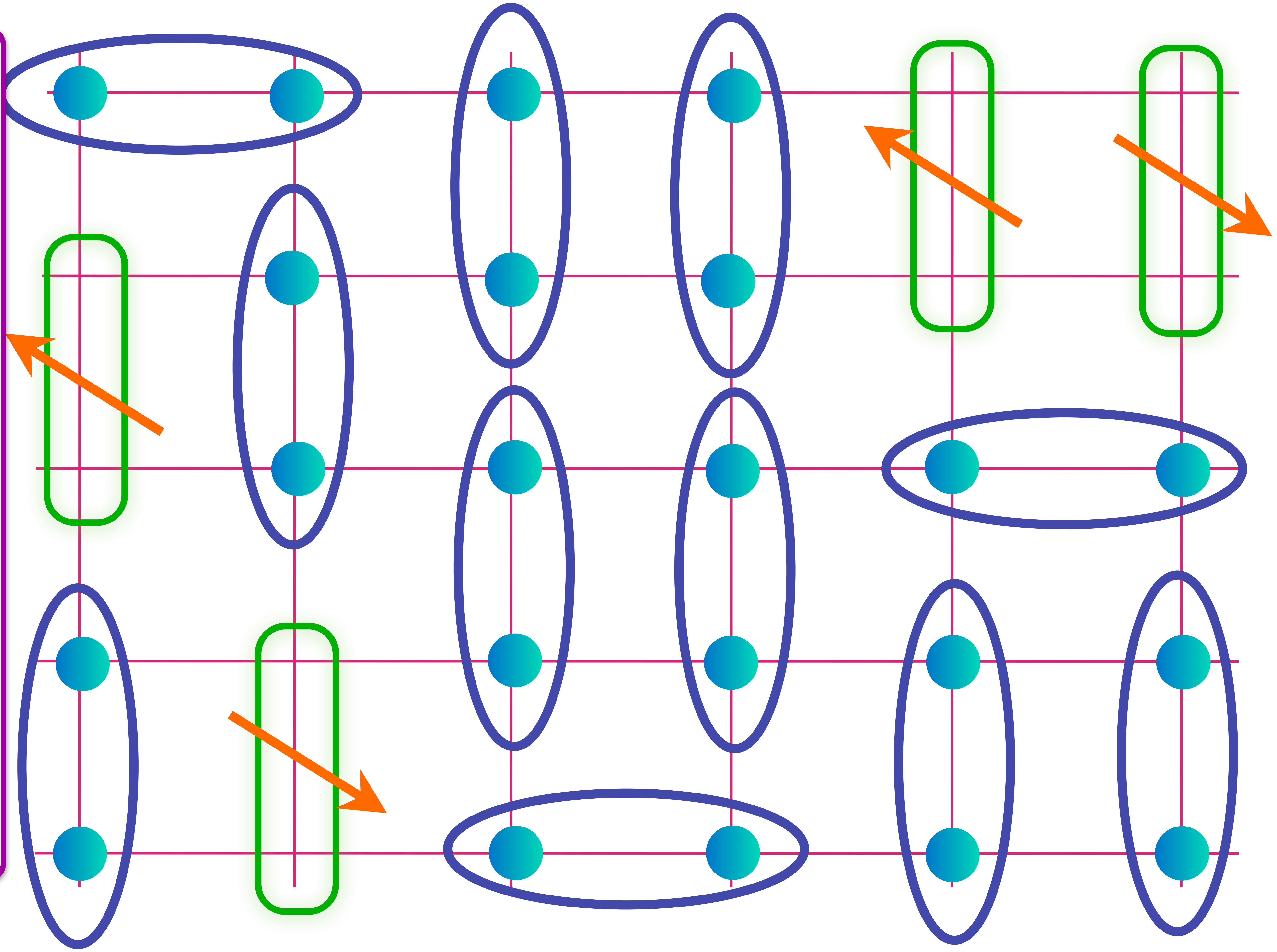

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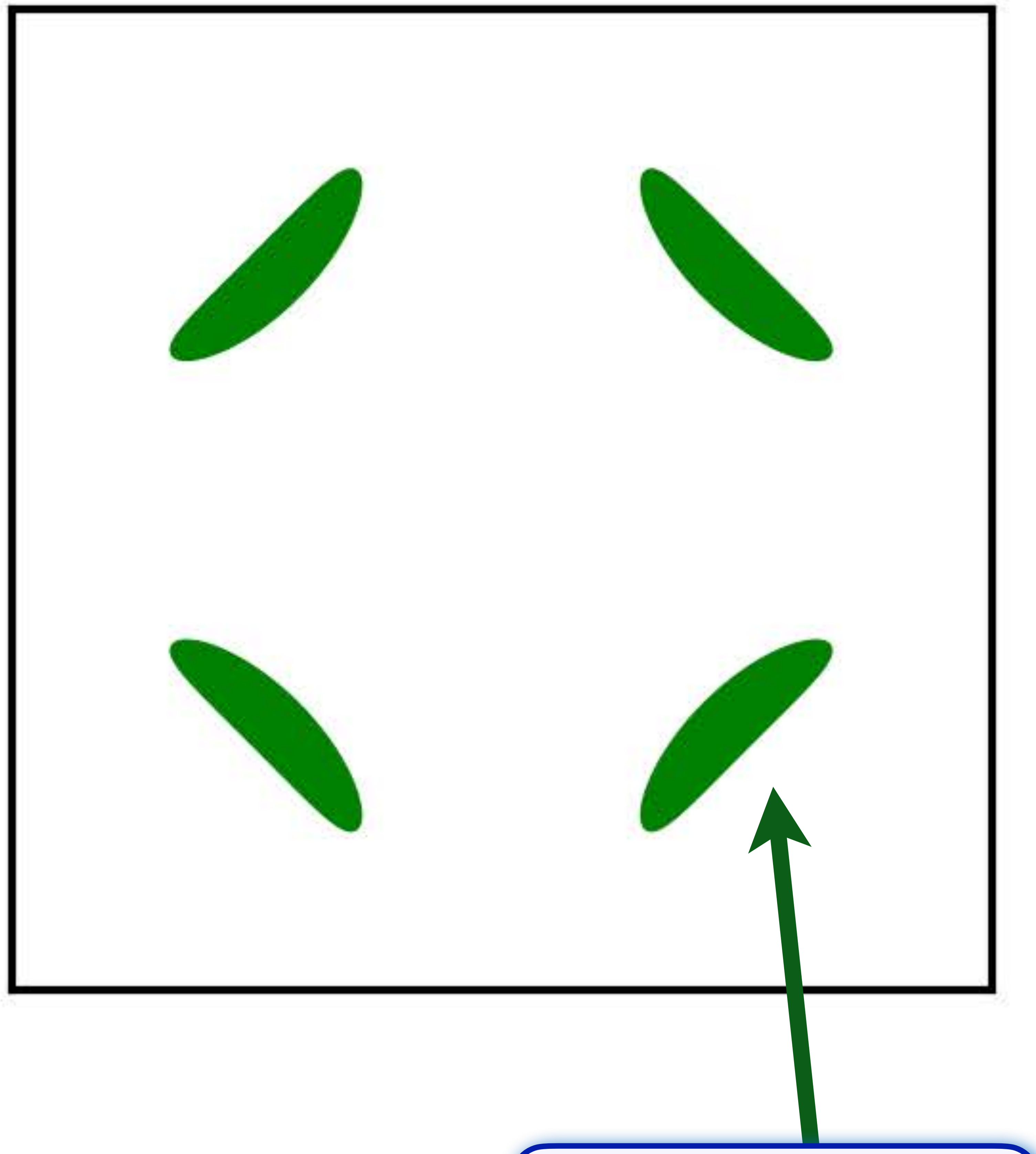

Area $p/8$

$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$ $\quad$ $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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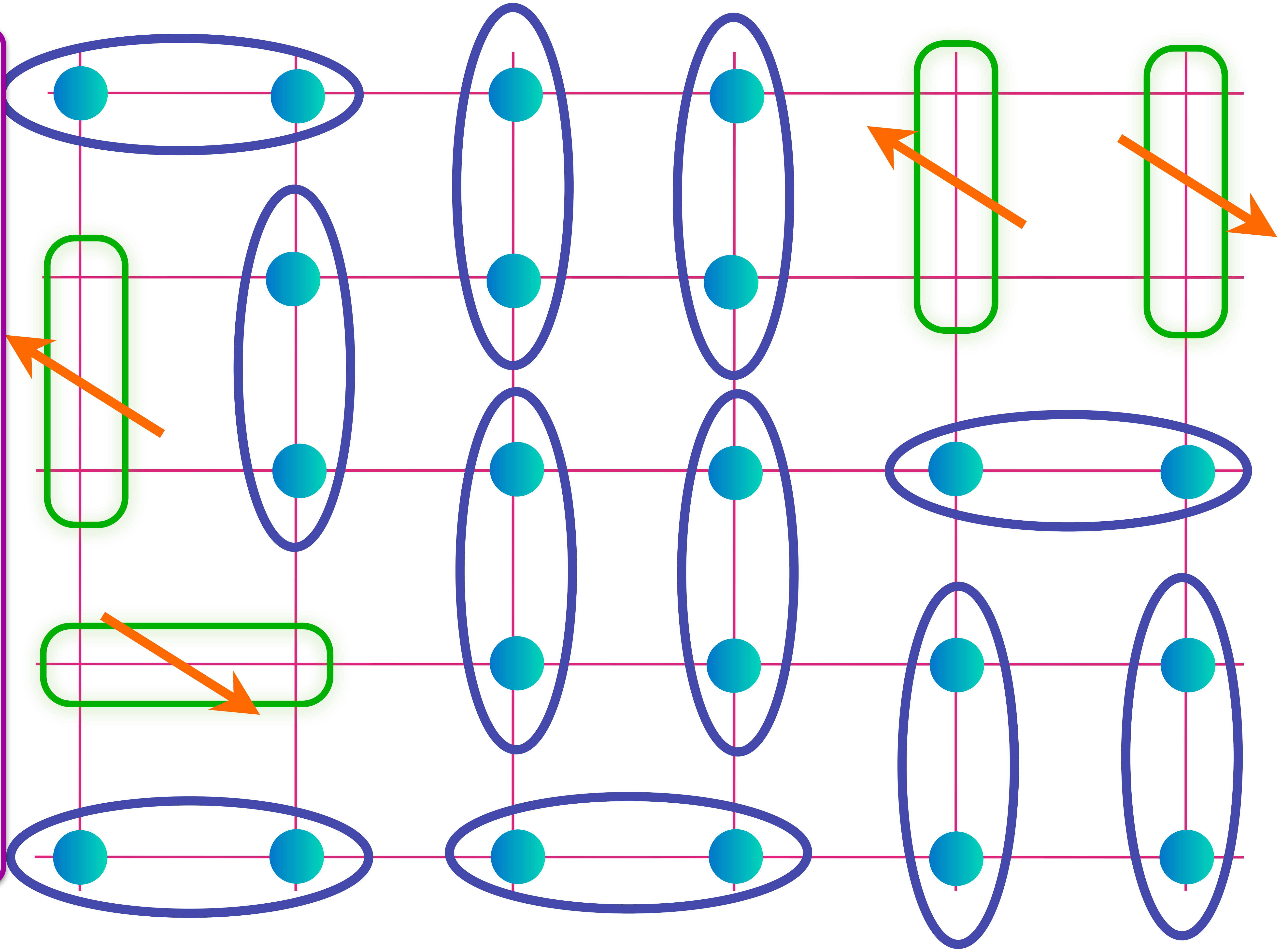

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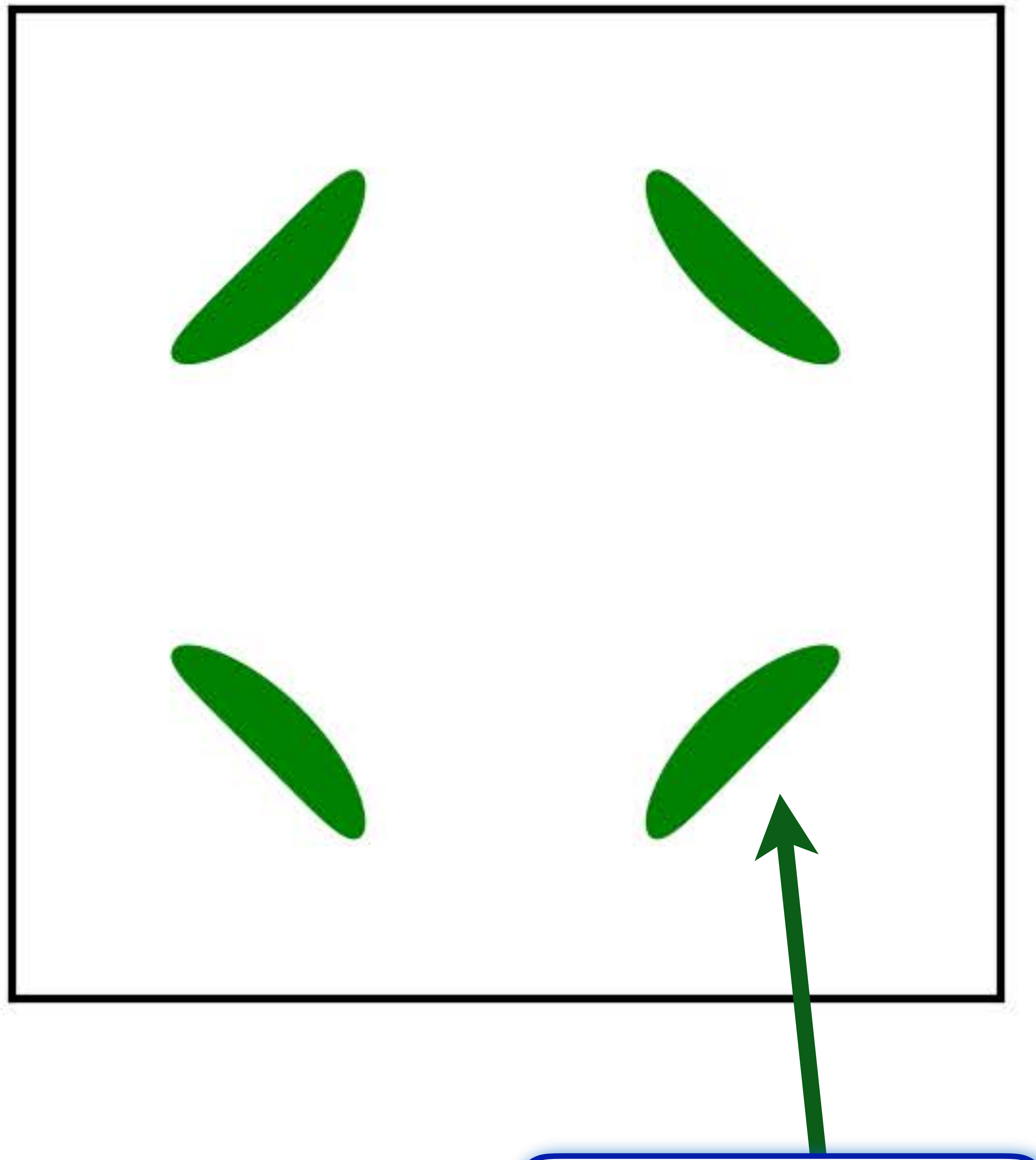
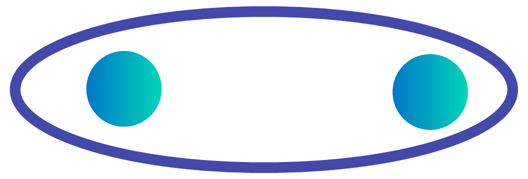
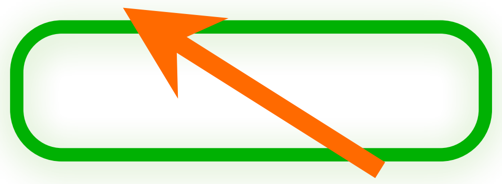


Area $p/8$

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$  $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

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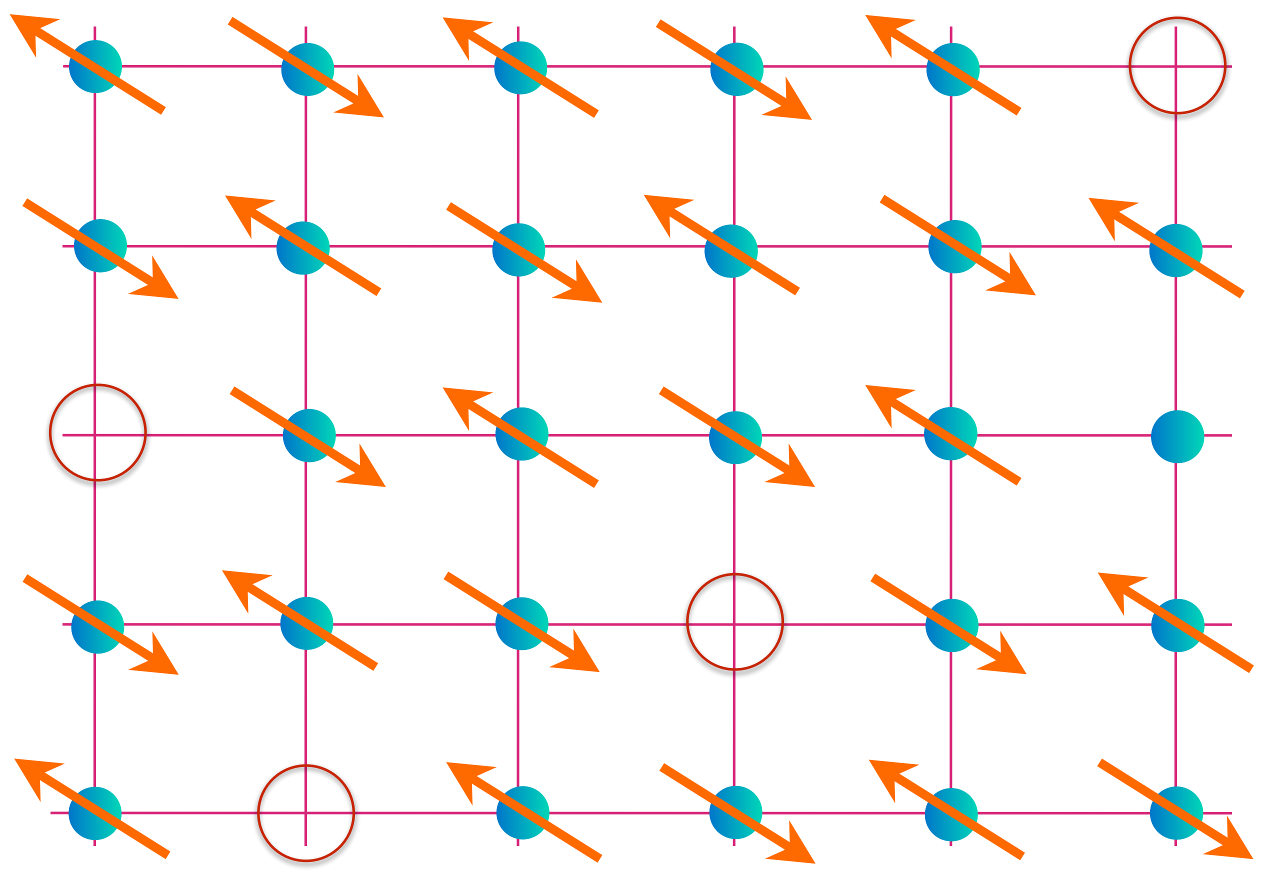
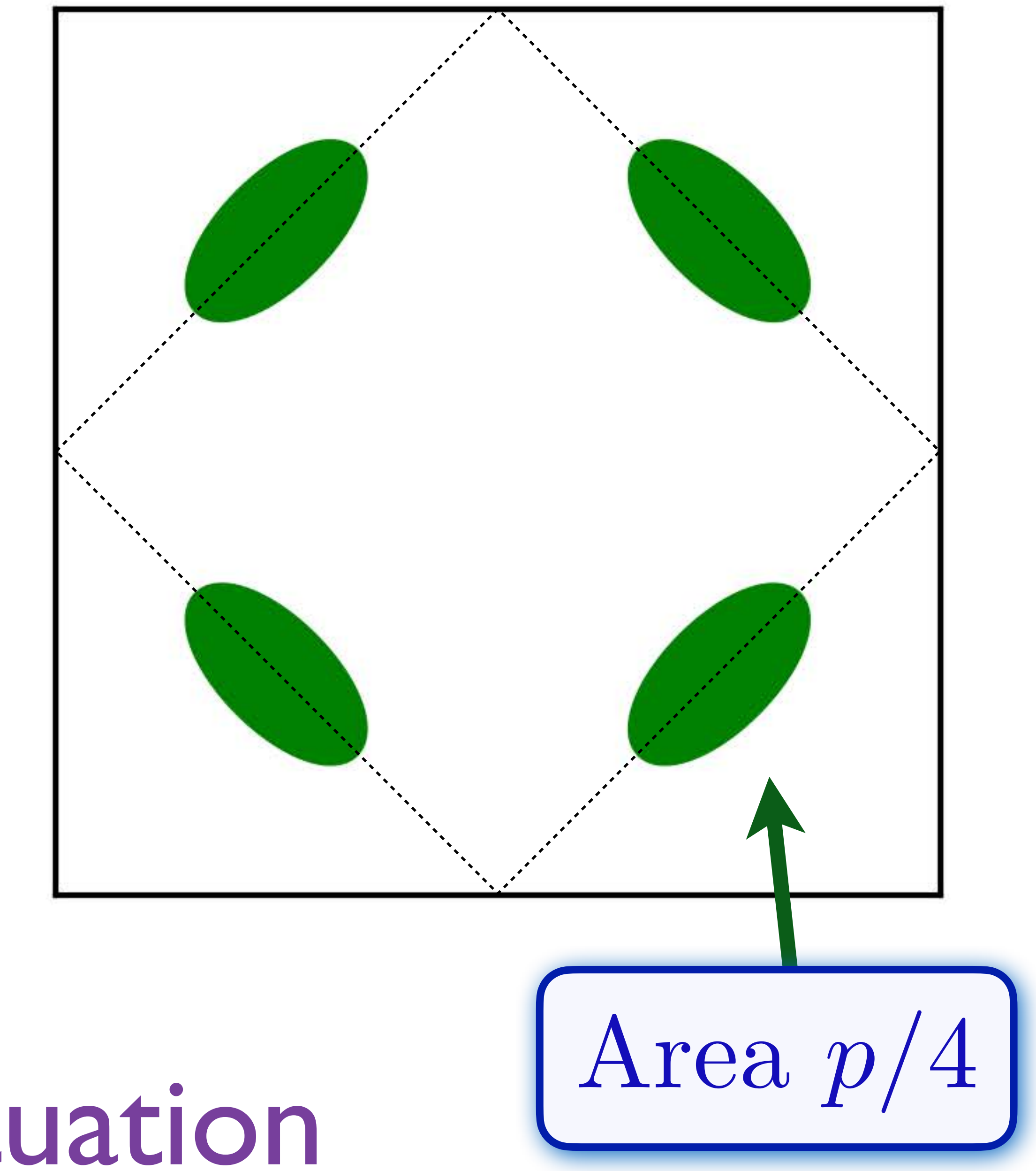
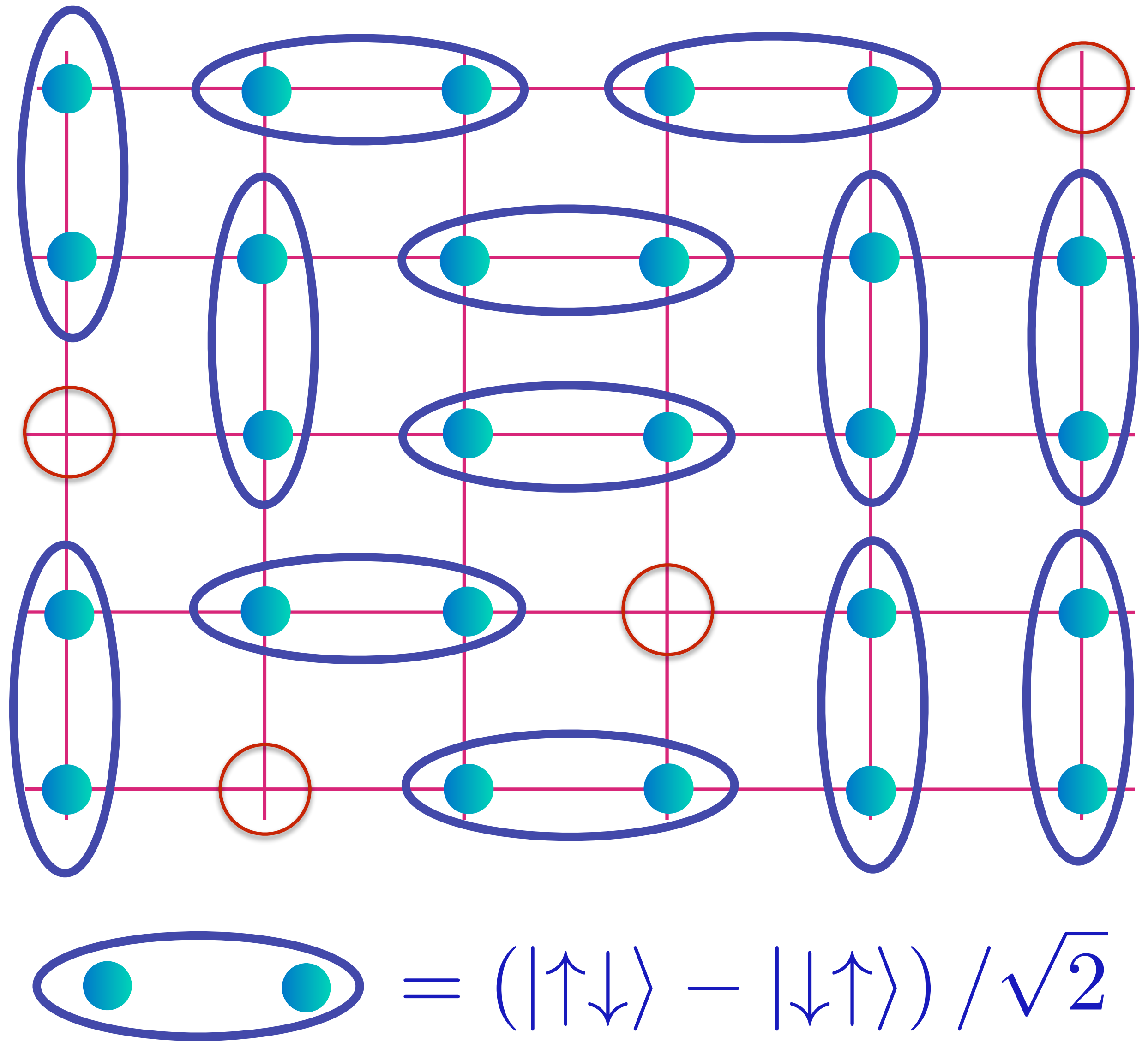
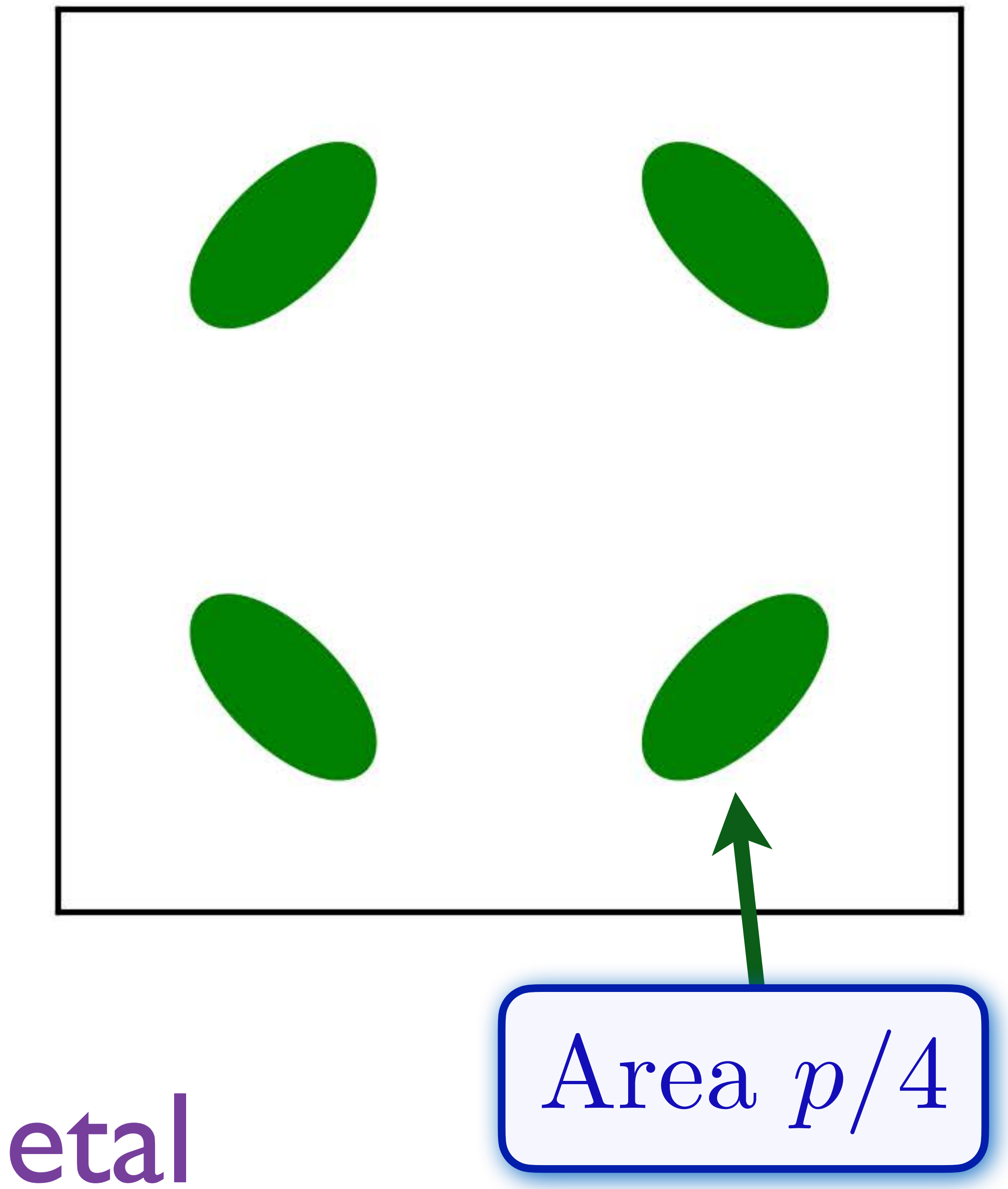




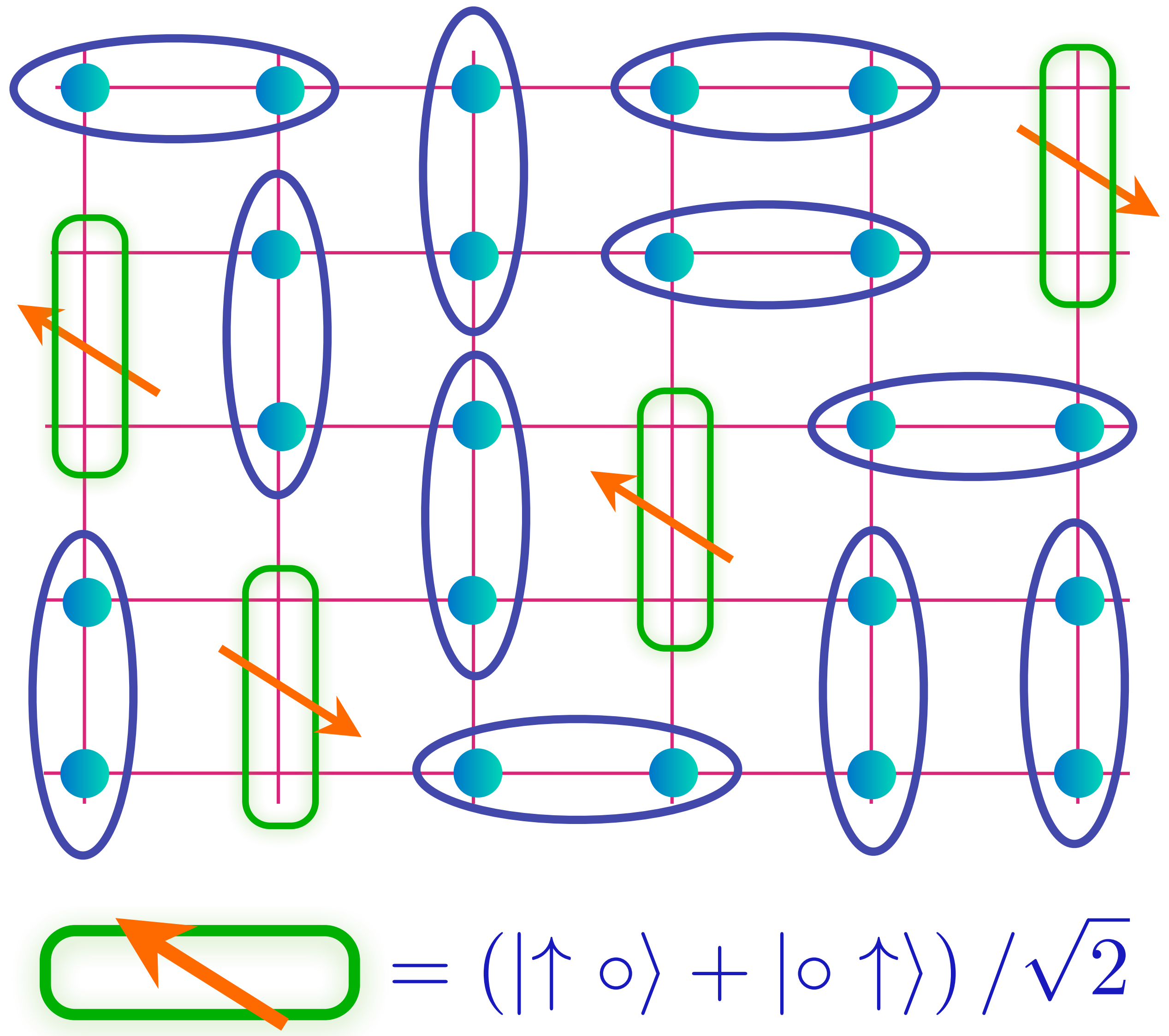
AF metal and SDW fluctuation



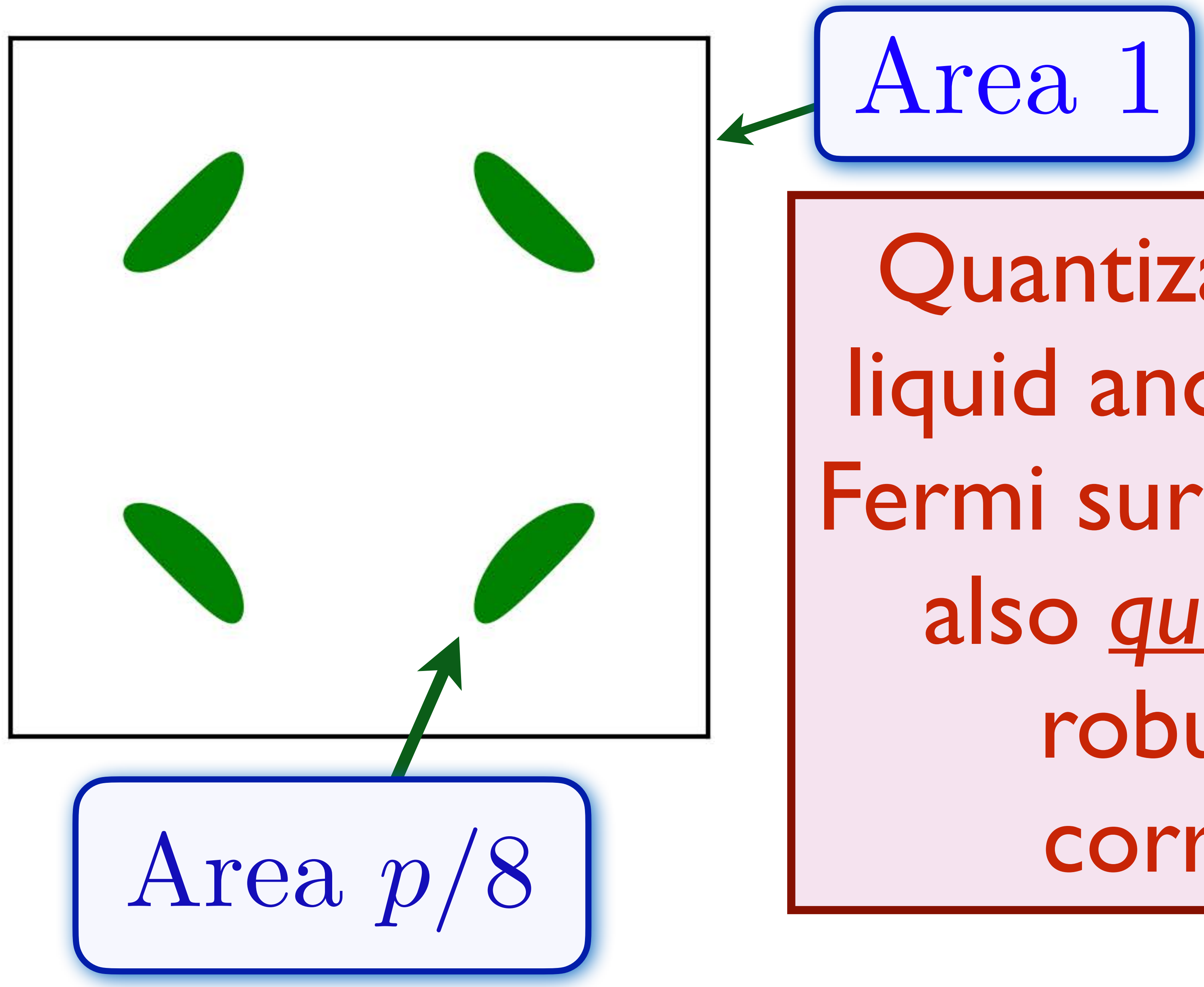


Holon metal

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003);
R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. S., PNAS **112**, 9552 (2015)
E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, S. S., PRB **105**, 075146 (2022)



FL*

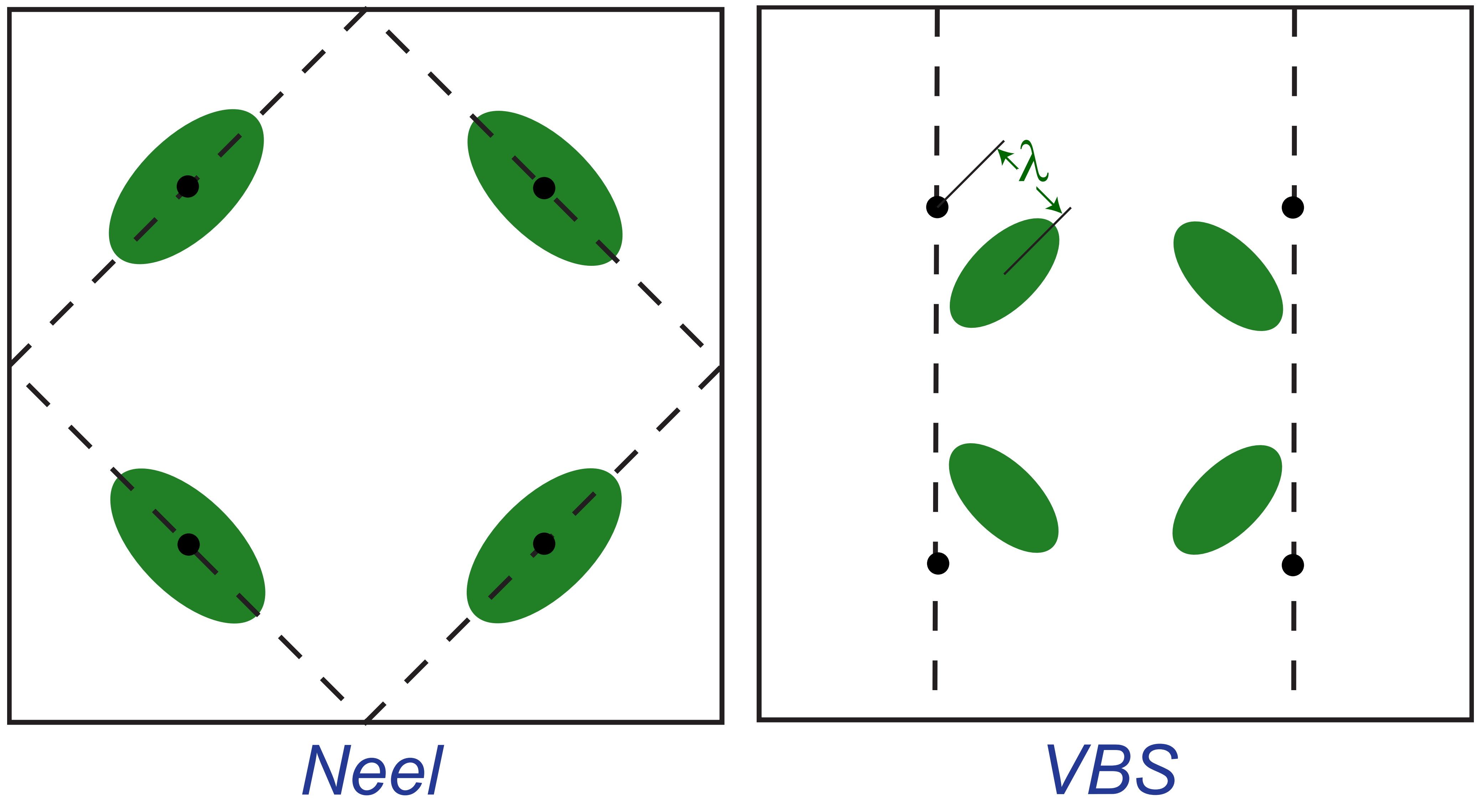


Quantization of spin liquid anomaly implies Fermi surface areas are also *quantized* and robust to all corrections.

PHYSICAL REVIEW B **75**, 235122 (2007)

Hole dynamics in an antiferromagnet across a deconfined quantum critical point

Ribhu K. Kaul,[1] Alexei Kolezhuk,[1,2] Michael Levin,[1] Subir Sachdev,[1] and T. Senthil[3,4]



The dashed line in the Néel phase indicates the boundary of the magnetic Brillouin zone. Only the Fermi surfaces within this zone contribute to the Luttinger counting, and so the area of each ellipse is $\mathcal{A}_F=(2\pi)^2\delta/4$. In the VBS phase, all four pockets are inequivalent, and so the area of each ellipse is $\mathcal{A}_F=(2\pi)^2\delta/8$.

Factor of 2 between SDW fluctuation and FL*

Observation of the Yamaji effect in the cuprate pseudogap

See also:

Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, *Nature Physics* **18**, 558 (2022)

Angle-dependent magnetoresistance (ADMR) of $La_{1.6-x}Nd_{0.4}Sr_xCuO_4$

Observation of the Yamaji effect in a cuprate superconductor

Mun K. Chan[1]✉, Katherine A. Schreiber[1], Oscar E. Ayala-Valenzuela[1], Eric D. Bauer[2], Arkady Shekhter[1] & Neil Harrison[1]

nature physics
21, 1753 (2025)
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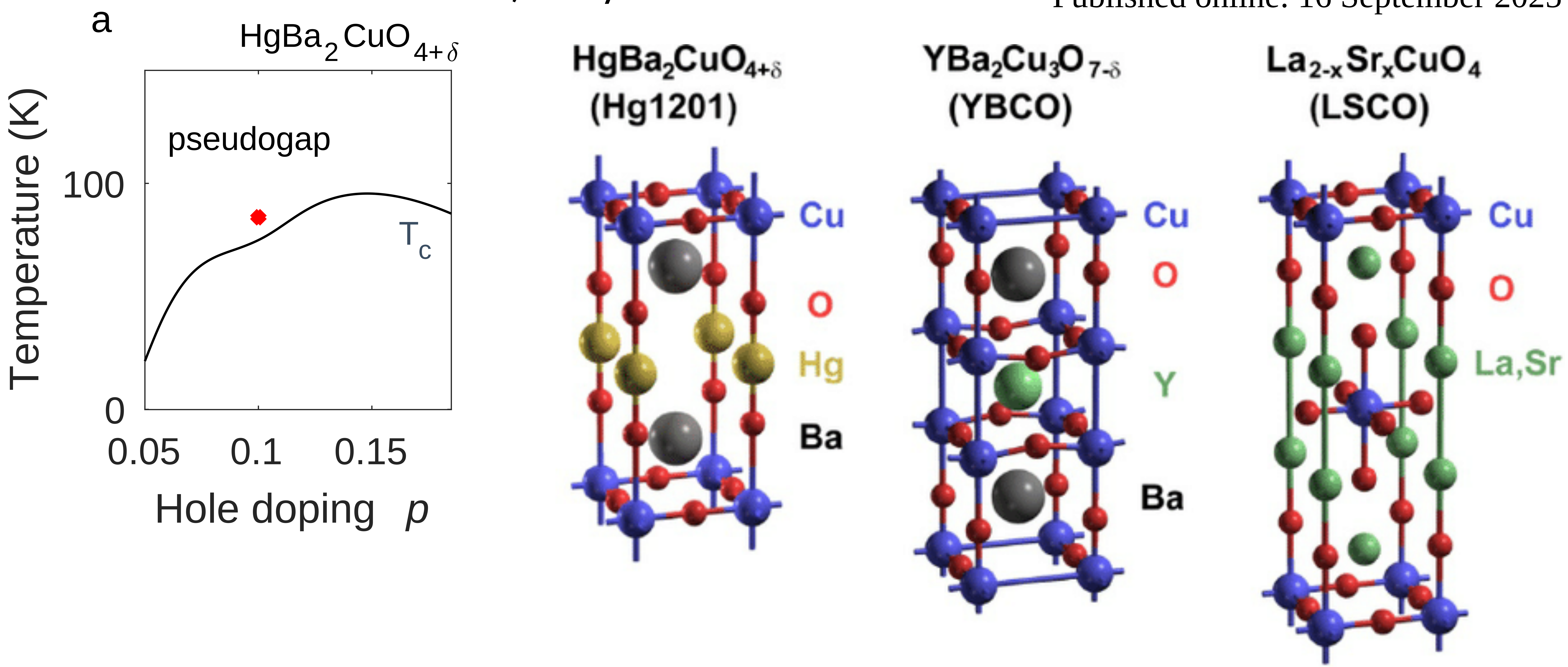

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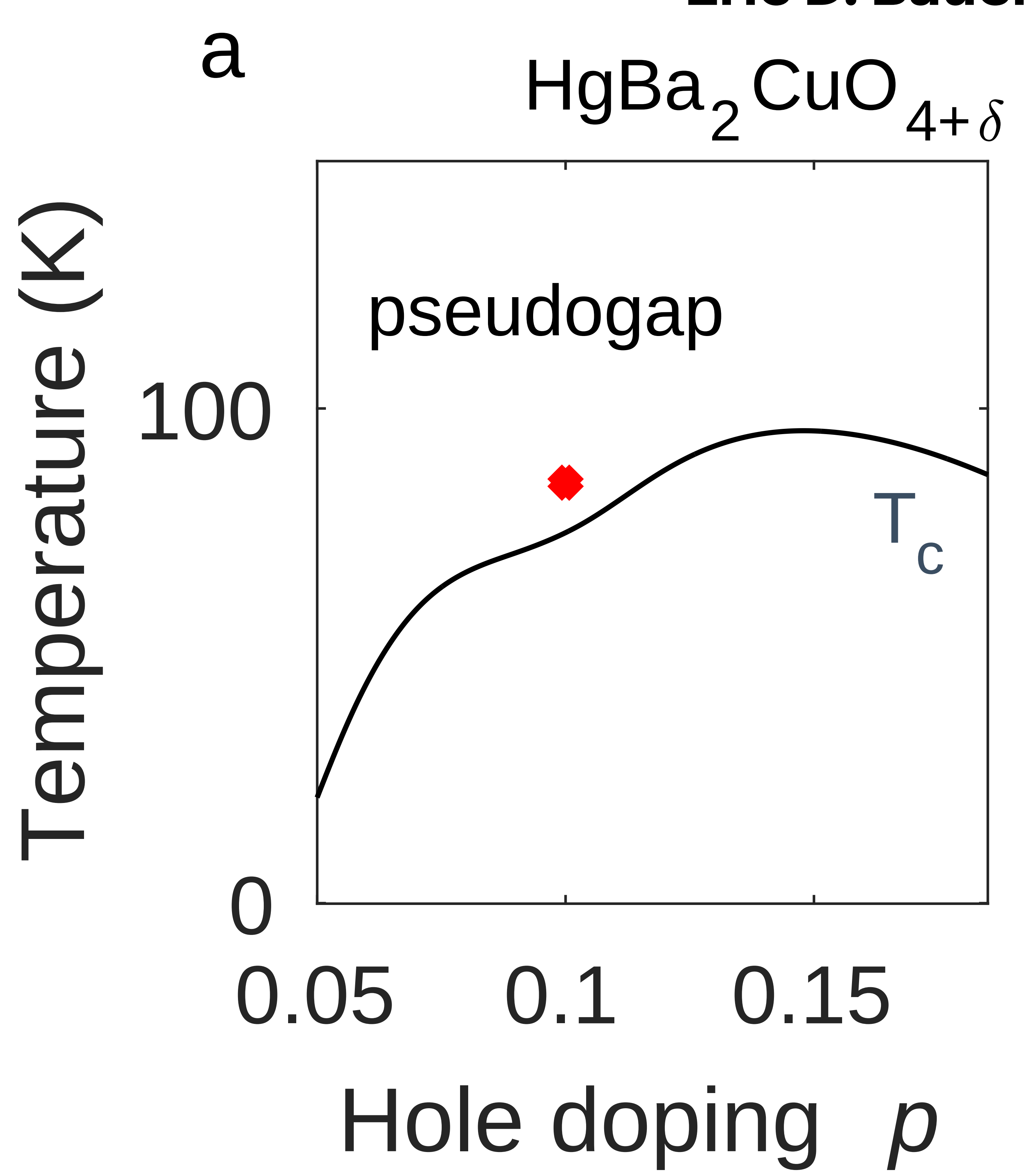

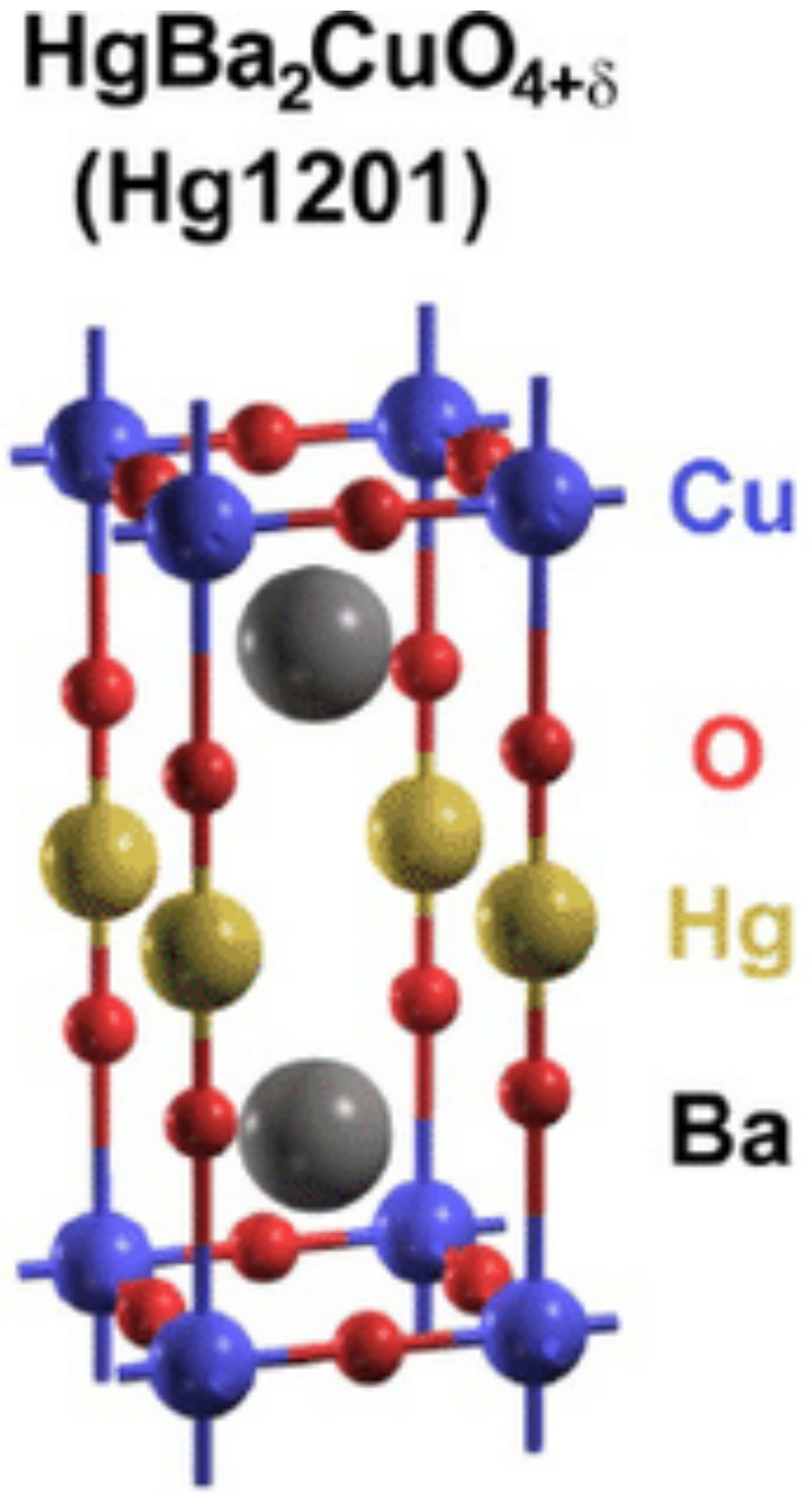

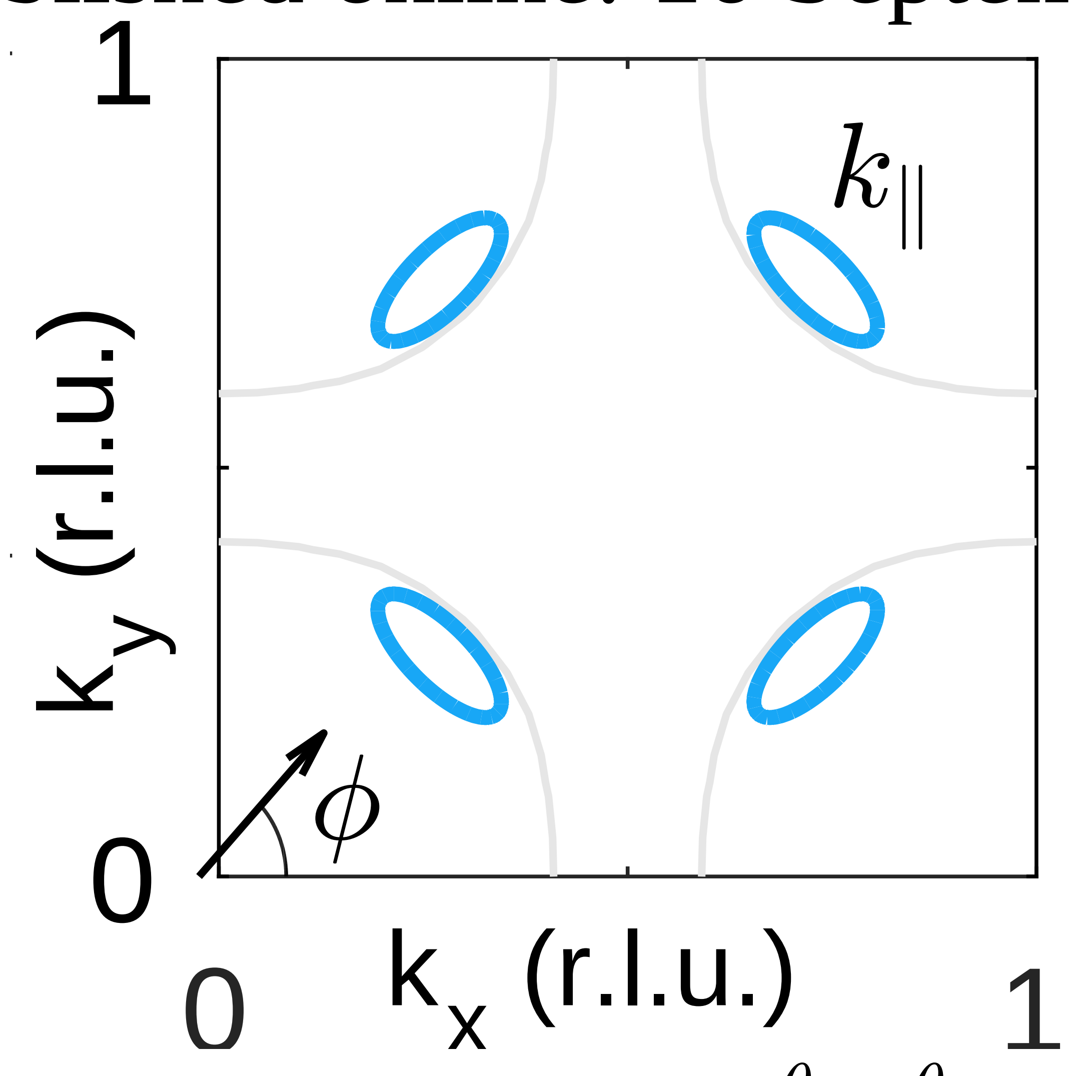

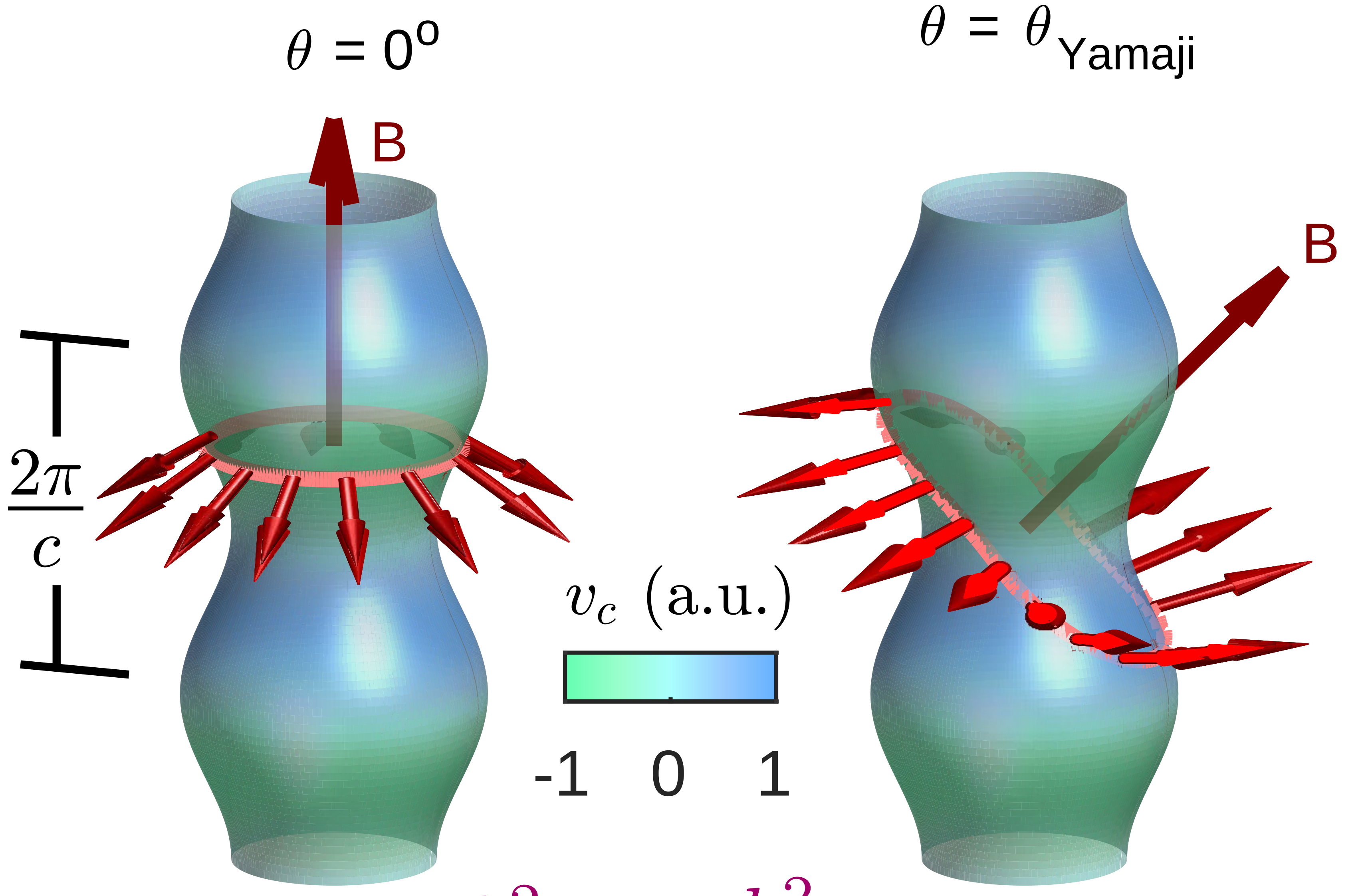

At the Yamaji angle, the orbits in the plane orthogonal to $\boldsymbol{B}$ have an area which is independent of momentum in the c direction, to first order in the hopping along the c direction.

K.Yamaji JPSJ **58**, 1520 (1989)

$$\epsilon(\mathbf{k}) = \frac{k_x^2}{2m_1} + \frac{k_y^2}{2m_2} - 2t_\perp \cos(k_z c)$$

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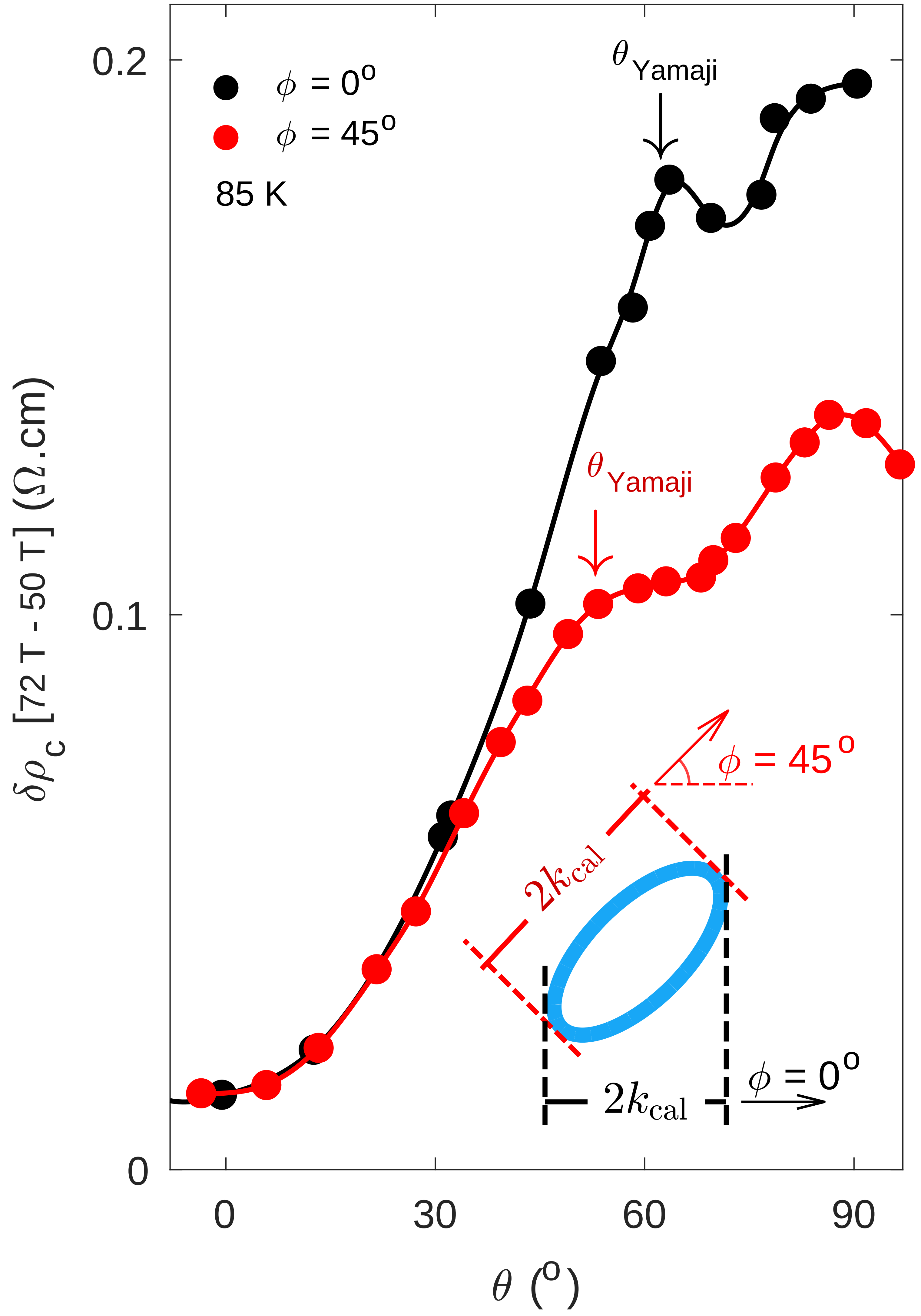

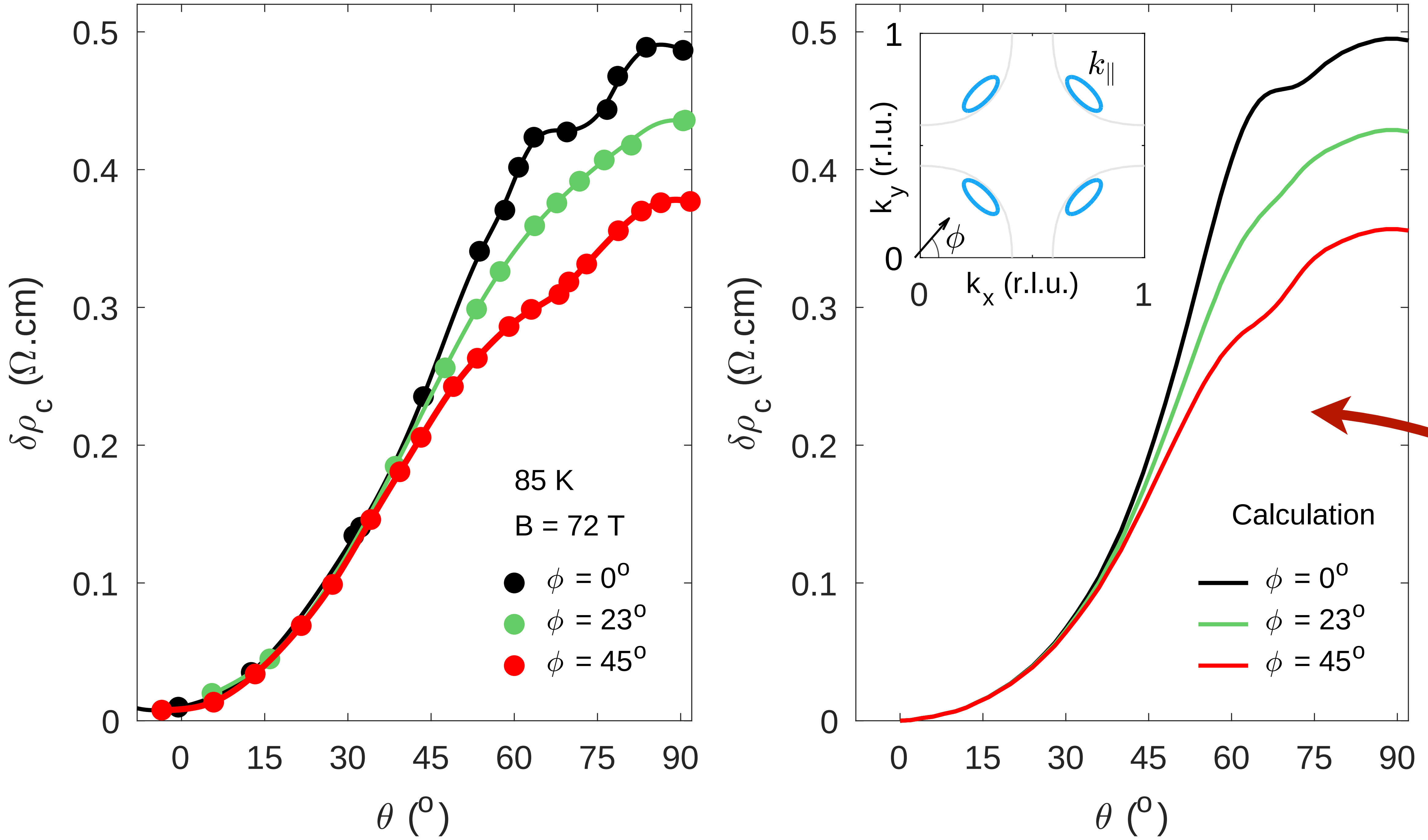

Doping
$p = 0.1$

The observation of the Yamaji peak is evidence for small Fermi-surface pockets in the normal state of the pseudogap phase.

$$\frac{\partial f}{\partial t} + e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \mathbf{v} \left(-\frac{\partial f}{\partial \epsilon}\right) = -\frac{f - f_0}{\tau}$$

$$\mathbf{v} = \nabla_{\mathbf{k}} \epsilon(\mathbf{k}) \; ; \; f_0(\epsilon) = \frac{1}{e^{(\epsilon - \mu)/T} + 1}$$

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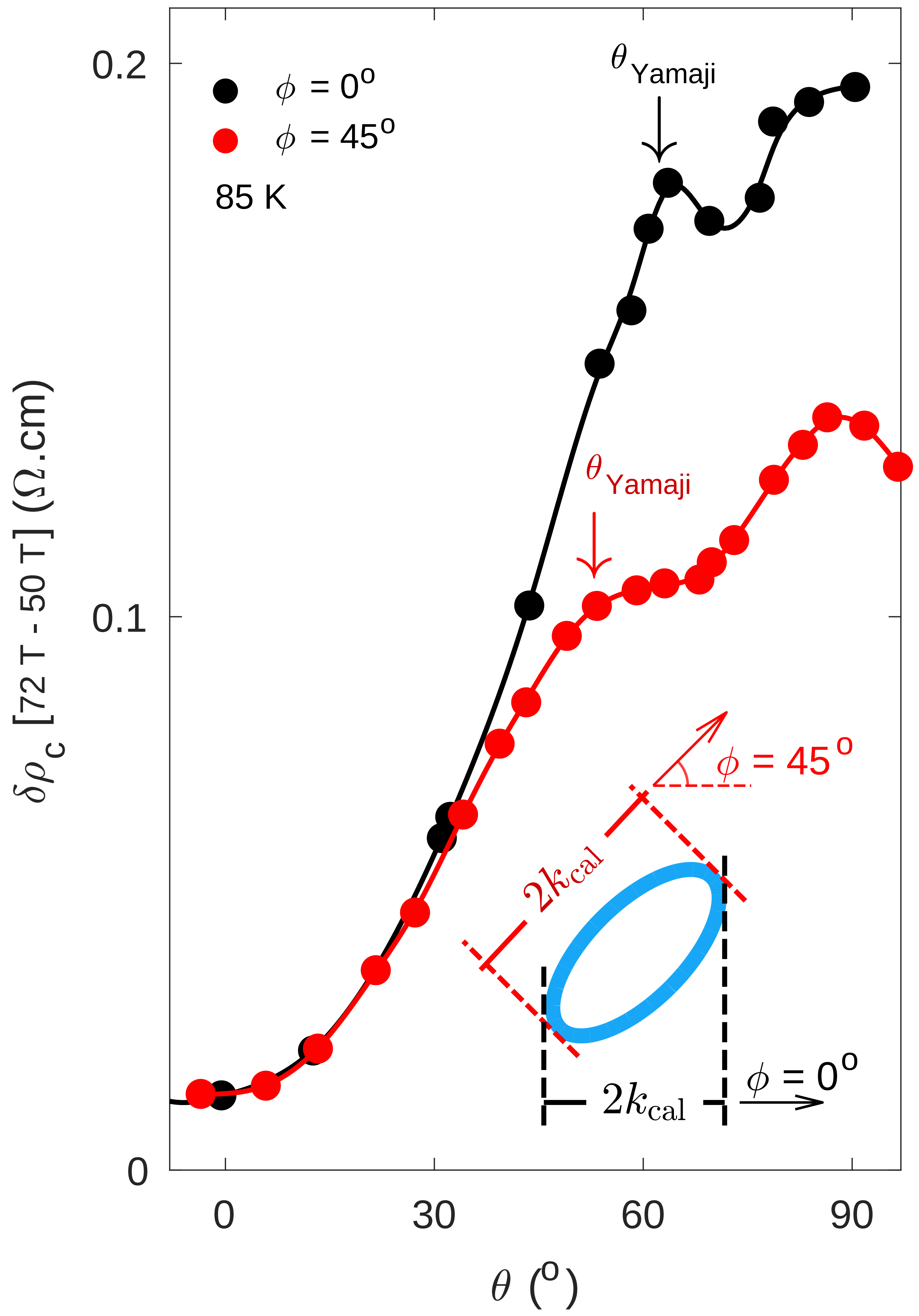

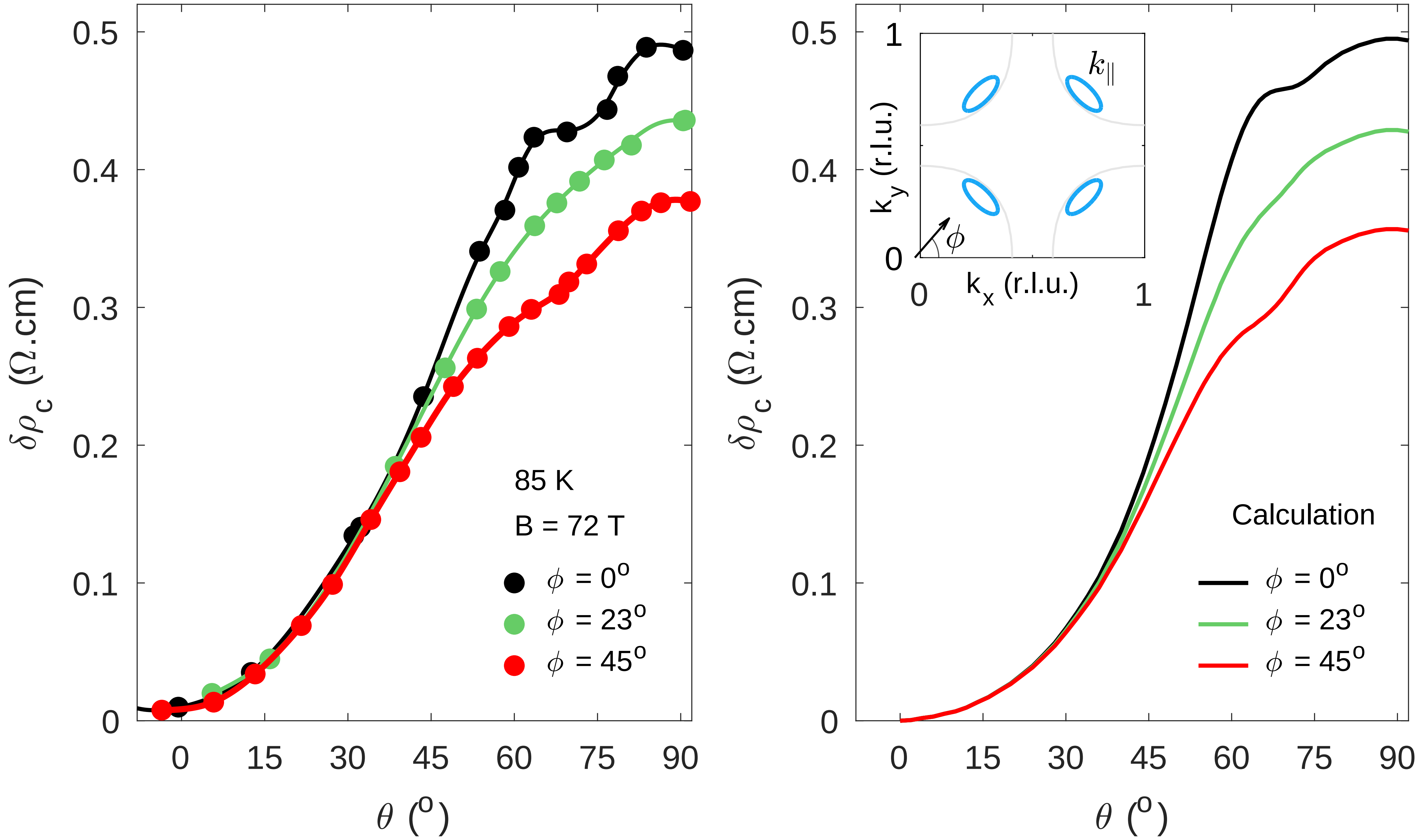

Doping
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Excellent evidence for hole pockets with coherent interlayer-transport.

Observation of the Yamaji effect in a cuprate superconductor

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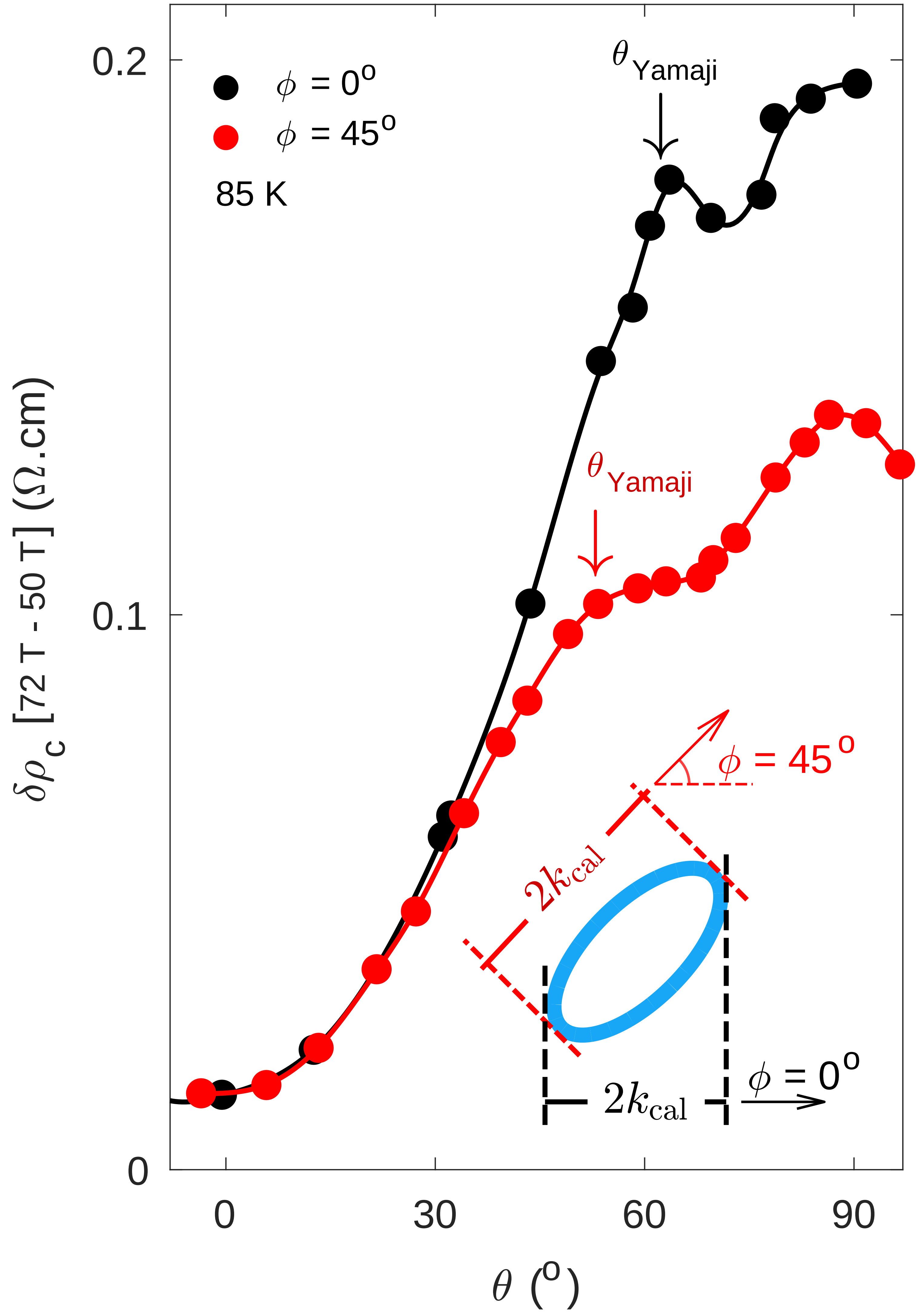

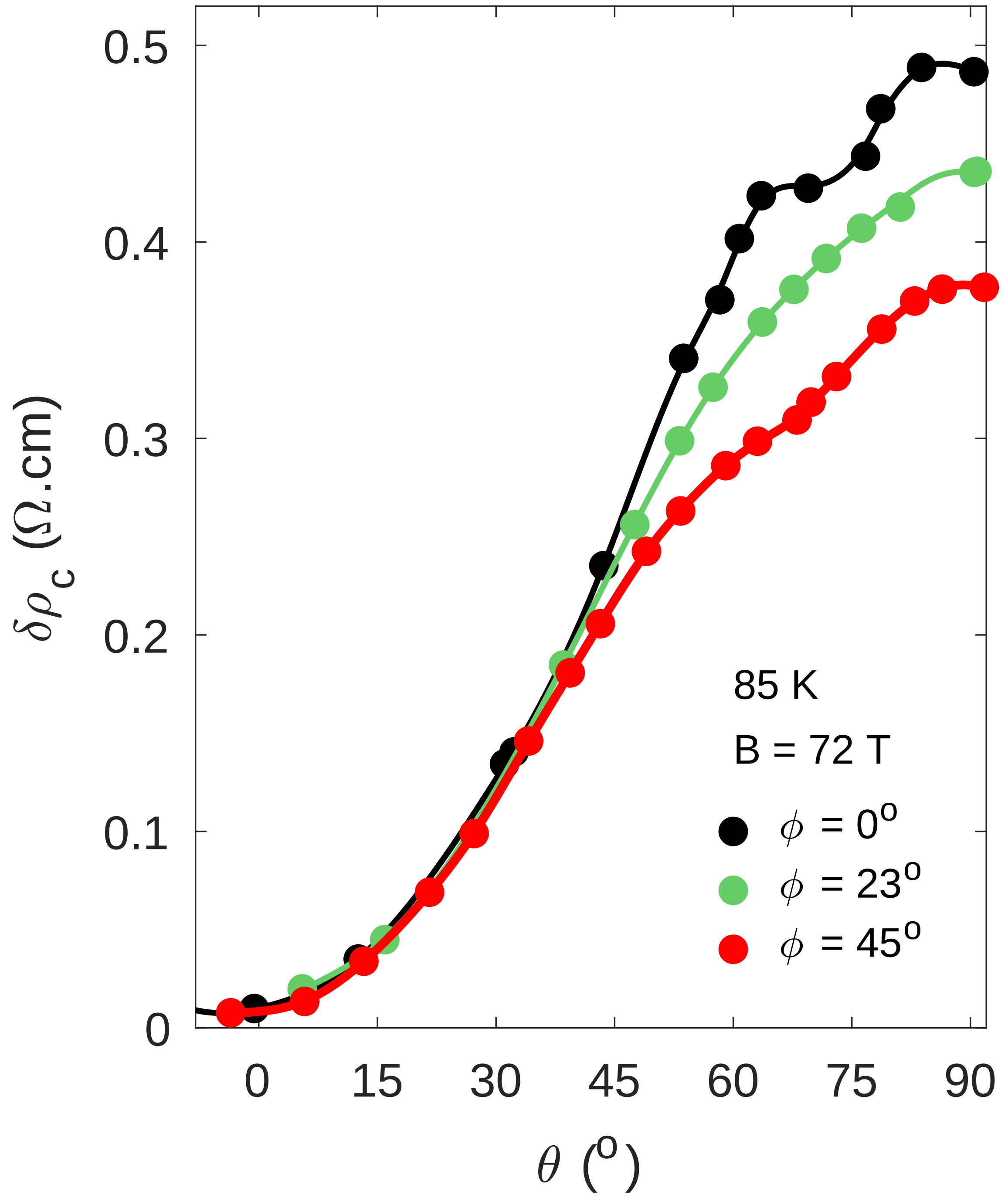

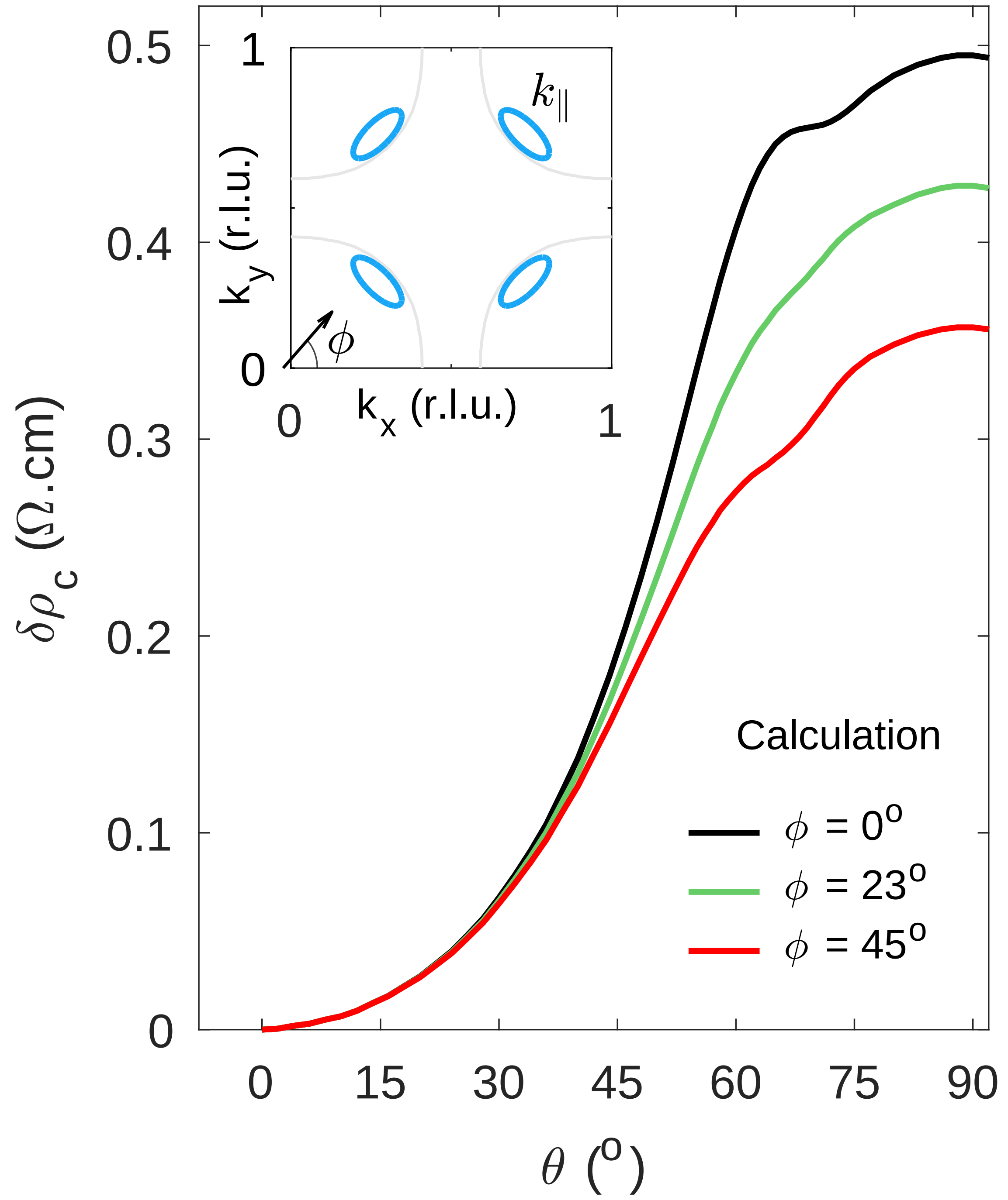

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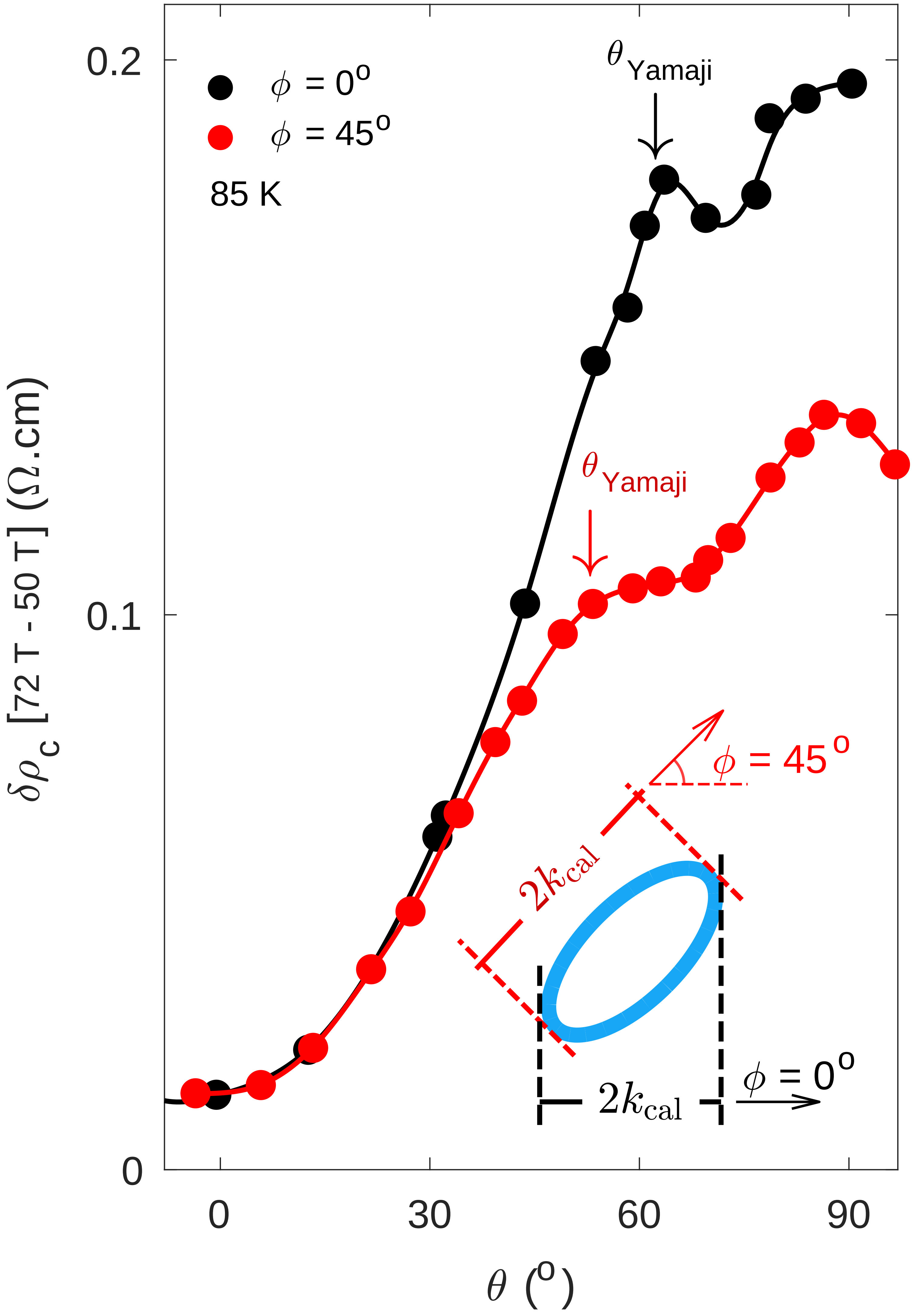

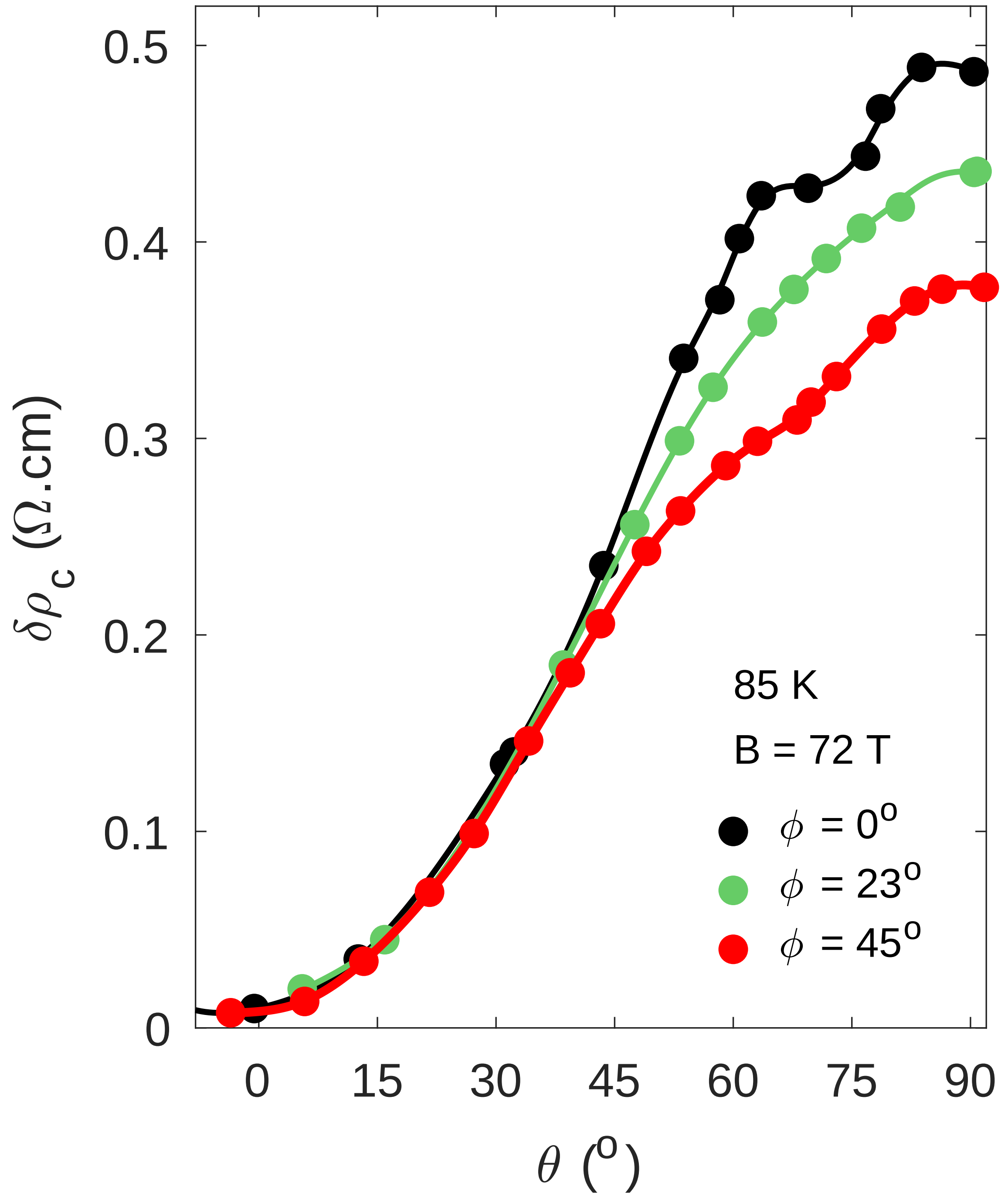

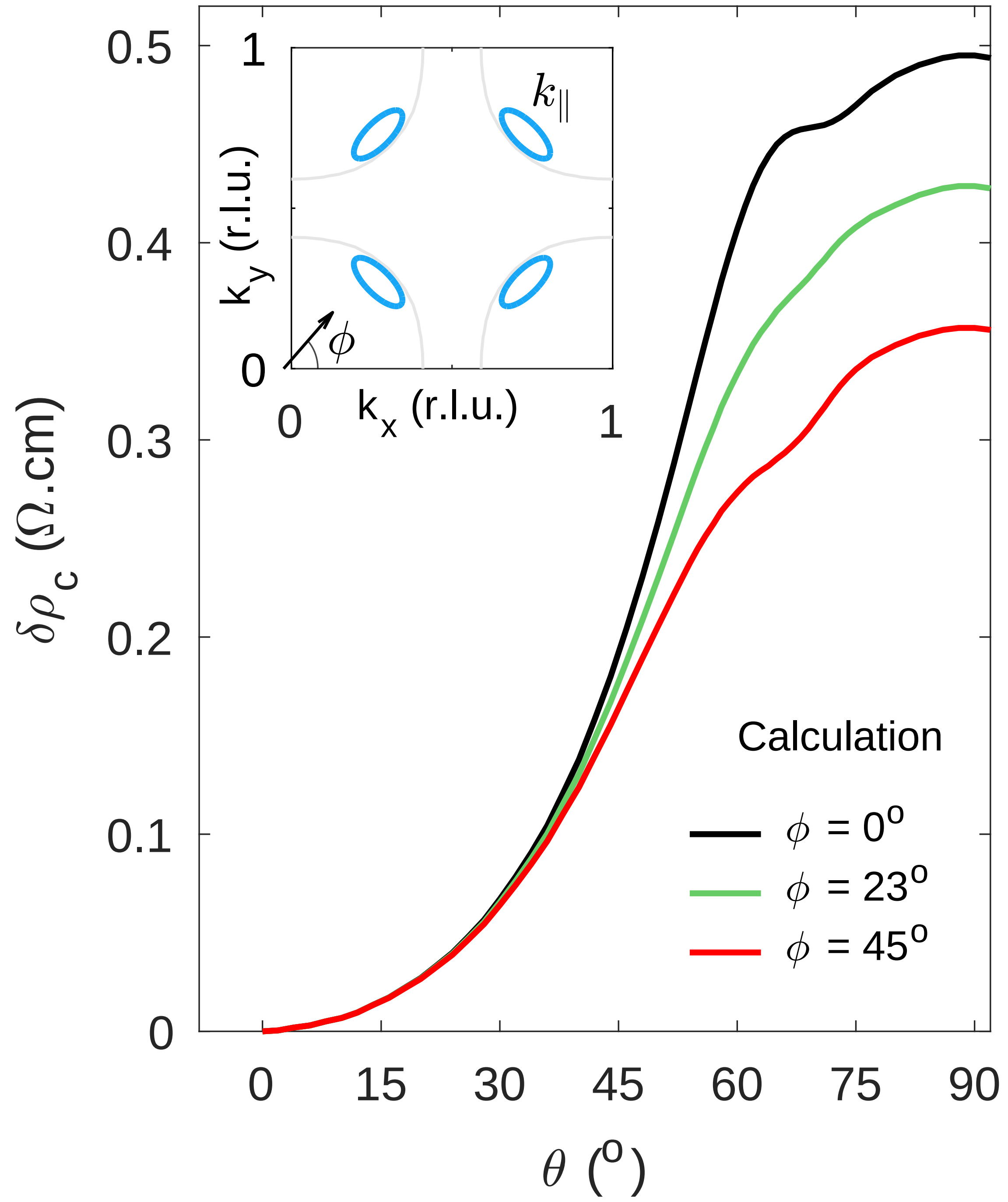

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Rules out holon metal
and possibly SDW metal

Observation of the Yamaji effect in a cuprate superconductor

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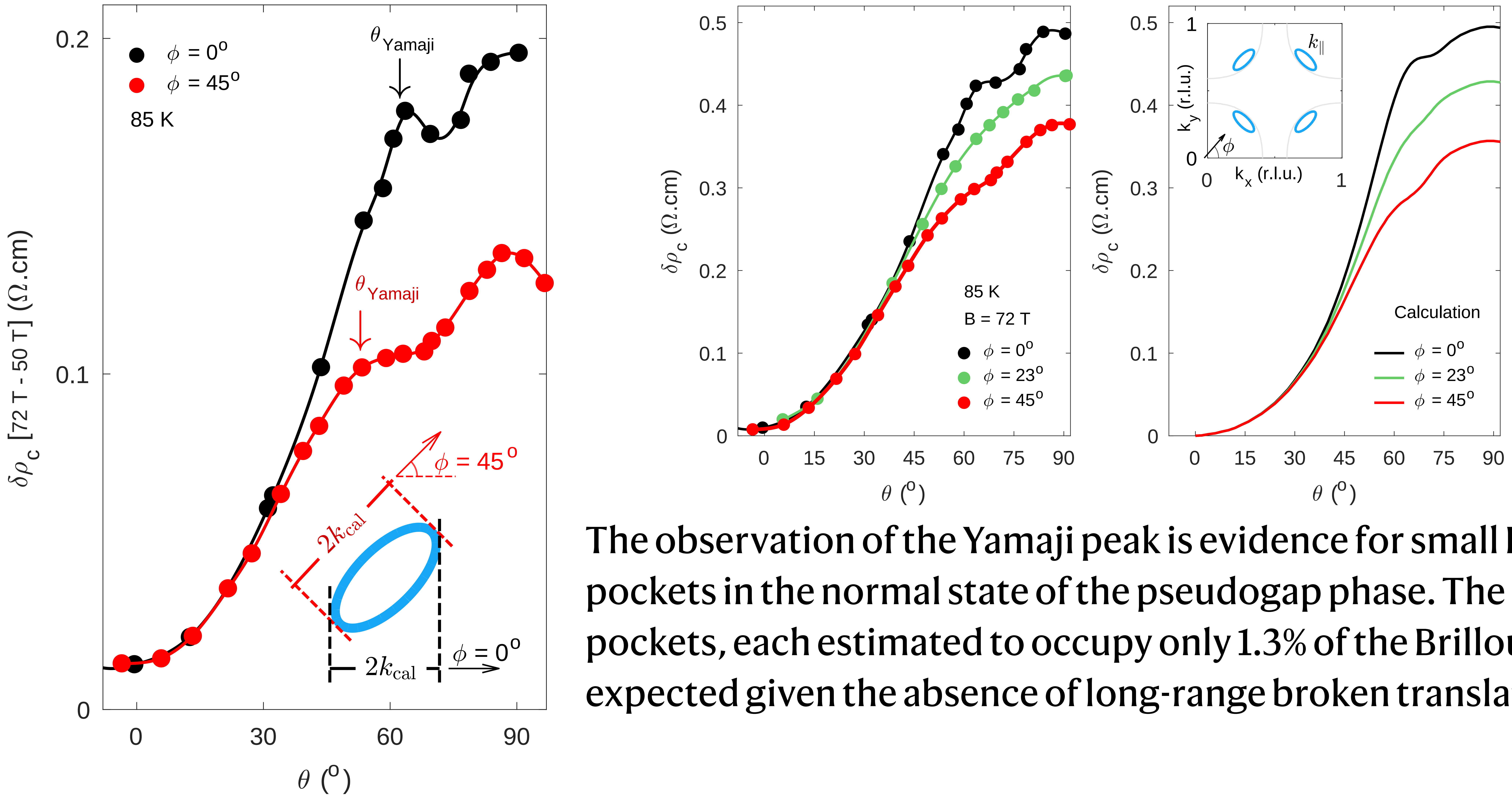

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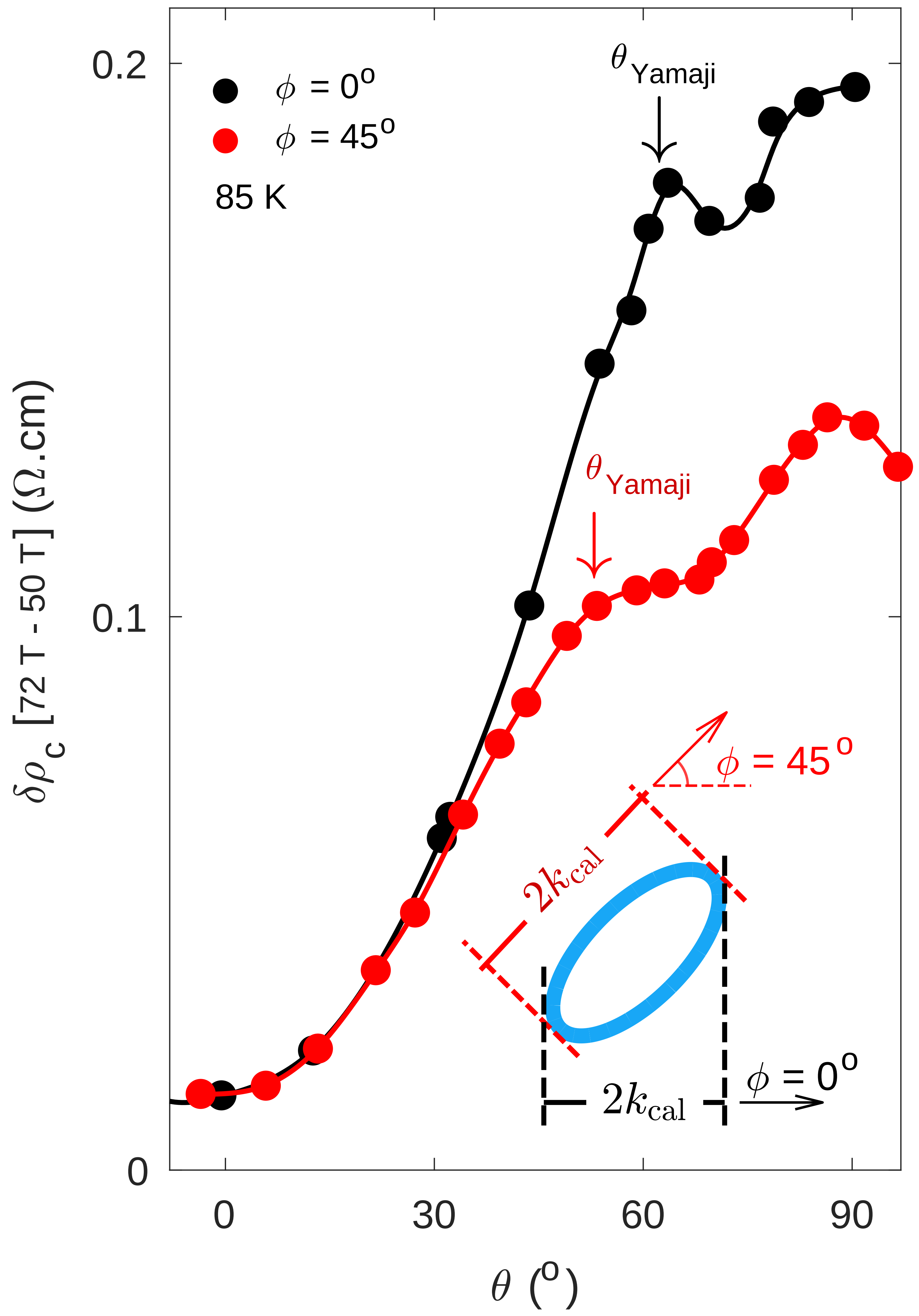

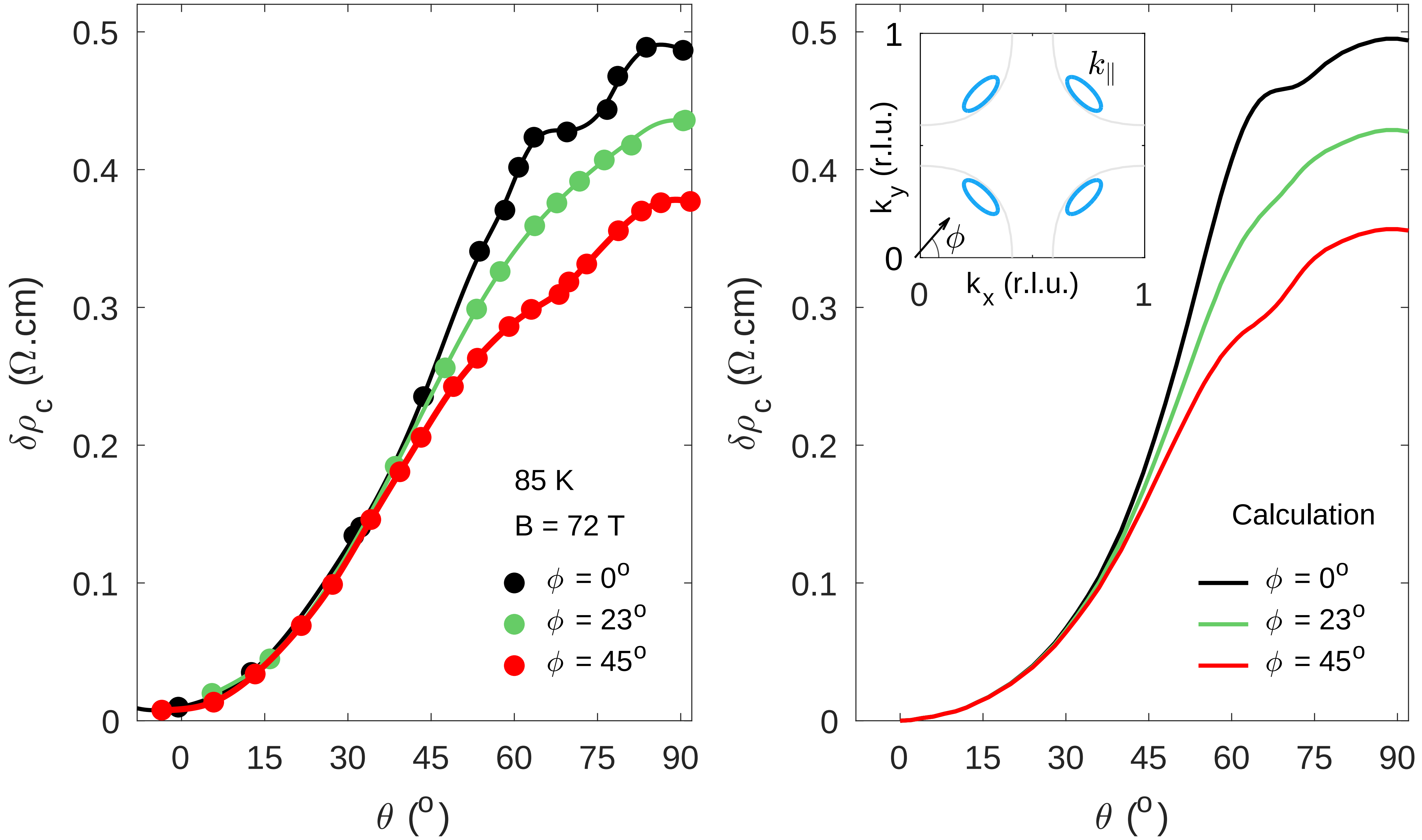

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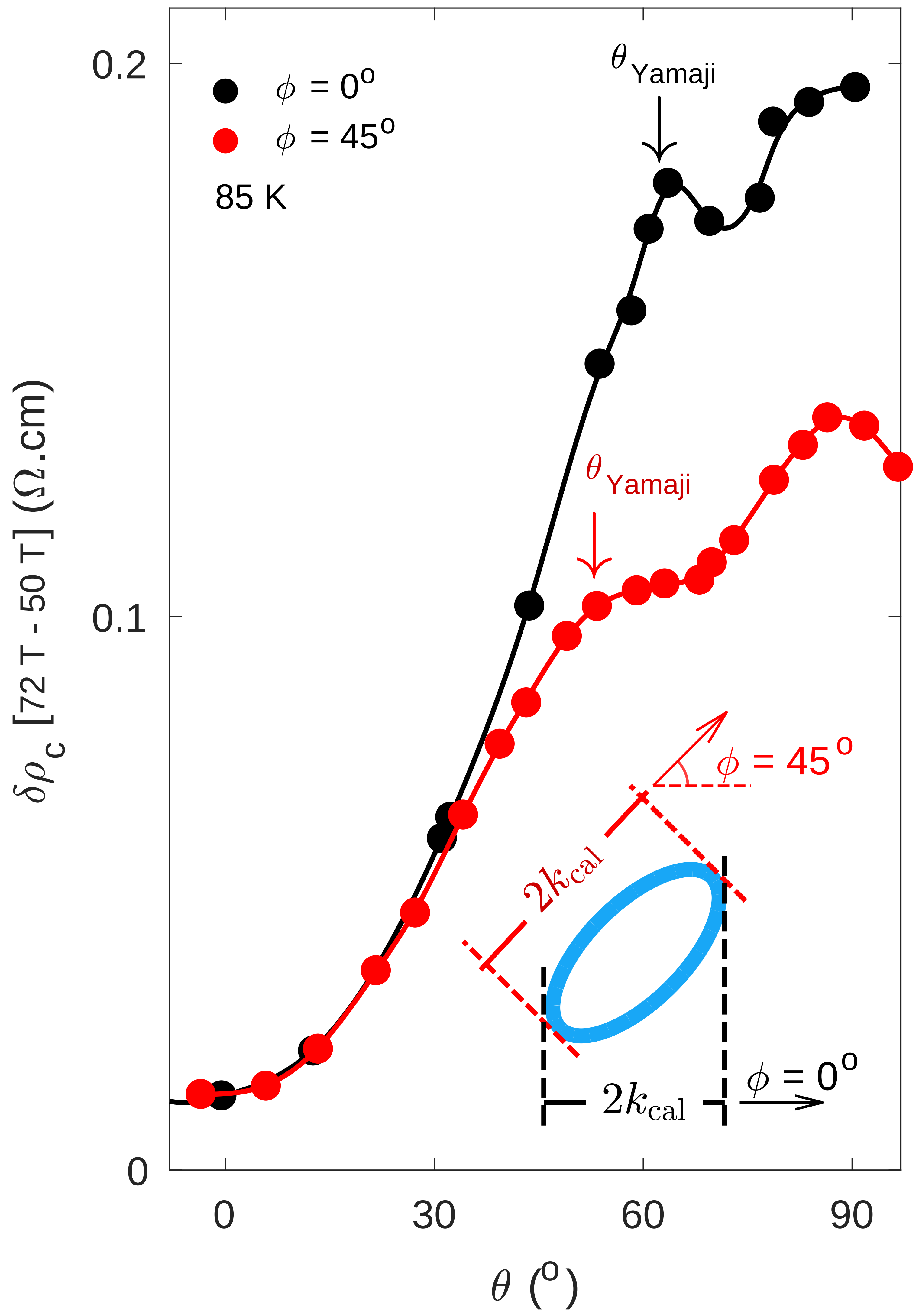
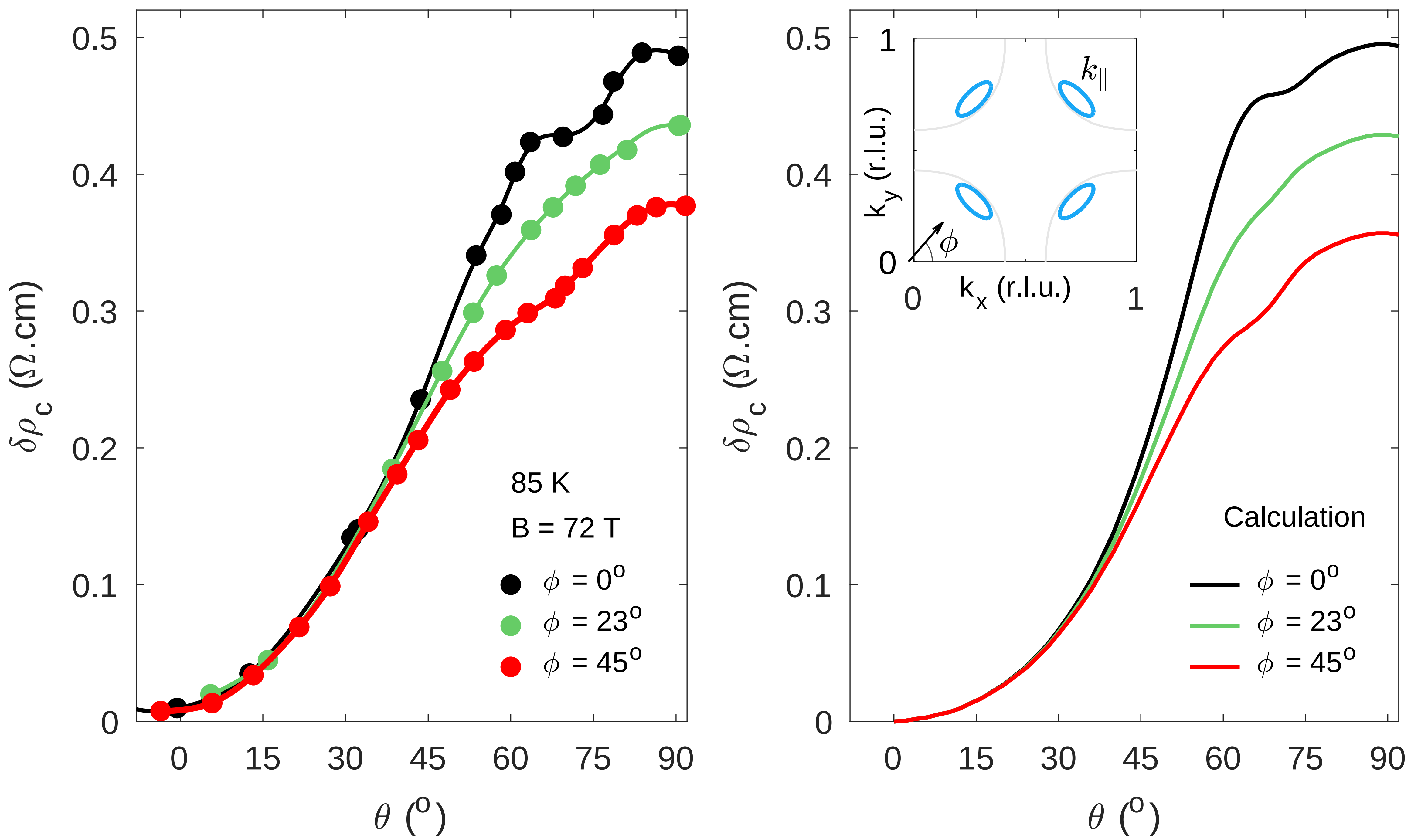
(was expected by us!)

Observation of the Yamaji effect in a cuprate superconductor

Mun K. Chan[1], Katherine A. Schreiber[1], Oscar E. Ayala-Valenzuela[1], Eric D. Bauer[2], Arkady Shekhter[1] & Neil Harrison[1]

nature physics
21, 1753 (2025)
Published online: 16 September 2025





Doping
$p = 0.1$

The observation of the Yamaji peak is evidence for small Fermi-surface pockets in the normal state of the pseudogap phase. The small size of the pockets, each estimated to occupy only 1.3% of the Brillouin zone area, is not expected given the absence of long-range broken translational symmetry.

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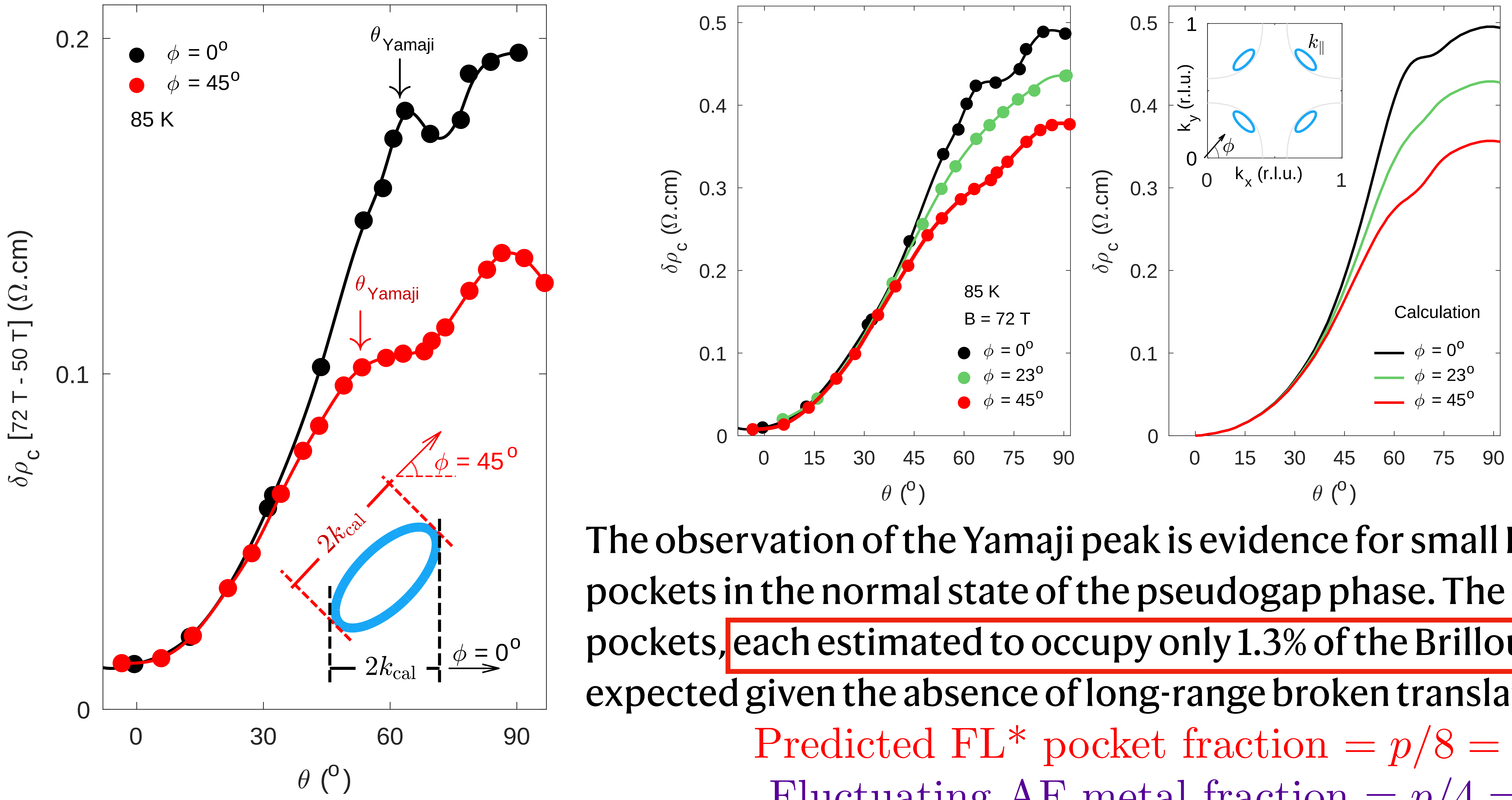

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Predicted FL* pocket fraction $= p/8 = 1.25\%$!

Fluctuating AF metal fraction $= p/4 = 2.5\%$.

(p/8 also in Yang-Rice-Zjang ansatz, Peter Johnson photoemission, and Jenny Hoffman and Seamus Davis STMs; Stanescu-Kotliar)

Jing-Yu Zhao, S. Chatterjee, S. S., Ya-Hui Zhang, arXiv:2510.13943

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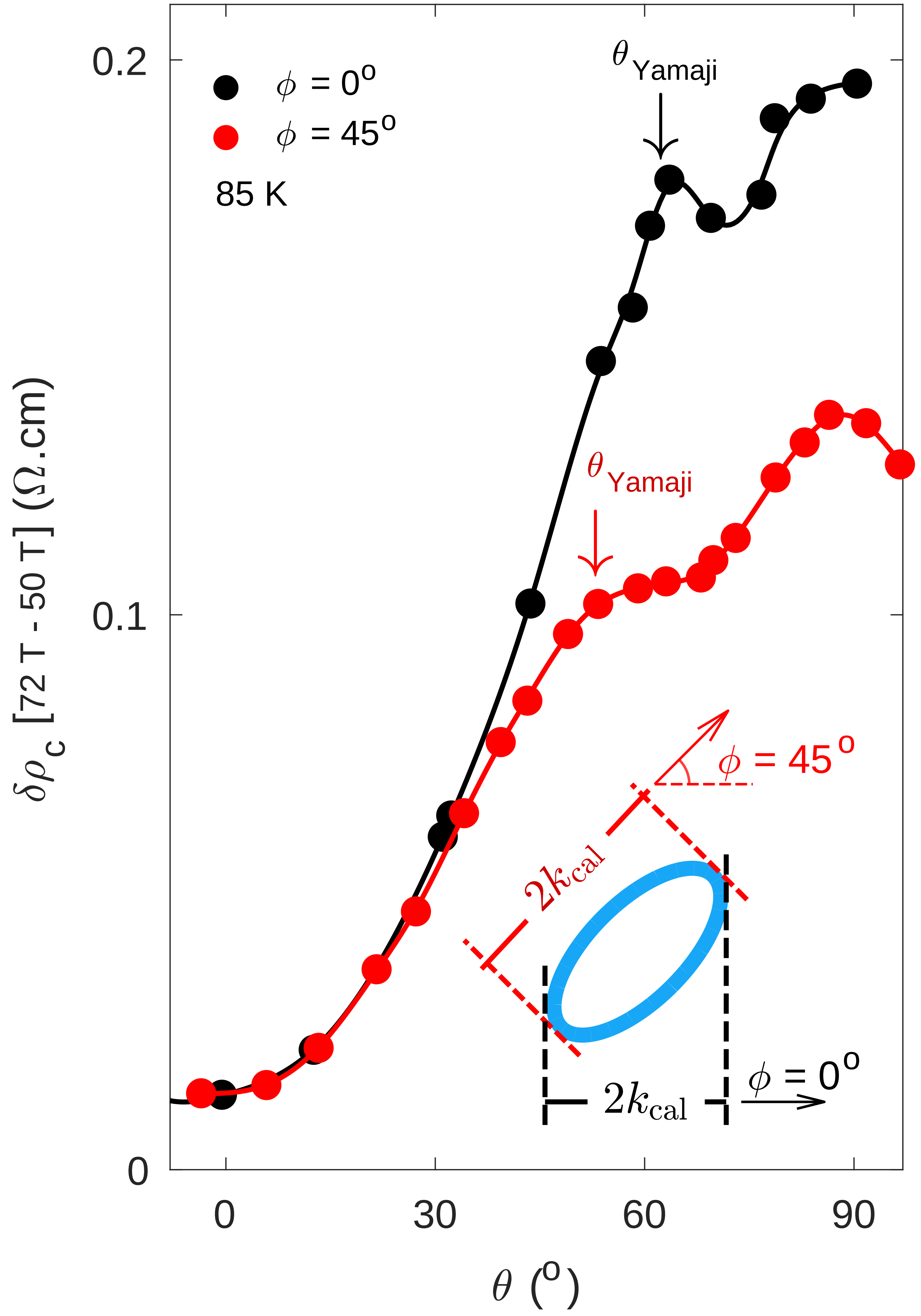

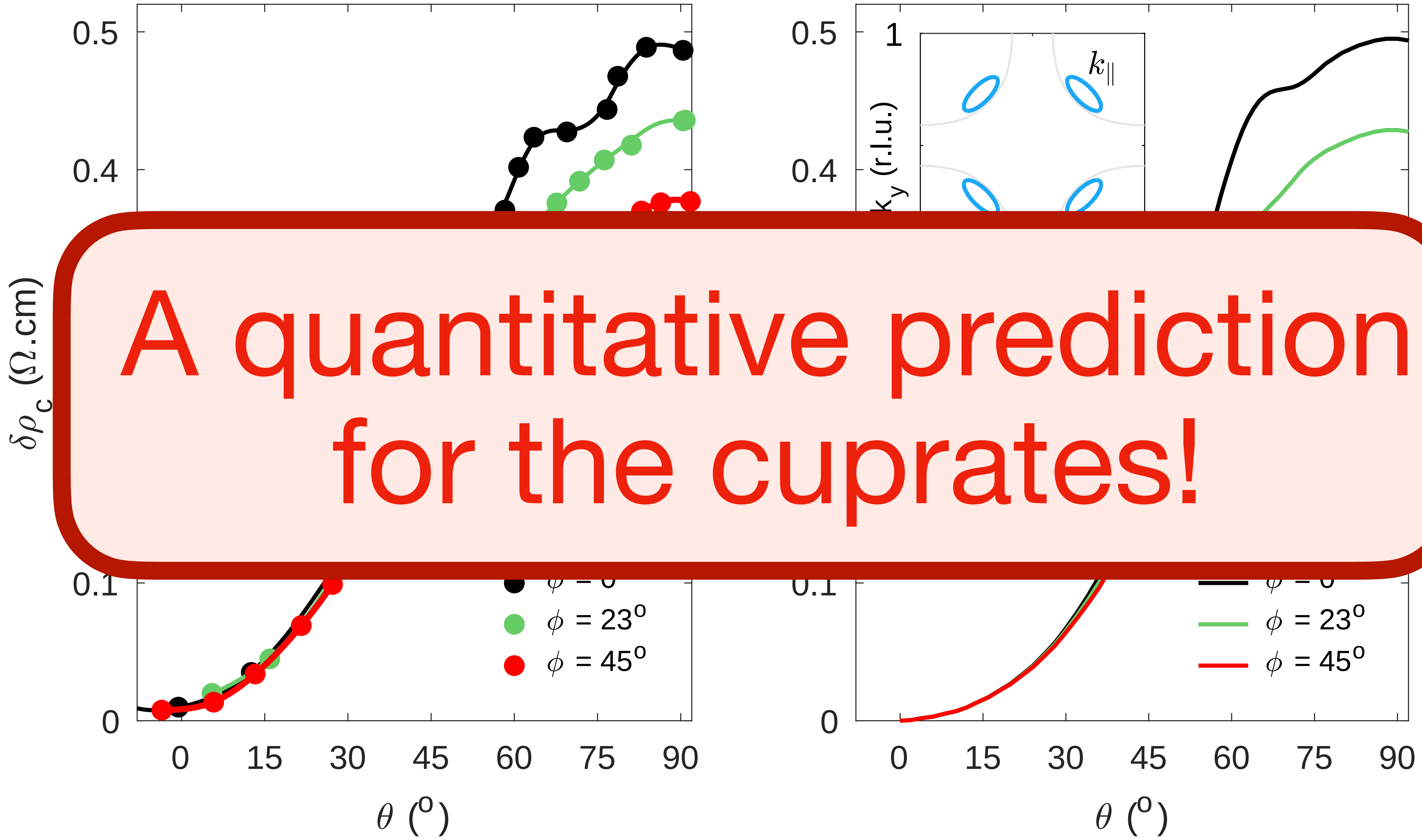

A quantitative prediction for the cuprates!

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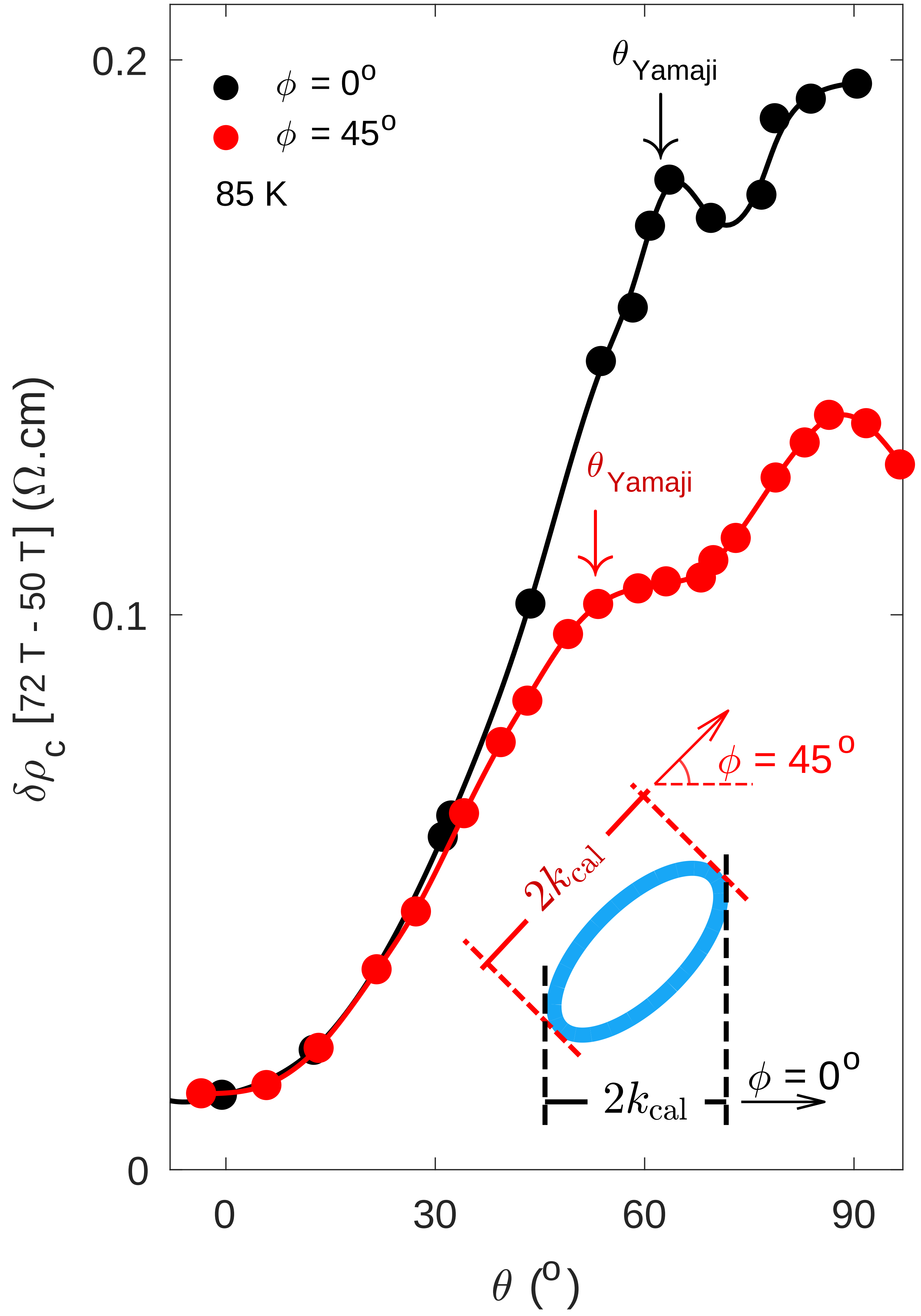

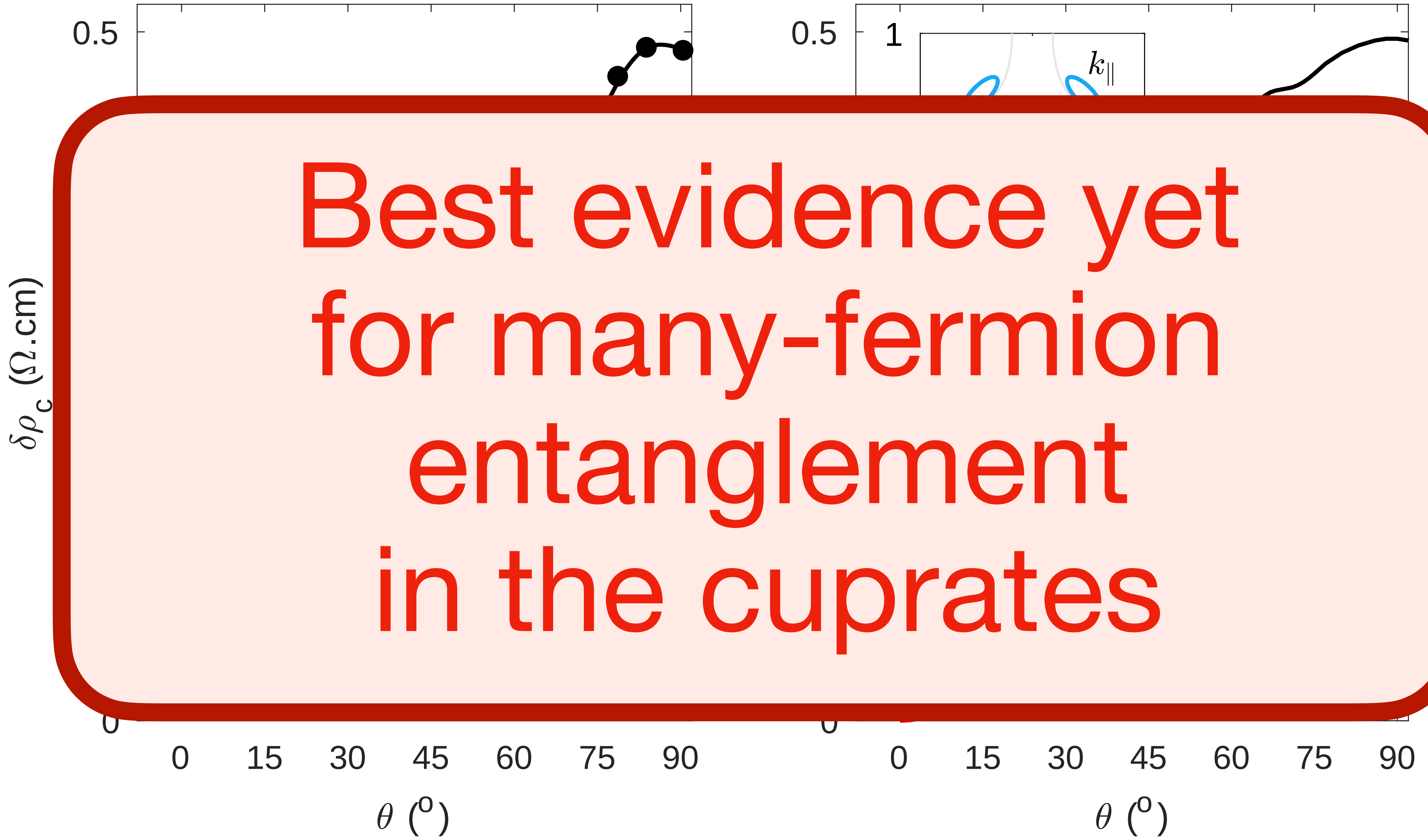

Best evidence yet for many-fermion entanglement in the cuprates

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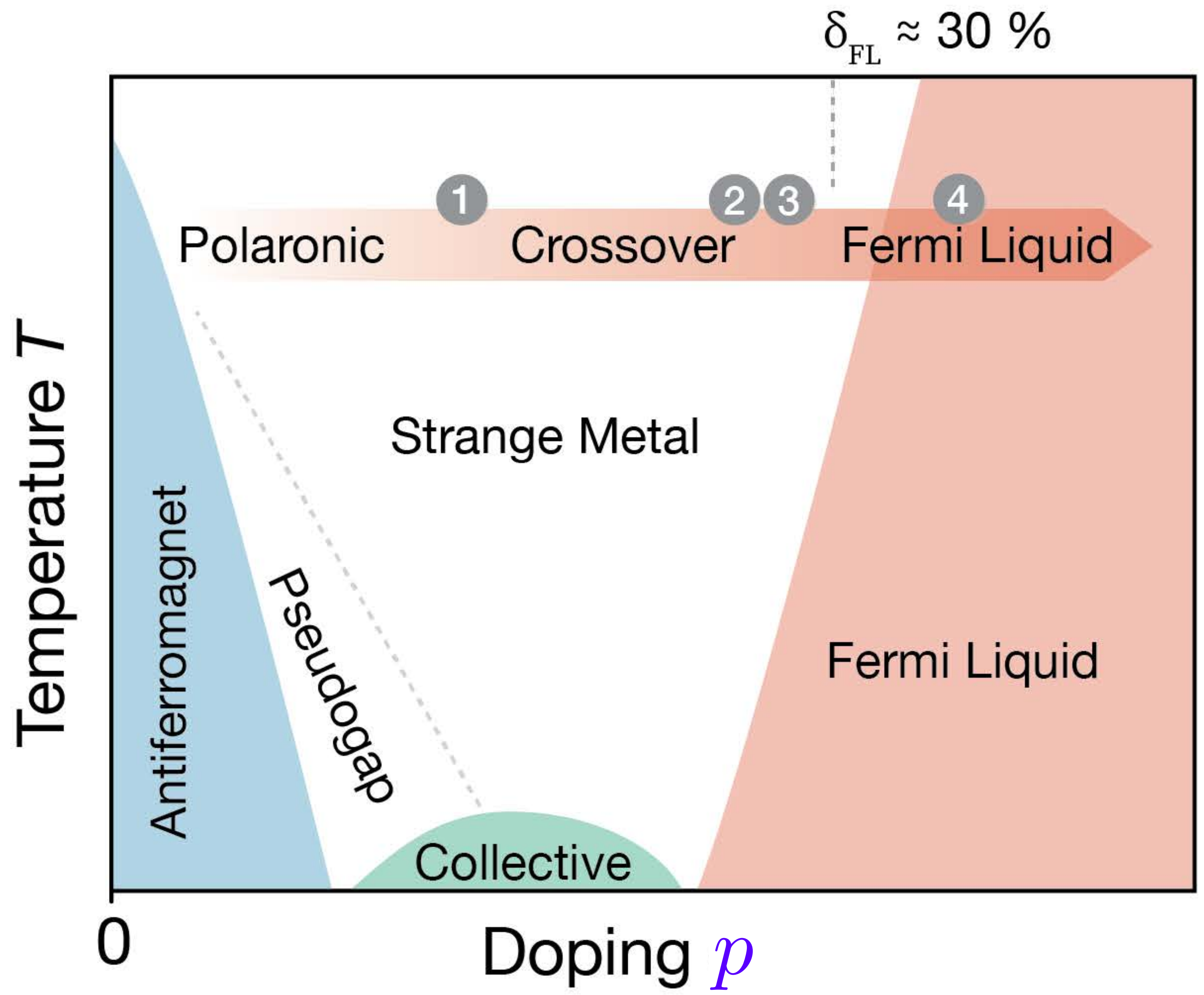
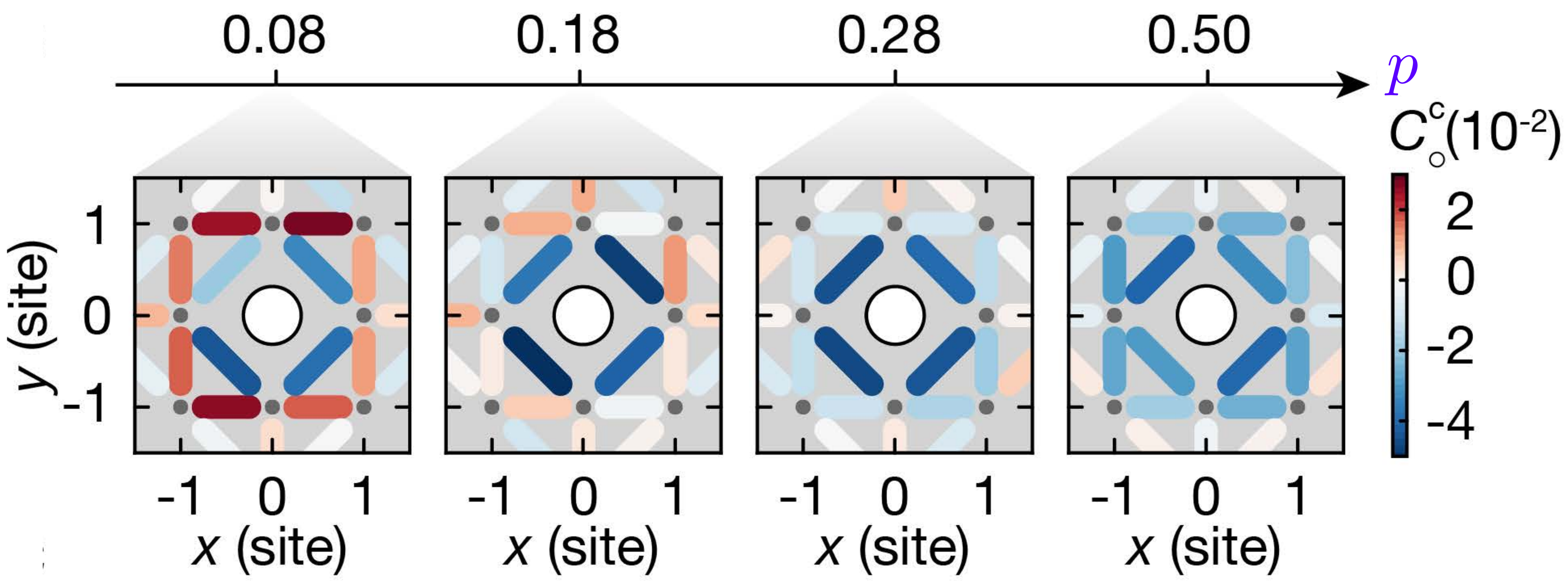
Wavefunction for FL*
and
observations on
ultracold atoms





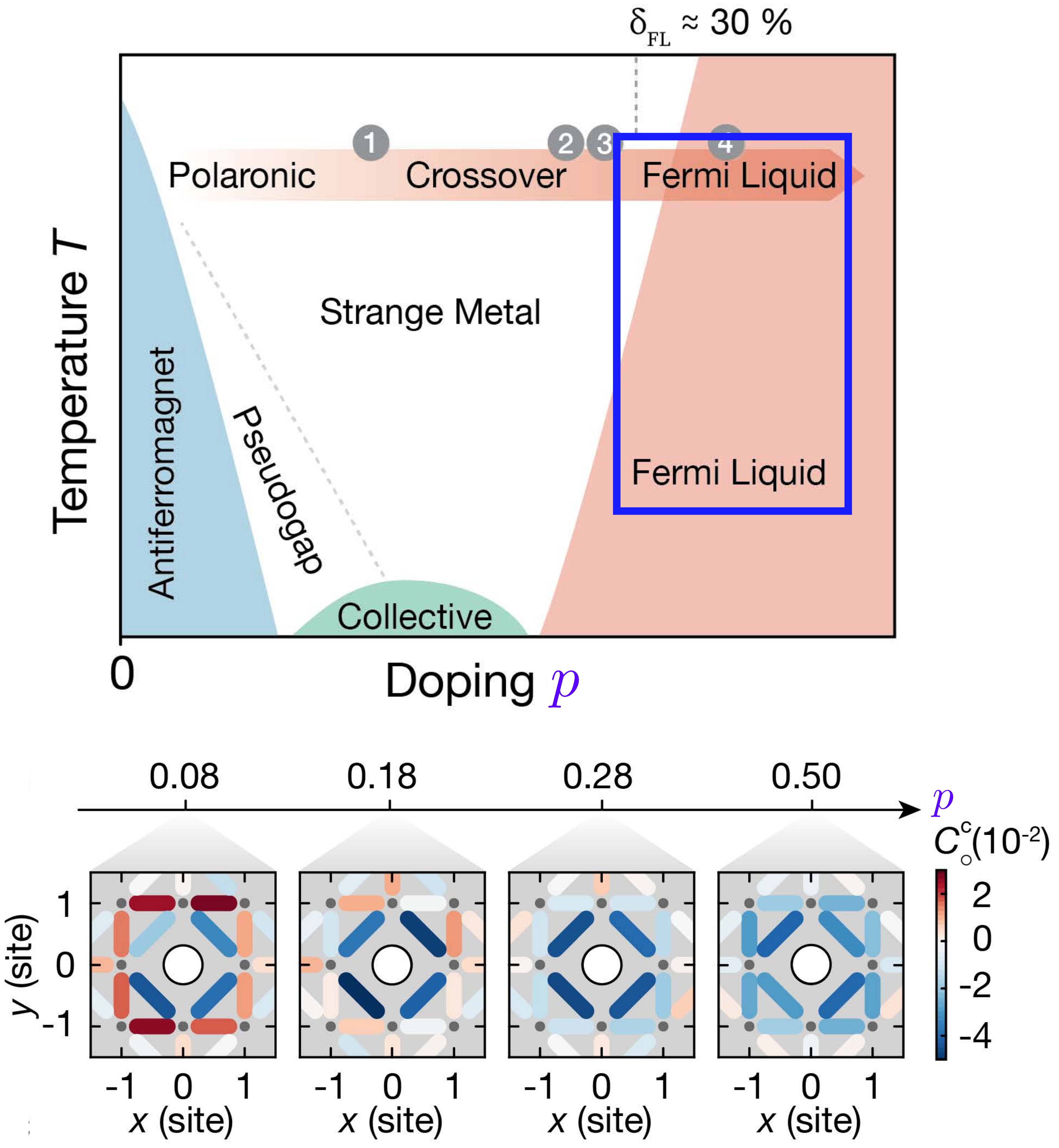
Microscopic evolution of doped Mott insulators from polaronic metal to Fermi liquid

Joannis Koepsell, Dominik Bourgund, Pimonpan Sompet, Sarah Hirthe, Annabelle Bohrdt, Yao Wang, Fabian Grusdt, Eugene Demler, Guillaume Salomon, Christian Gross, Immanuel Bloch

Science **374** (2021) 82

Chalopin….Bloch, PNAS **123**, e2525539123 (2026)

Max Planck Institute of Quantum Optics, Garching



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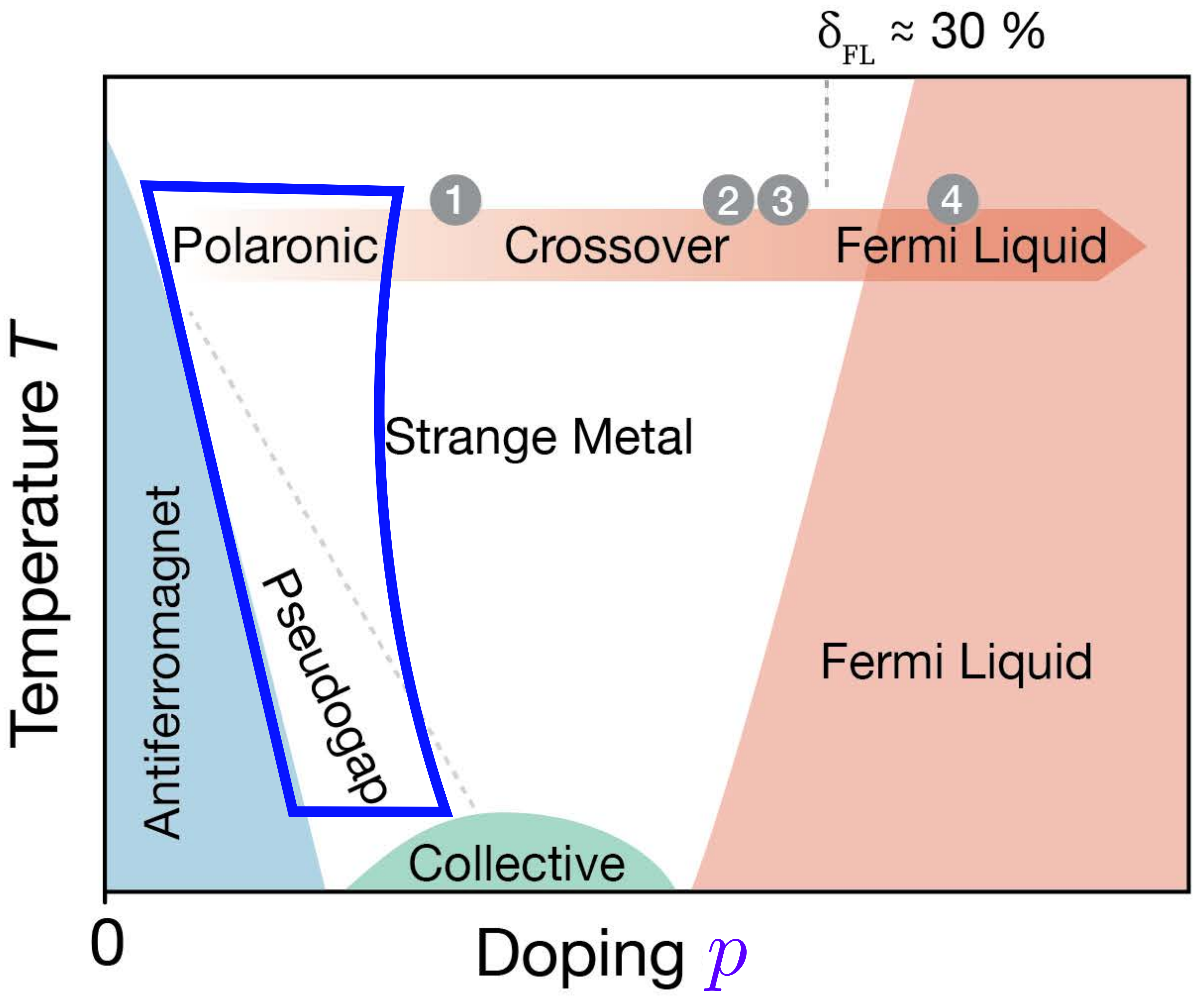

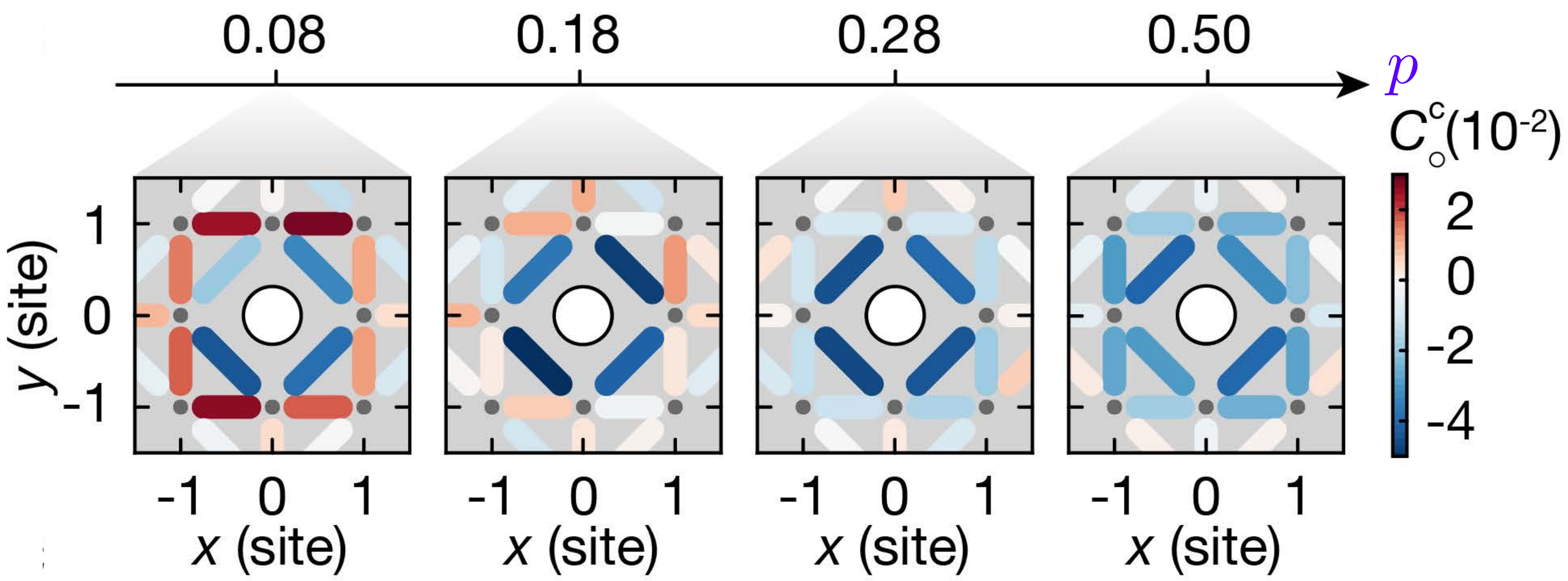

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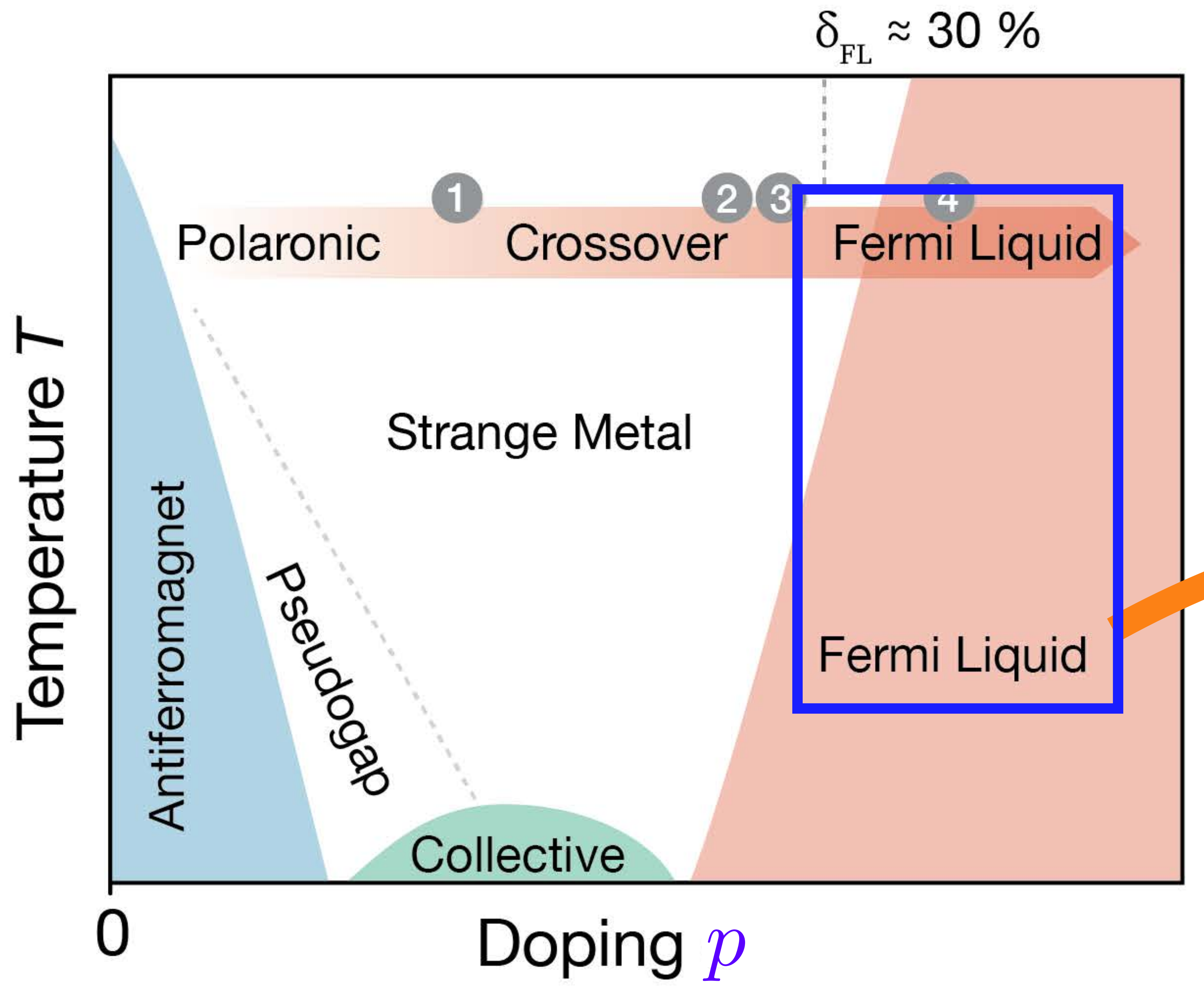
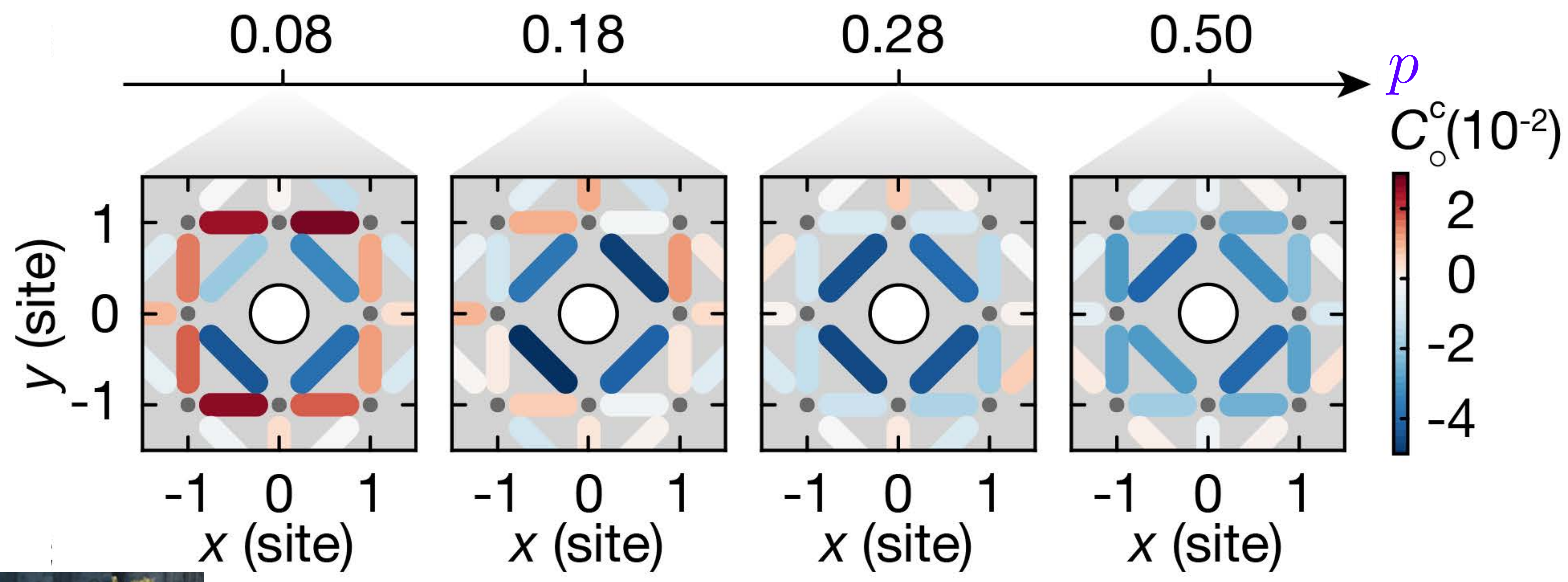
Ancilla Layer Model

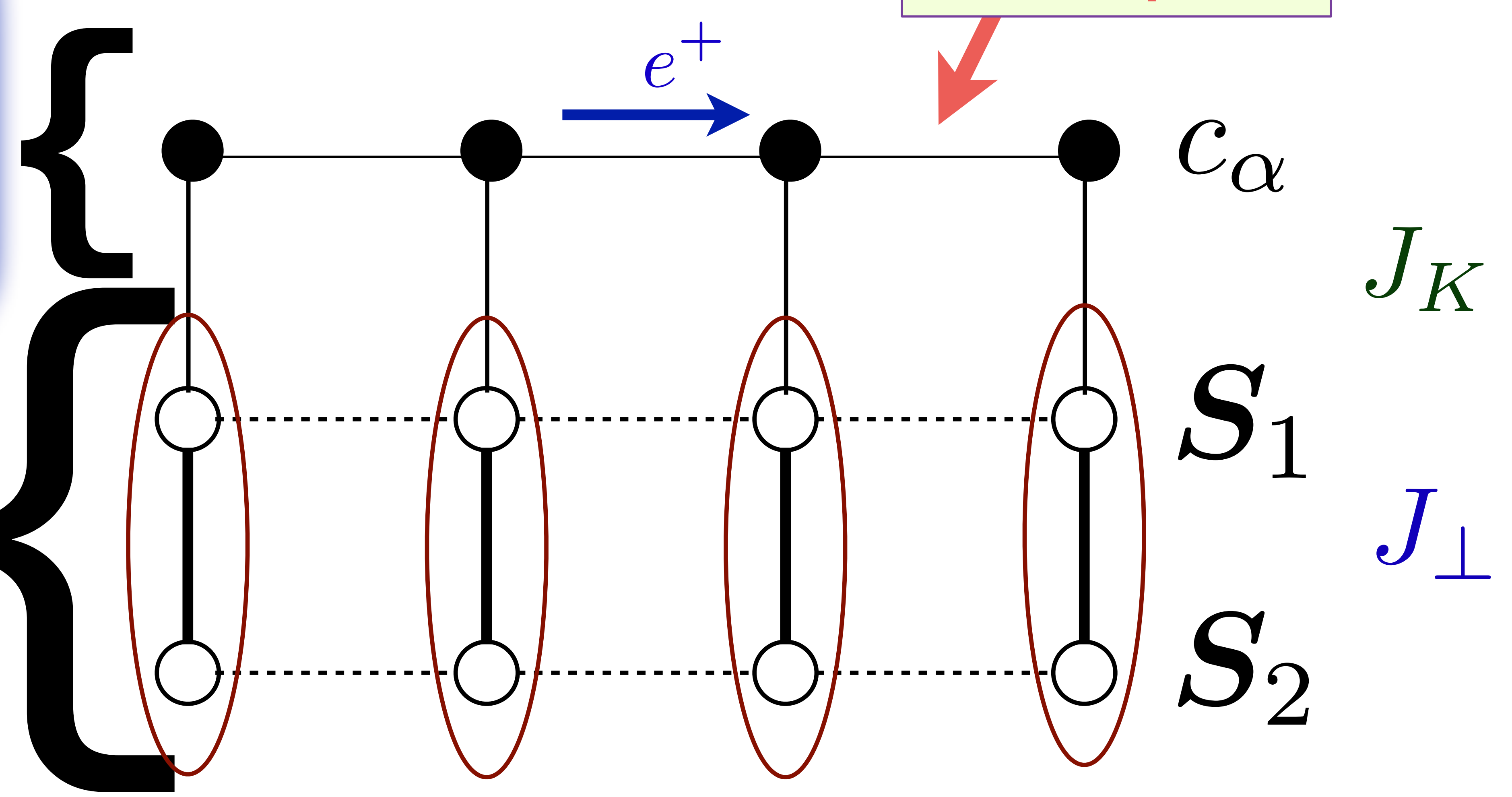




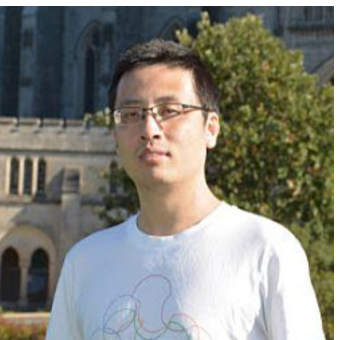
Fermi liquid. Fermi surface encloses area $(1+p)/2$.

Rung singlets of ancilla spins $\boldsymbol{S}_1$, $\boldsymbol{S}_2$.

Holes of density 1+p



$$\text{Fermi liquid} = \begin{array}{c} \text{FL of } c_\alpha \\ \oplus \\ \textit{Trivial, gapped} \text{ state of } \boldsymbol{S}_1 \text{ and } \boldsymbol{S}_2 \end{array}$$



Ya-Hui Zhang

Ya-Hui Zhang and S. Sachdev, PRR **2**, 023172 (2020)
E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, S. Sachdev, PRB **105,** 075146 (2022)

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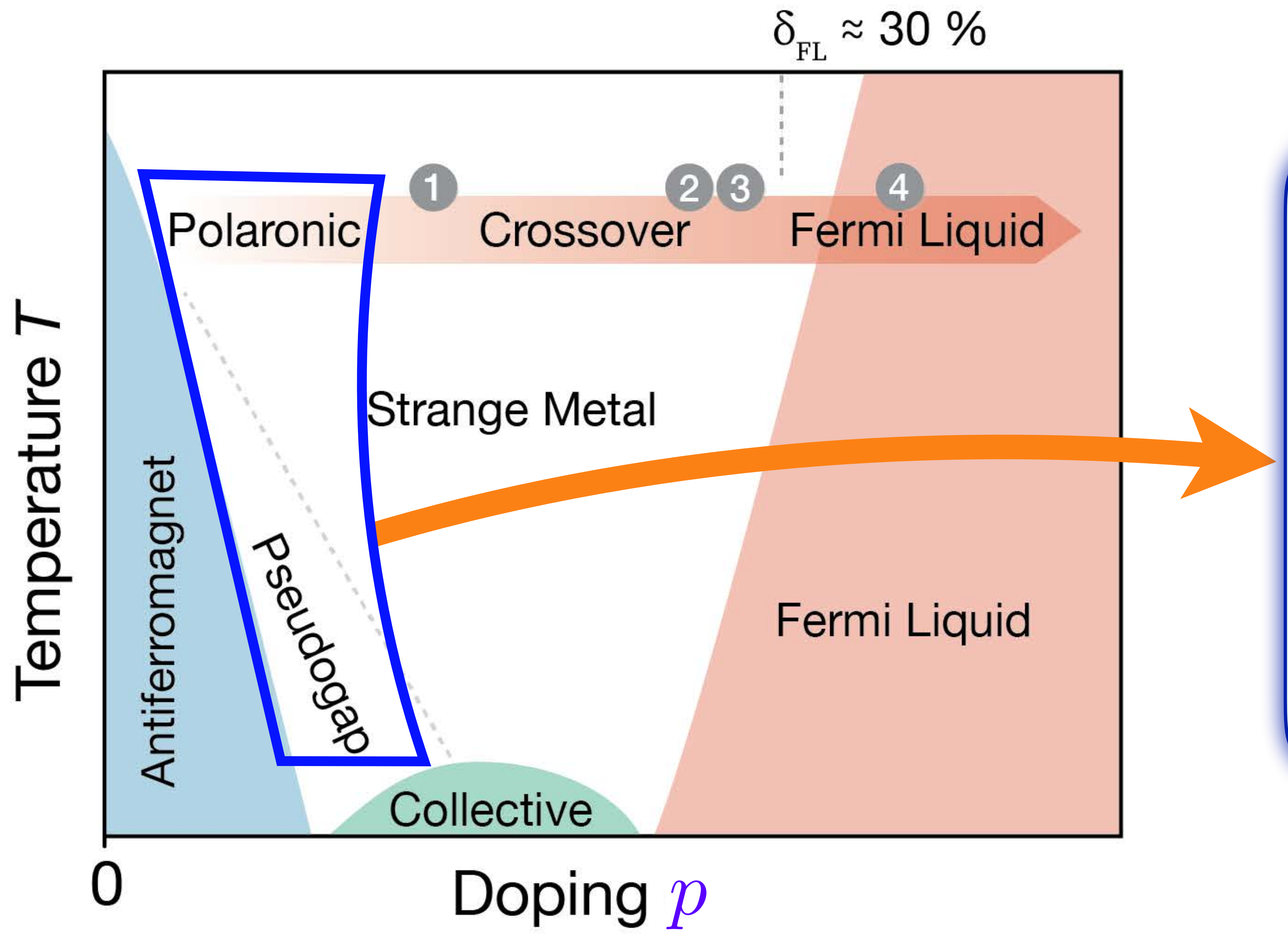
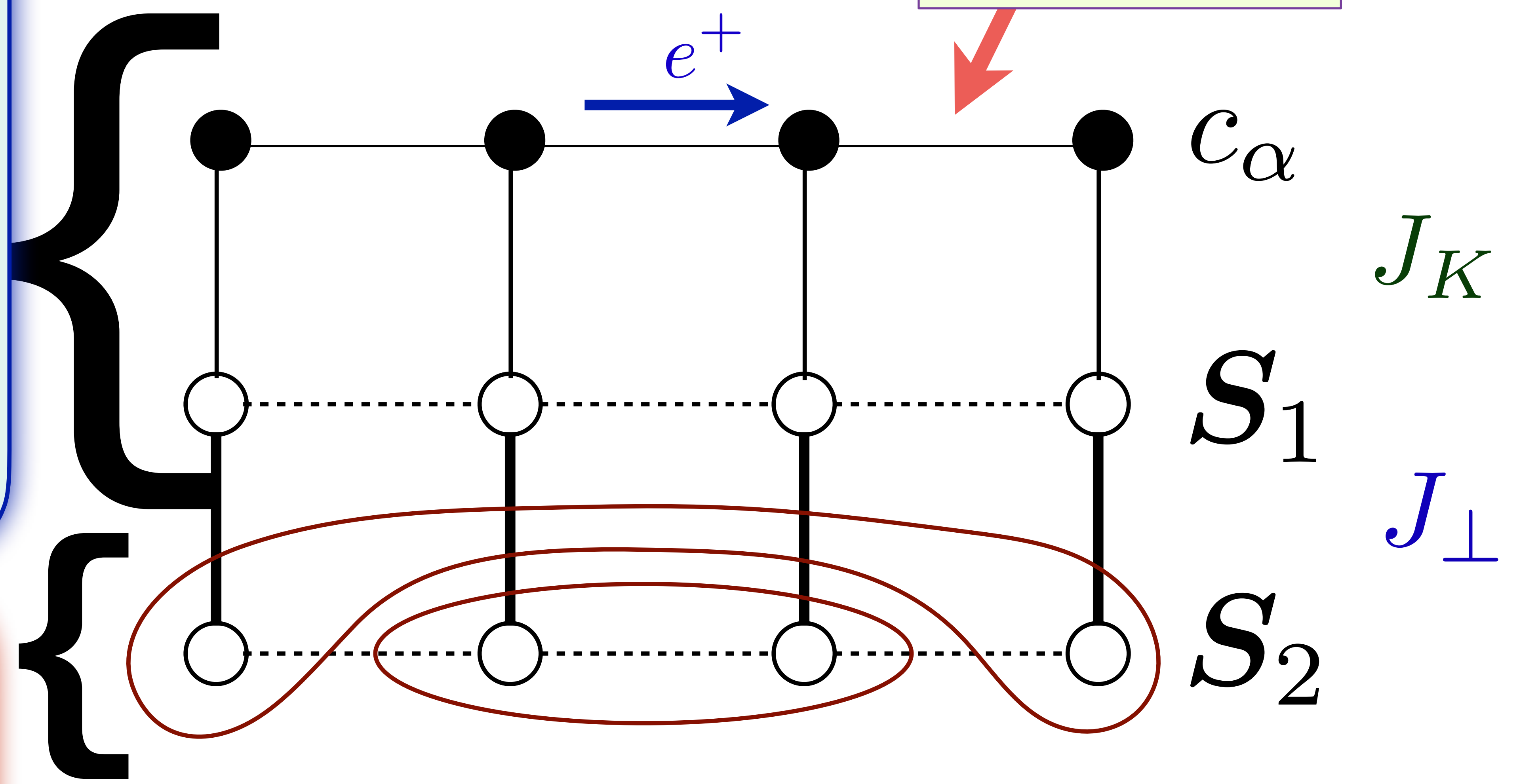


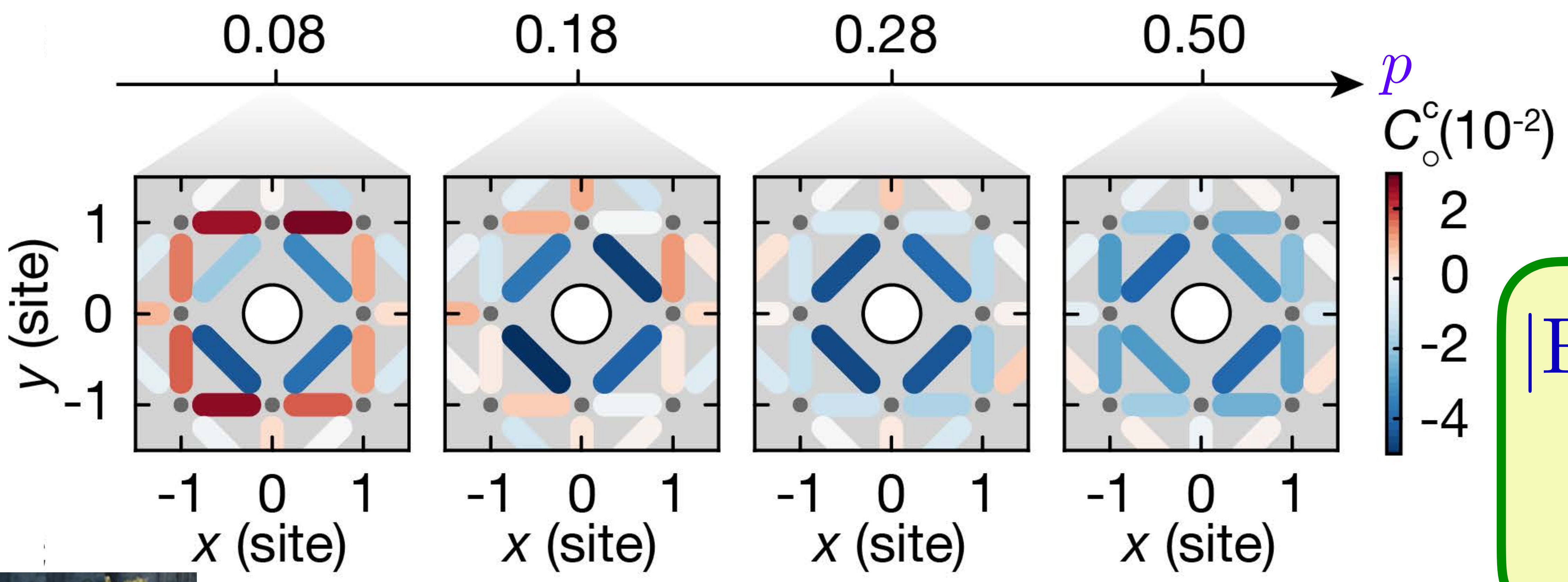
Kondo lattice heavy Fermi liquid. Hybridization $\Phi \sim f^\dagger_{1\alpha} c_\alpha$. Fermi surfaces enclose area $(1 + p + 1)/2 = p/2 \pmod 1$.

Holes of density 1+p

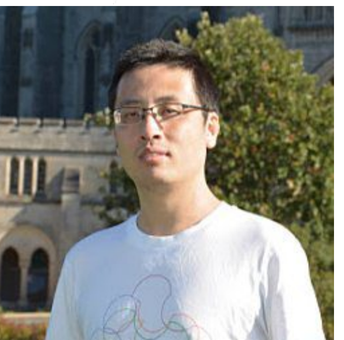


Your favorite spin liquid

$$\boldsymbol{S}_{1i} = \frac{1}{2} f^\dagger_{1i\alpha} \boldsymbol{\sigma}_{\alpha\beta} f_{1i\beta}, \quad \boldsymbol{S}_{2i} = \frac{1}{2} f^\dagger_{i\alpha} \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta}$$



$$|\text{FL}*\rangle_{\text{Hubbard}} = [\text{Projection onto rung singlets of } \boldsymbol{S}_1, \boldsymbol{S}_2] \bowtie |\text{Slater determinant of } (c, f_1)\rangle \otimes |\text{Slater determinant of } f\rangle$$



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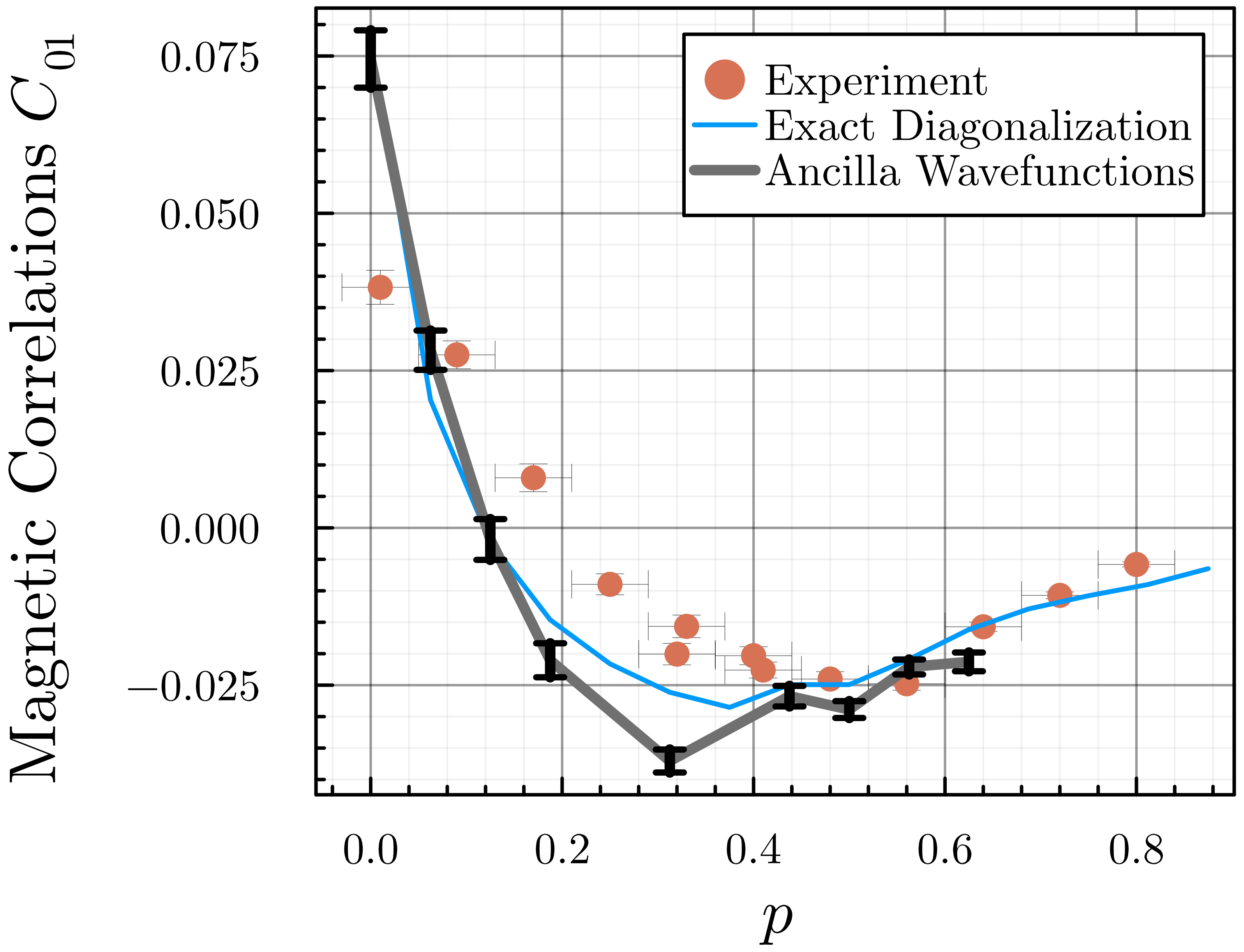
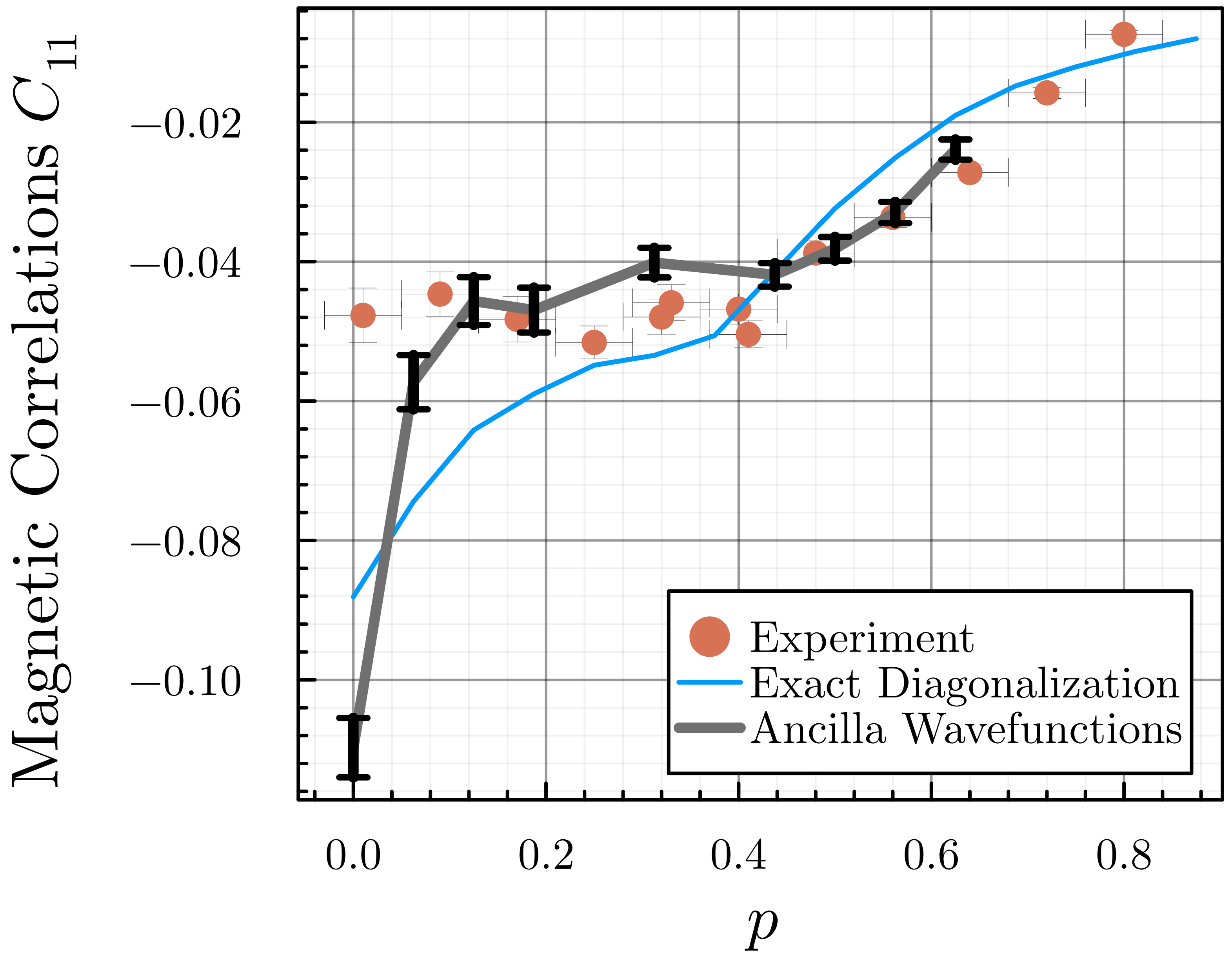
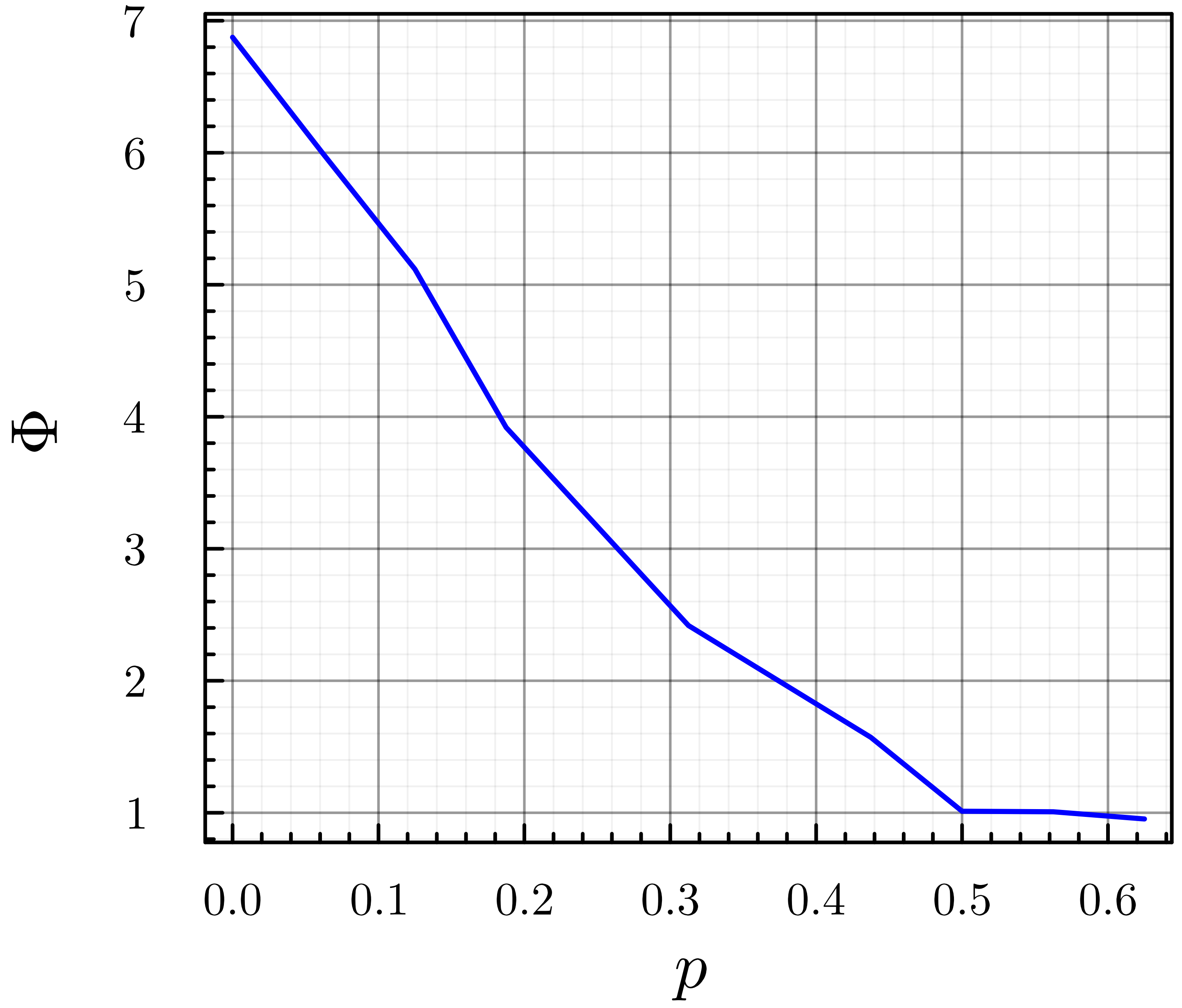
Ancilla wavefunction for FL* of Hubbard model










Hybridization (Higgs boson) $\Phi \sim f_{1\alpha}^\dagger c_\alpha$.

FL*: $\Phi \neq 0$; FL: $\Phi = 0$

Φ: variational parameter with Hubbard Hamiltonian

$$|\mathrm{FL}*\rangle_{\mathrm{Hubbard}} = [\text{Projection onto rung singlets of } \boldsymbol{S}_1, \boldsymbol{S}_2] \bowtie |\text{Slater determinant of } (c, f_1)\rangle \otimes |\text{Slater determinant of } f\rangle$$

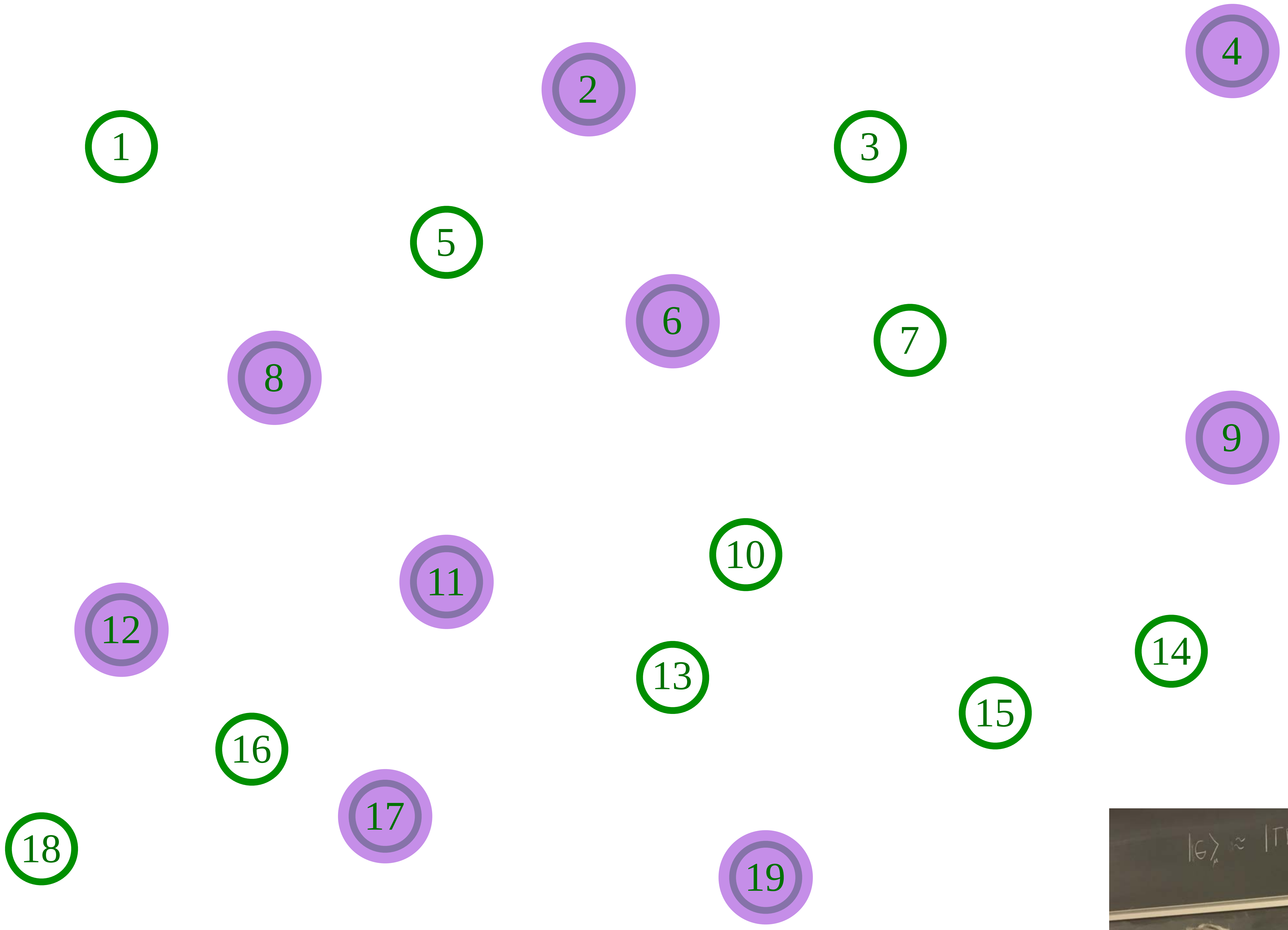
L. Shackleton and Shiwei Zhang, arXiv:2408.02190

Tobias Müller, Yasir Iqbal, S.S., Ronny Thomale, PNAS **122**, e2504261122 (2025)

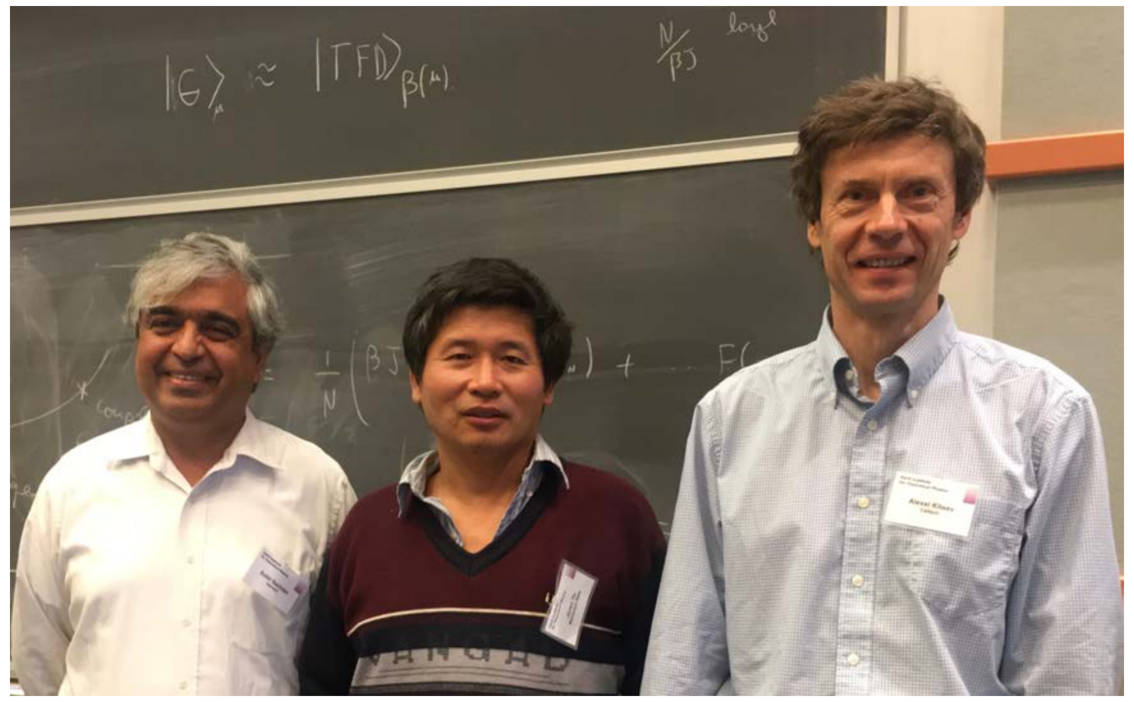
Sachdev-Ye-Kitaev liquids

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)



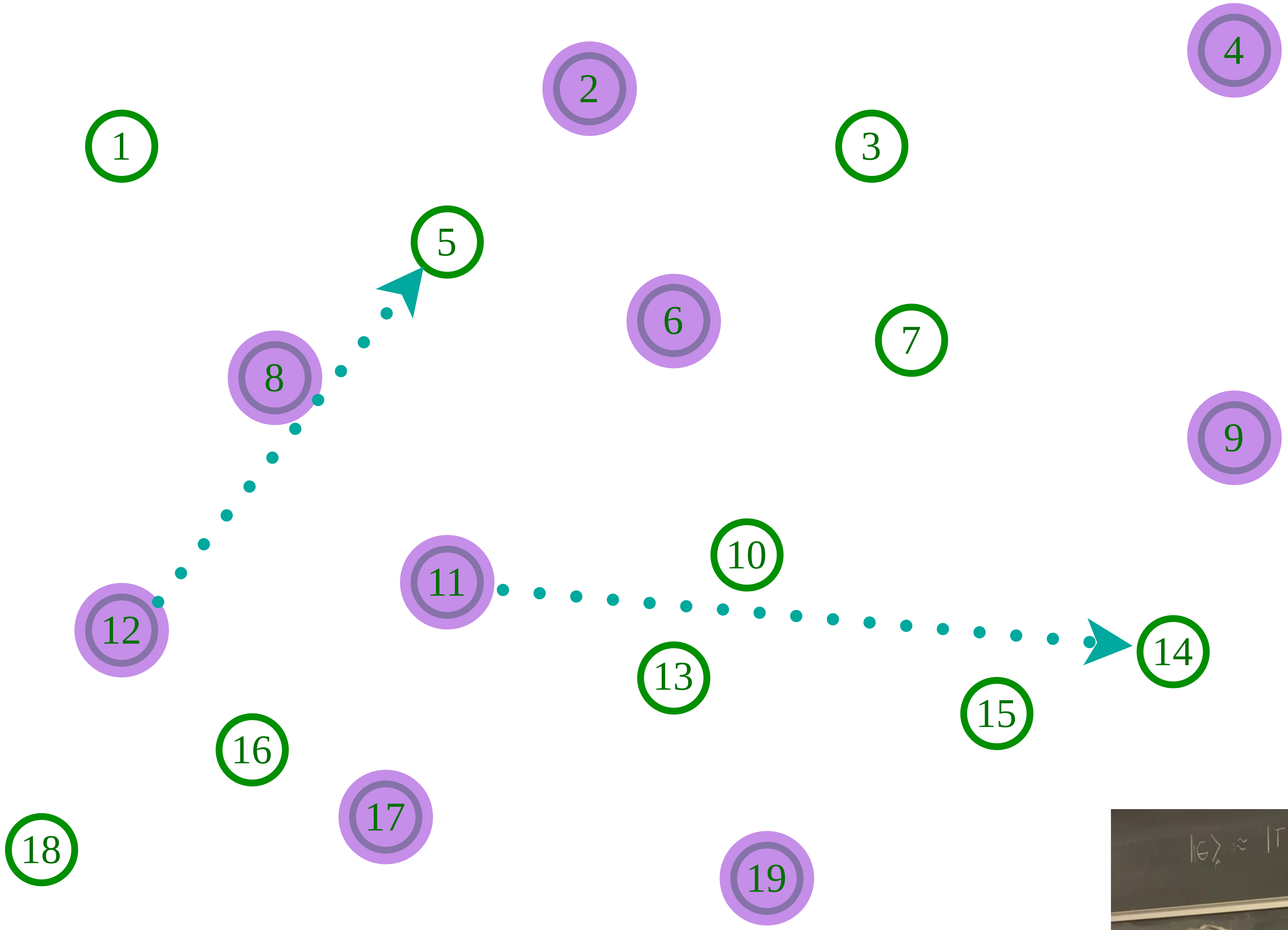
Place electrons randomly on some sites



The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



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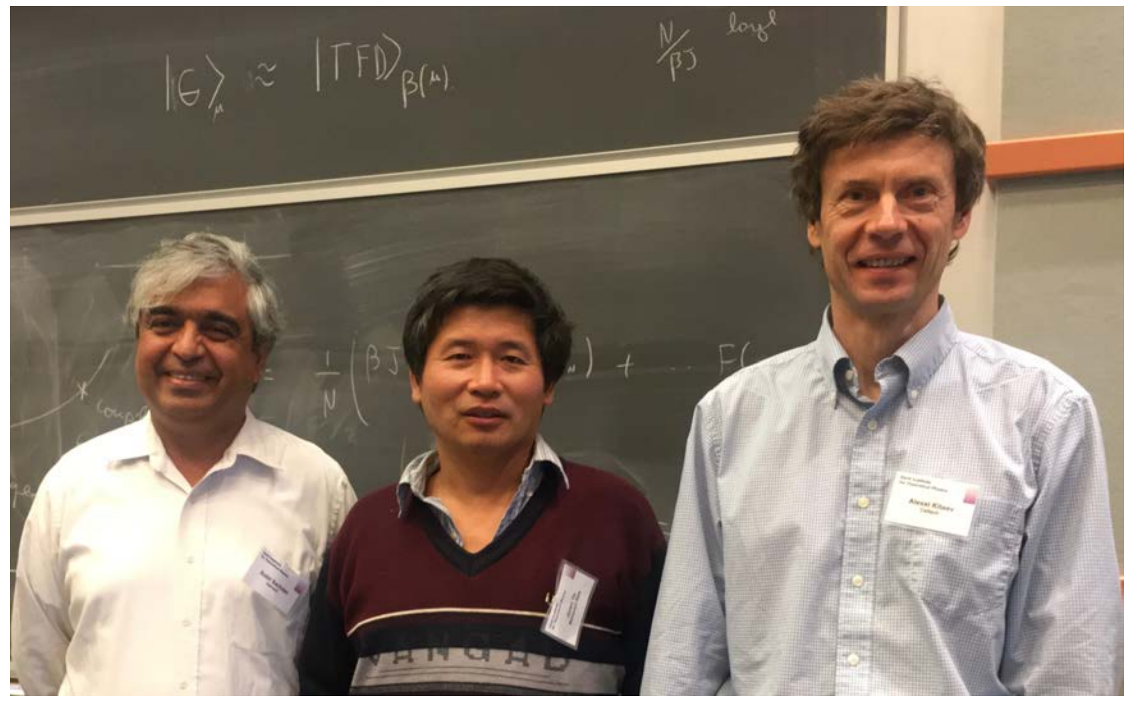

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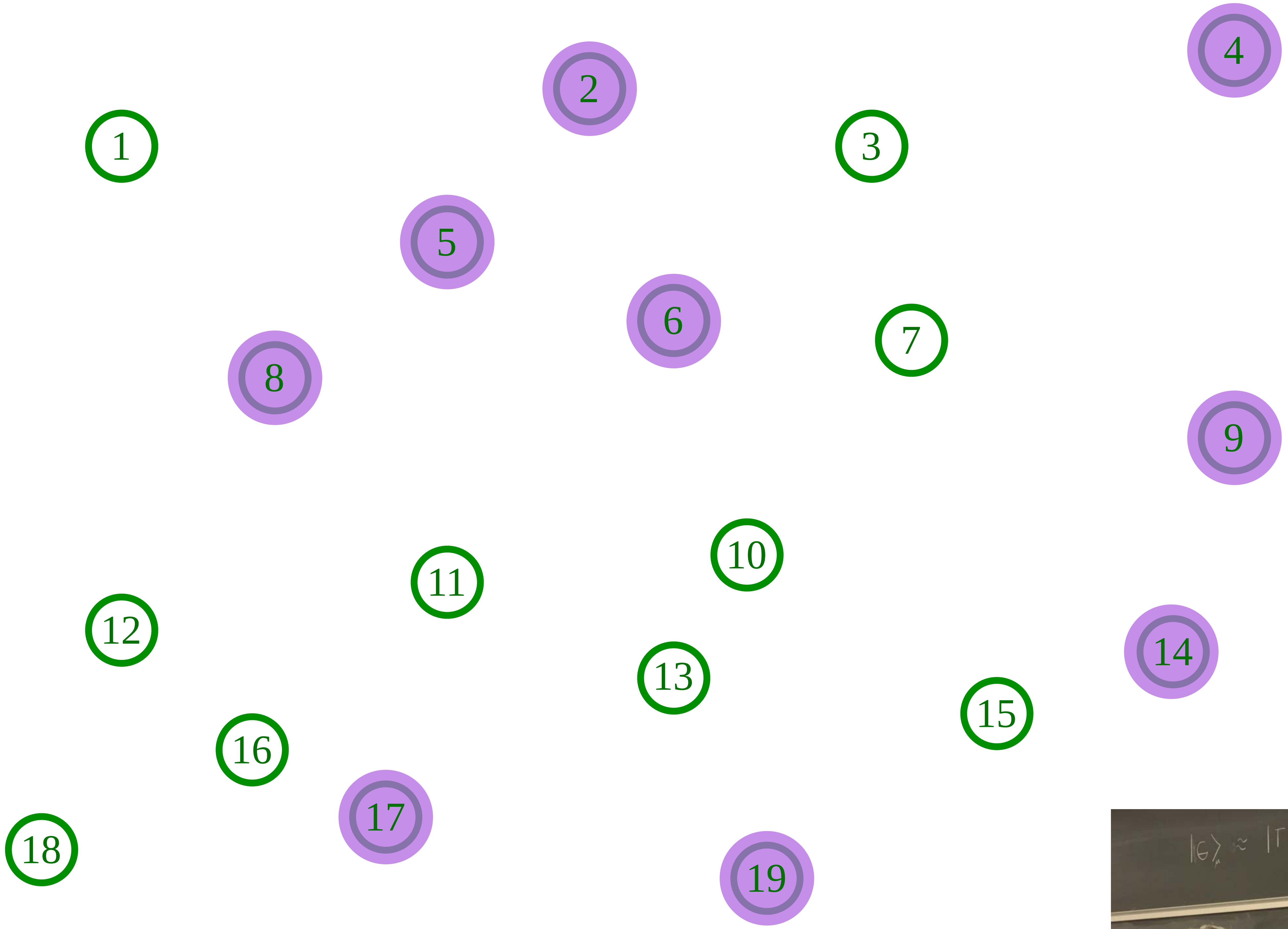
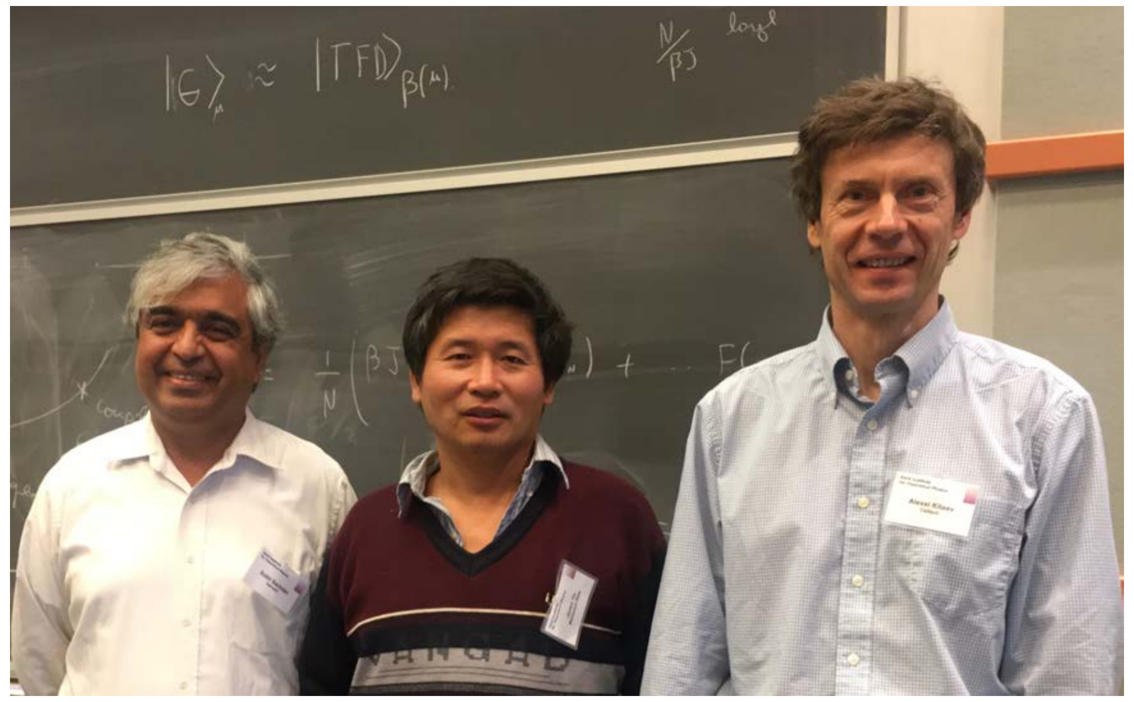


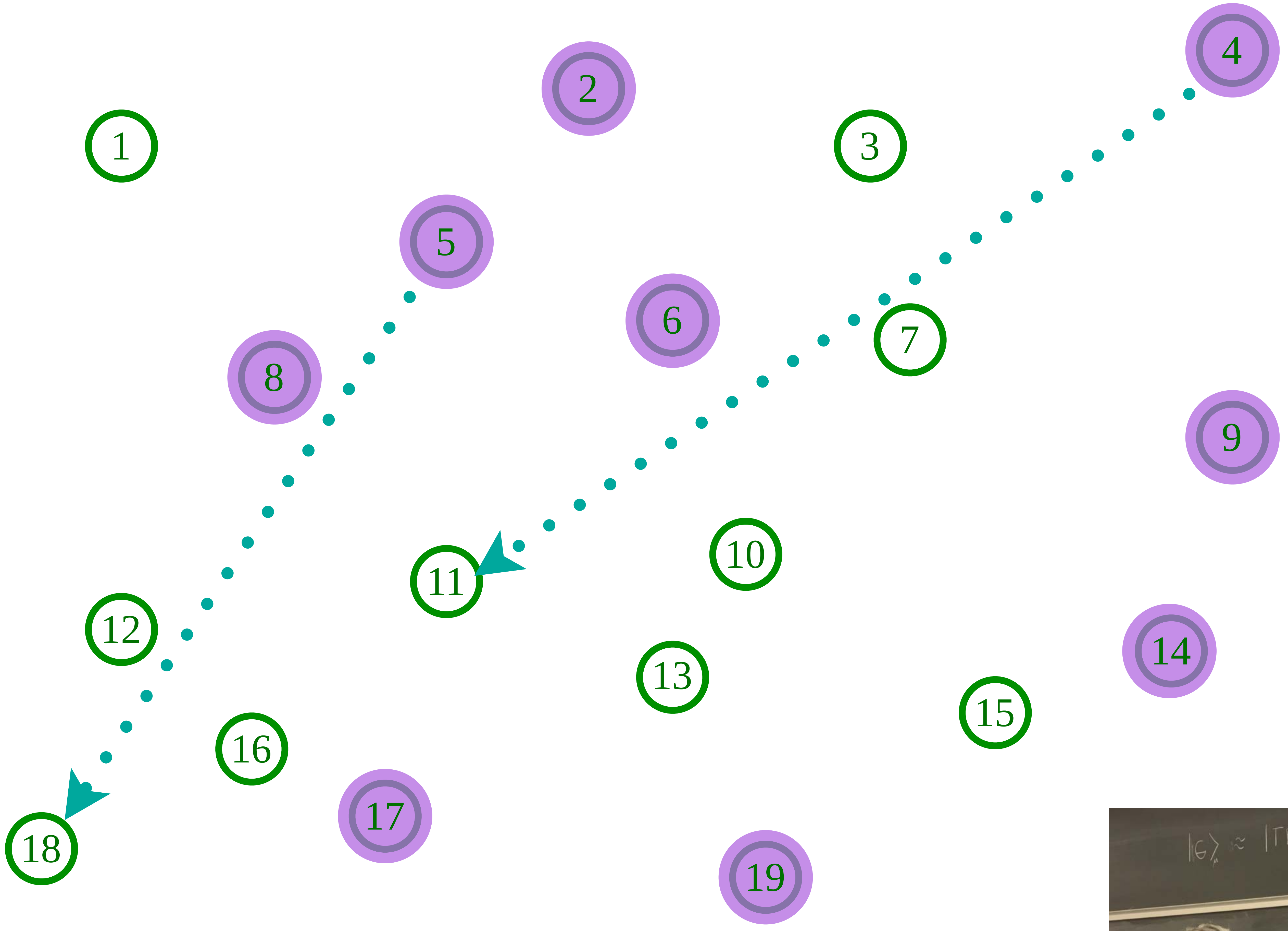
Entangle electrons pairwise randomly



The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$



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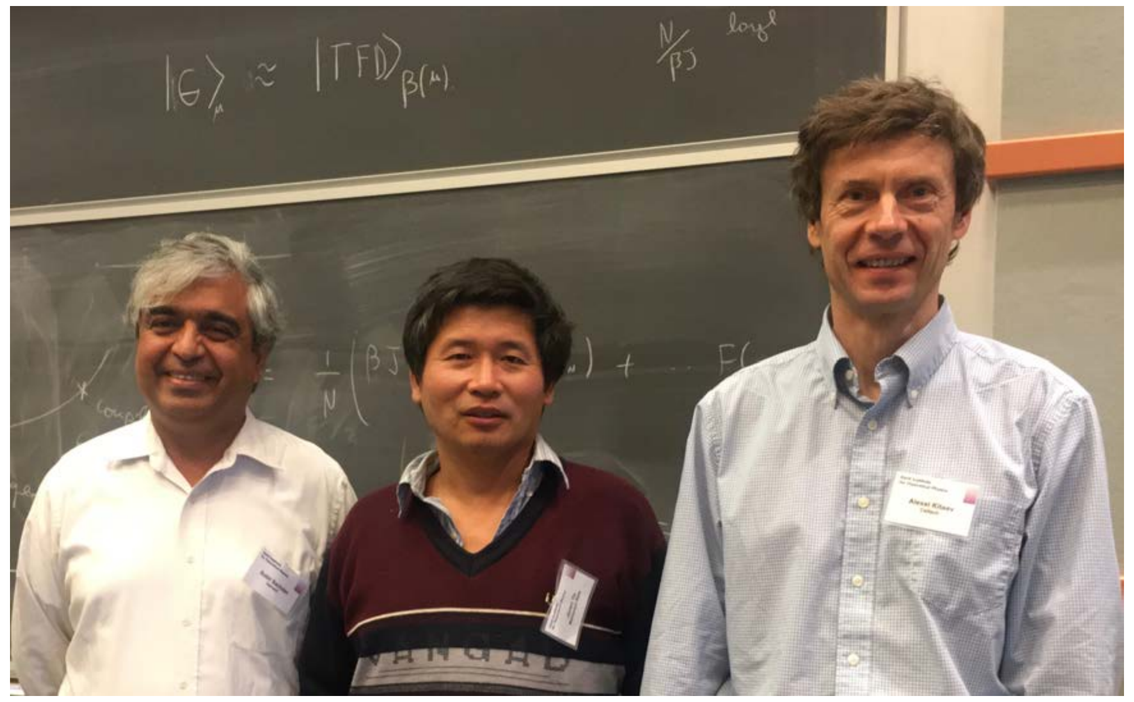

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Sachdev, Ye (1993); Kitaev (2015)

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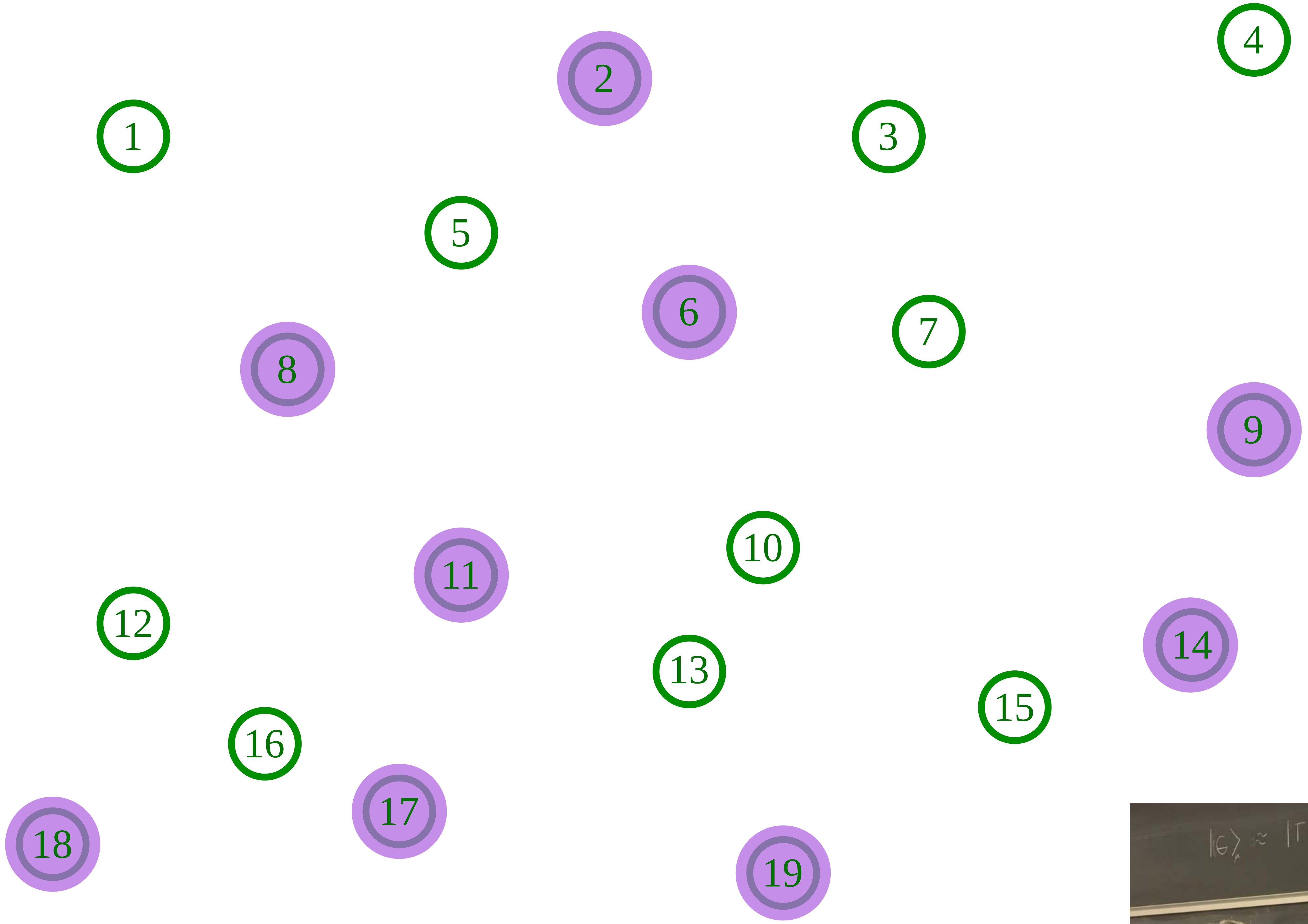

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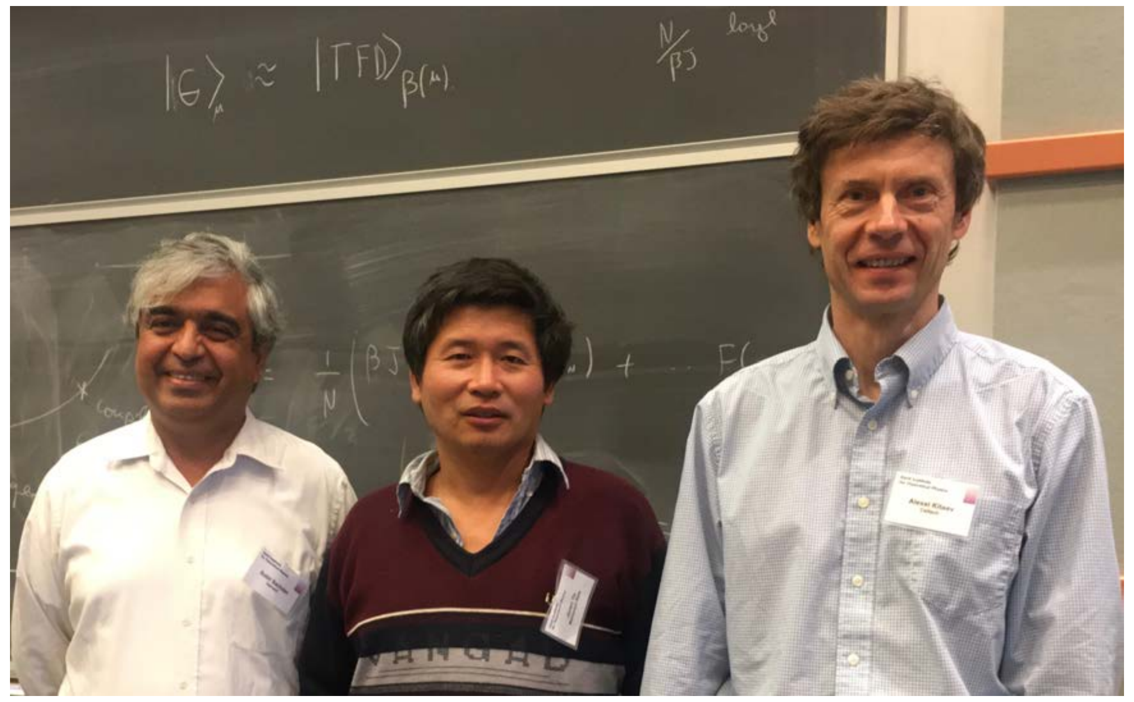
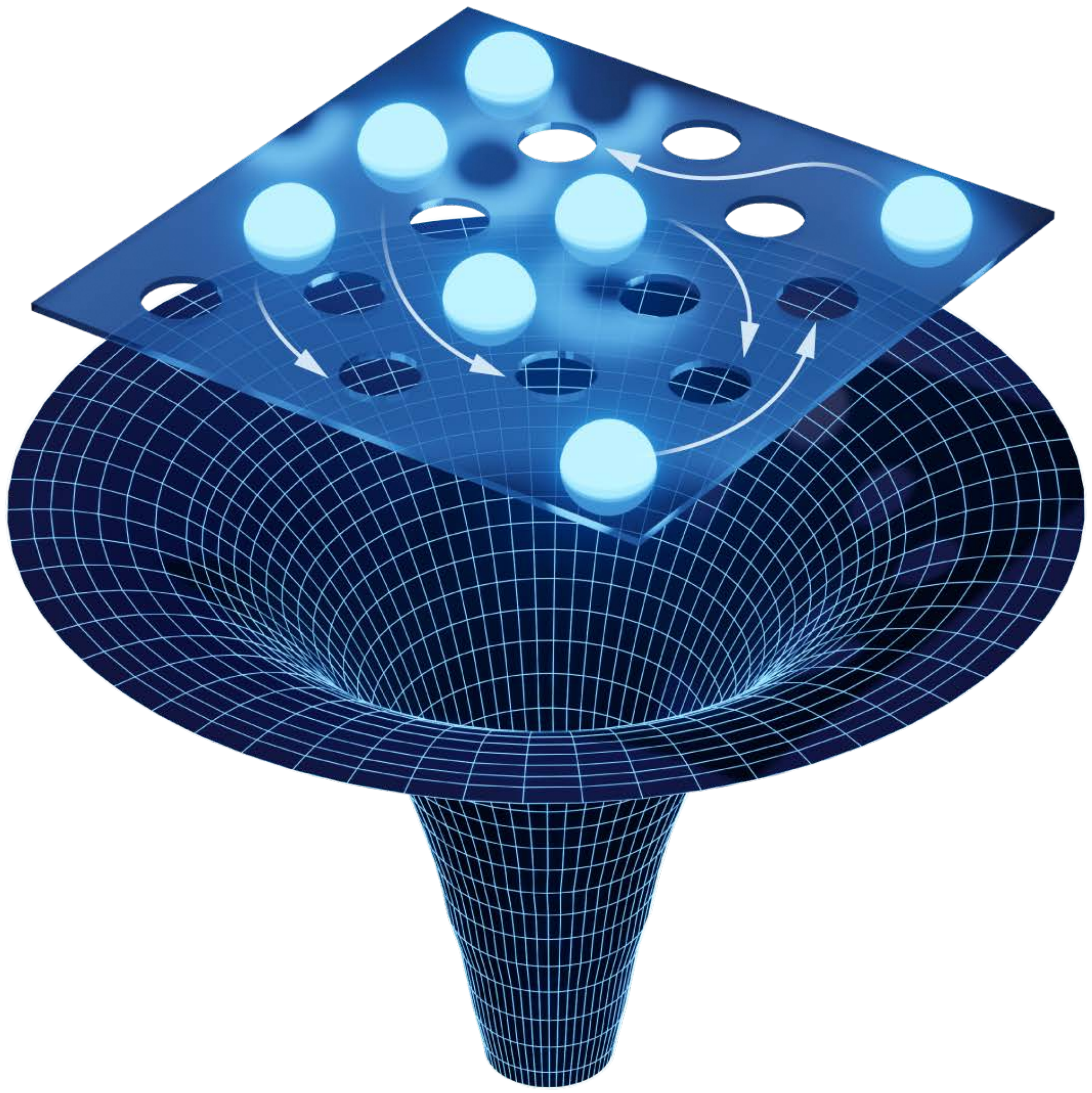


The Sachdev-Ye-Kitaev (SYK) models

Sachdev, Ye (1993); Kitaev (2015)

Solvable models of multi-particle quantum entanglement with mobile fermions.



Yields a metal whose excitations are not particle-like i.e. no bosons, fermions, anyons….

Current is carried by an "entangled quantum soup"

Yukawa-Sachdev-Ye-Kitaev model

$$\mathcal{H} = -\mu \sum_i c_i^\dagger c_i + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \Phi_\ell^2\right) + \frac{1}{N}\sum_{ij\ell} g_{ij\ell} c_i^\dagger c_j \Phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean.

W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)
J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)
A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)
E. Marcus and S. Vandoren, JHEP 01, 166 (2018)
Yuxuan Wang, PRL **124**, 017002 (2020)
I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, PRB **105**, 235111 (2022)
Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)
W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)
I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

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with $g_{ij\ell}$ independent random numbers with zero mean. The large N equations for the Green's functions and self energies of the fermions (G, Σ) and bosons (D, Π) are

$$G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad D(i\omega_n) = \frac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)}$$

$$\Sigma(\tau) = g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau)$$

Make the low frequency ansatz

$$G(i\omega) \sim -i\mathrm{sgn}(\omega)|\omega|^{-(1-2\Delta)} \quad , \quad D(i\omega) \sim |\omega|^{1-4\Delta} \quad , \quad \frac{1}{4} < \Delta < \frac{1}{2}$$

A consistent solution exists for

$$\frac{4\Delta - 1}{2(2\Delta - 1)[\sec(2\pi\Delta) - 1]} = 1 \quad , \quad \Delta = 0.42037\ldots$$

I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
See also Yuxuan Wang, PRL **124**, 017002 (2020)

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$$\Sigma(\tau) = g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau)$$

At $T > 0$, solutions are fully characterized by a universal frequency-dependent relaxation time,

$$\frac{\hbar}{\tau(\omega)} = k_B T \, \Phi_\tau \left(\frac{\hbar\omega}{k_B T} \right)$$

where Φ_τ is a known universal function.

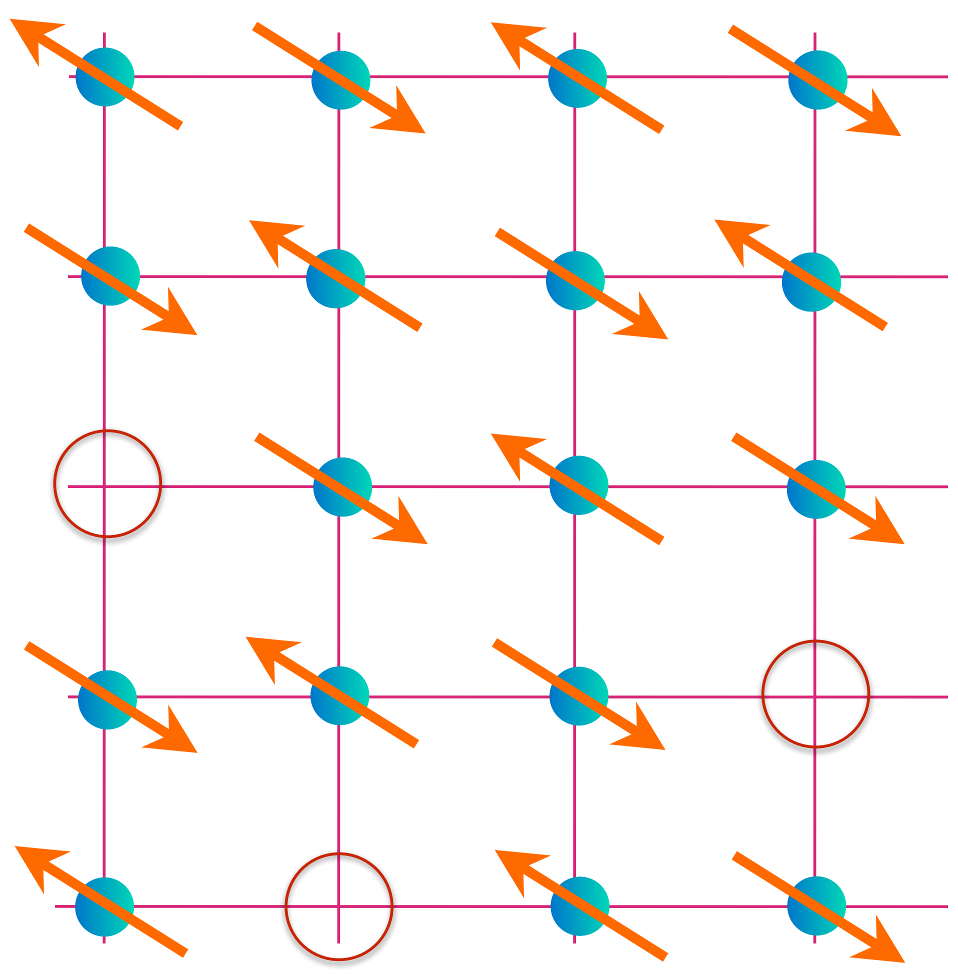
From FL* to FL
via
the strange metal
using the 2D-YSYK model

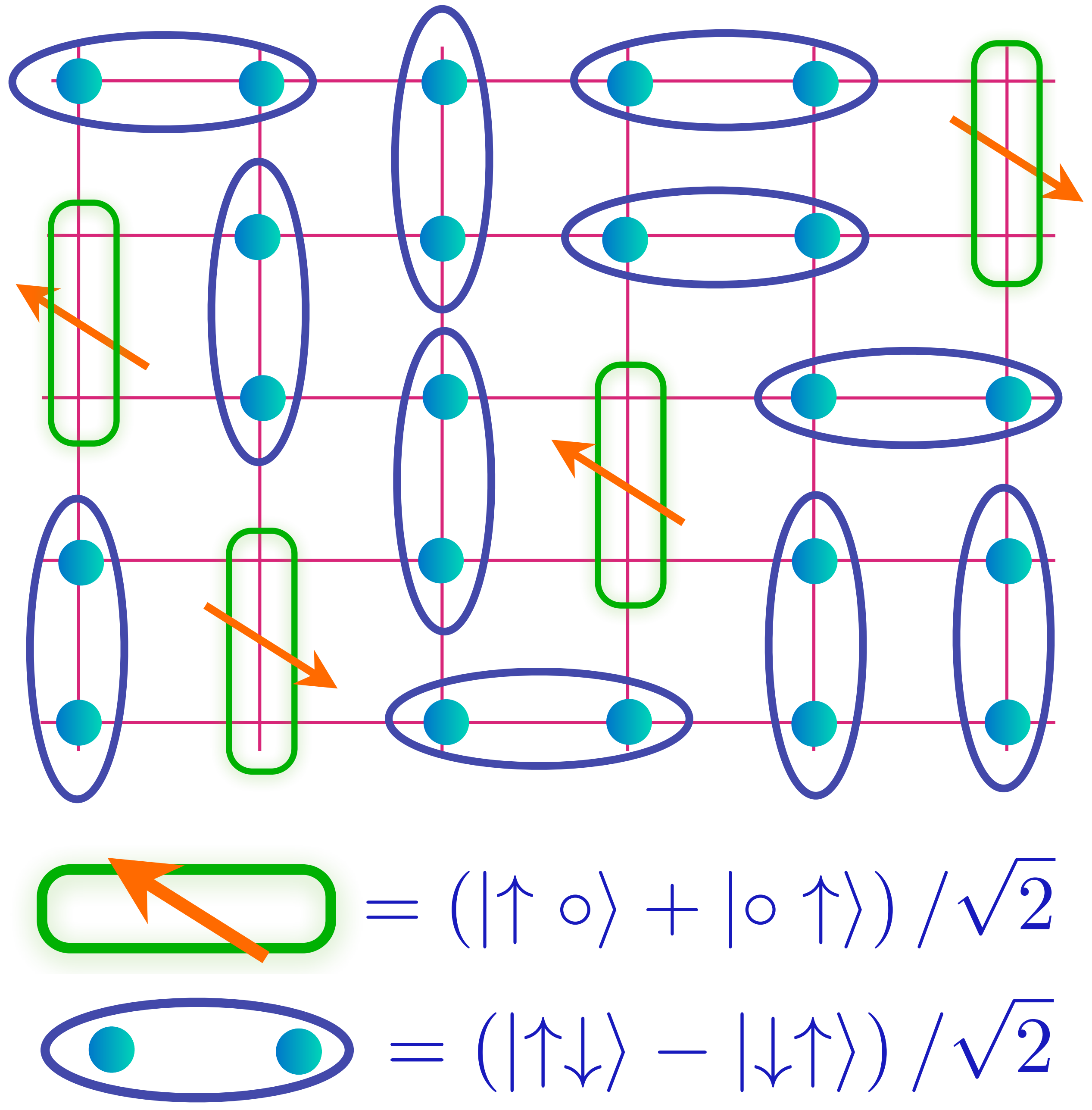
AF Metal

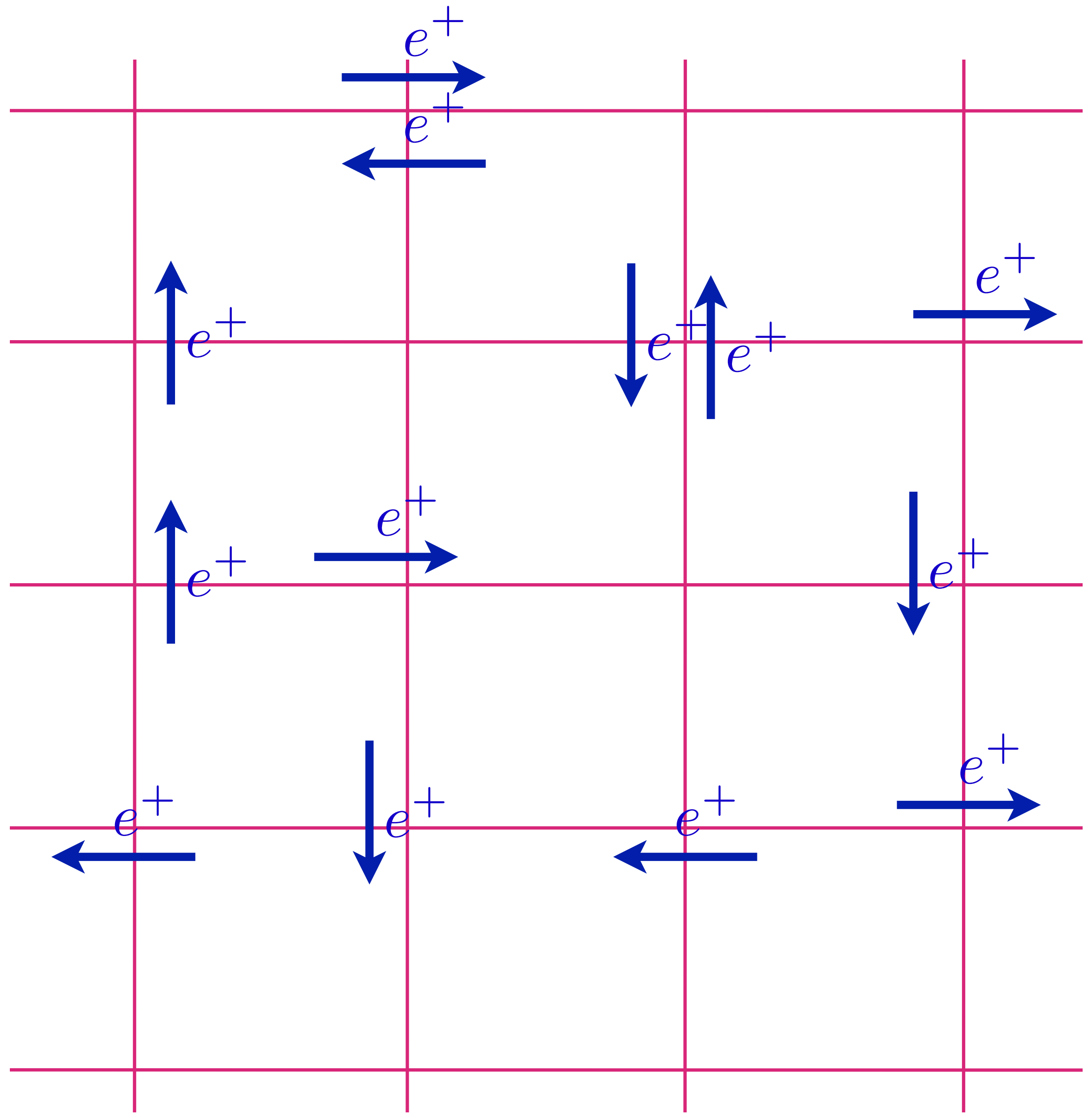
FL*

FL

Quantum phase transition between two metals (FL* and FL) at $p = p_c$, with no symmetry breaking.







Carrier density p

$\langle (-1)^{\boldsymbol{r}} \boldsymbol{S}_{\boldsymbol{r}} \rangle \neq 0$

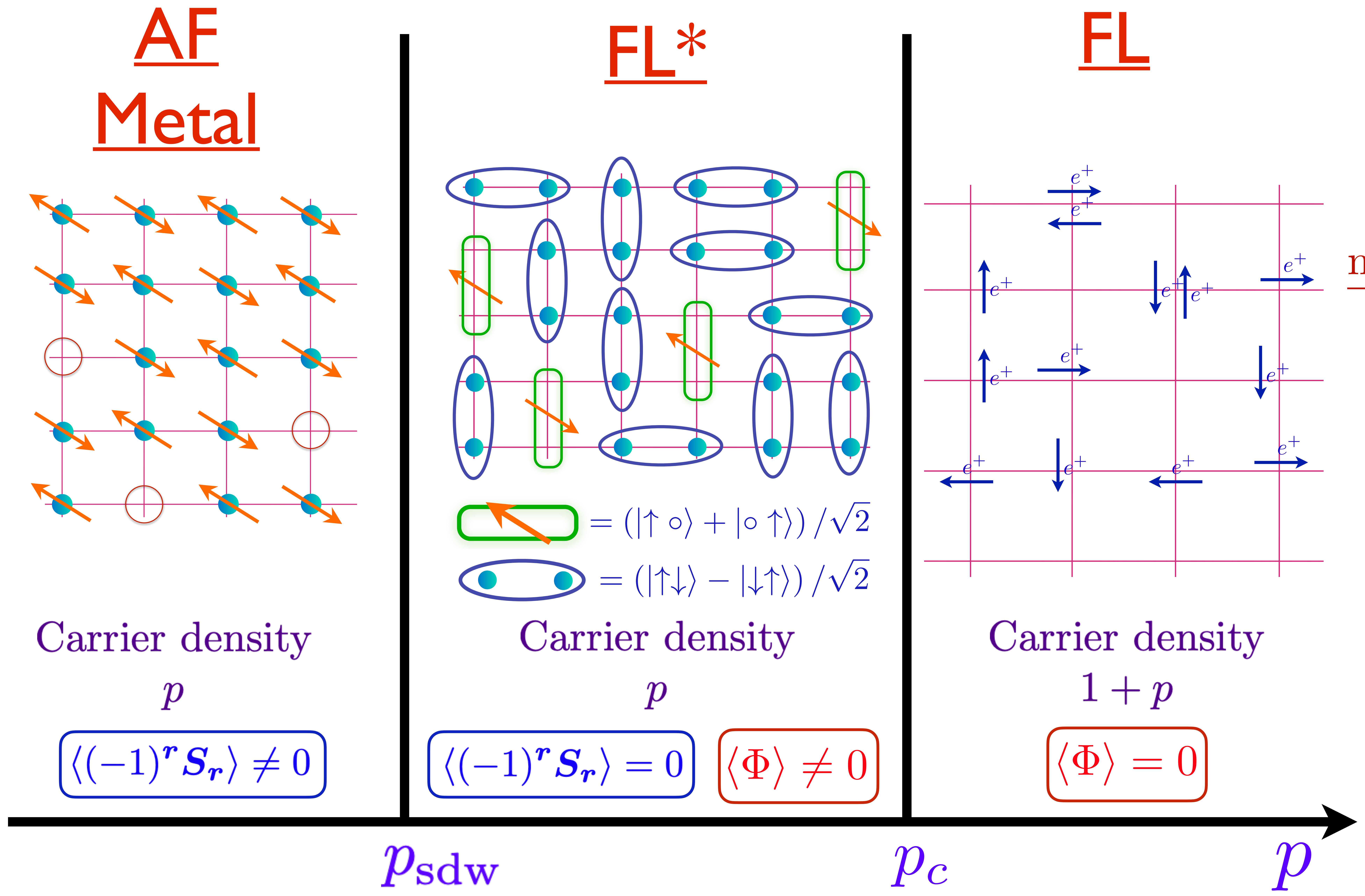
Carrier density p

$\langle (-1)^{\boldsymbol{r}} \boldsymbol{S}_{\boldsymbol{r}} \rangle = 0$

Carrier density $1 + p$

p_{sdw} p_c p

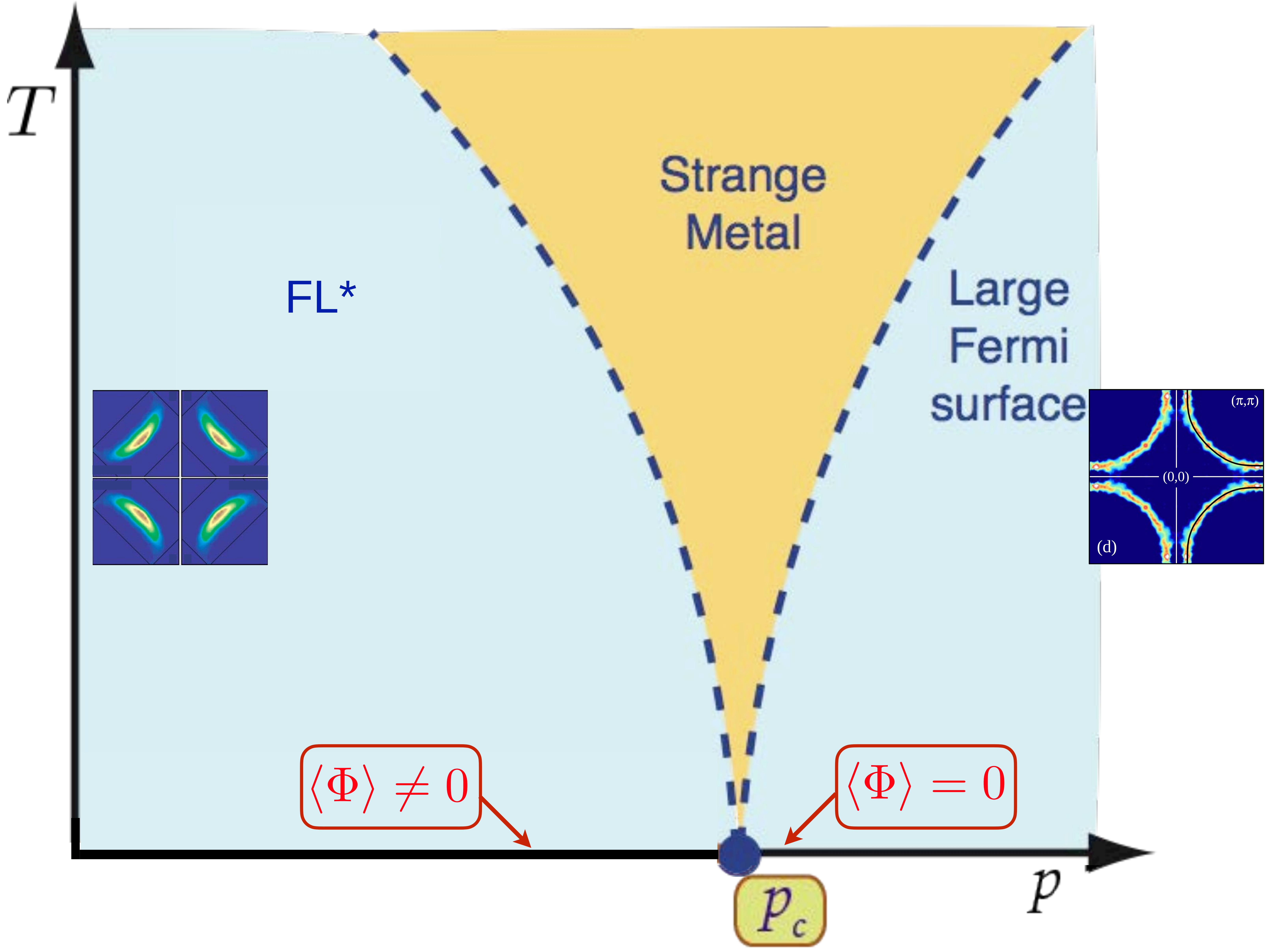
S. Sachdev, M.A. Metlitski and M. Punk, Journal of Physics Condensed Matter **24**, 294205 (2012)



Quantum phase transition between two metals (FL* and FL) at $p = p_c$, with no symmetry breaking.

Described by the condensation of a *Higgs* field Φ.

S. Sachdev, M.A. Metlitski and M. Punk, Journal of Physics Condensed Matter **24**, 294205 (2012)
Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)



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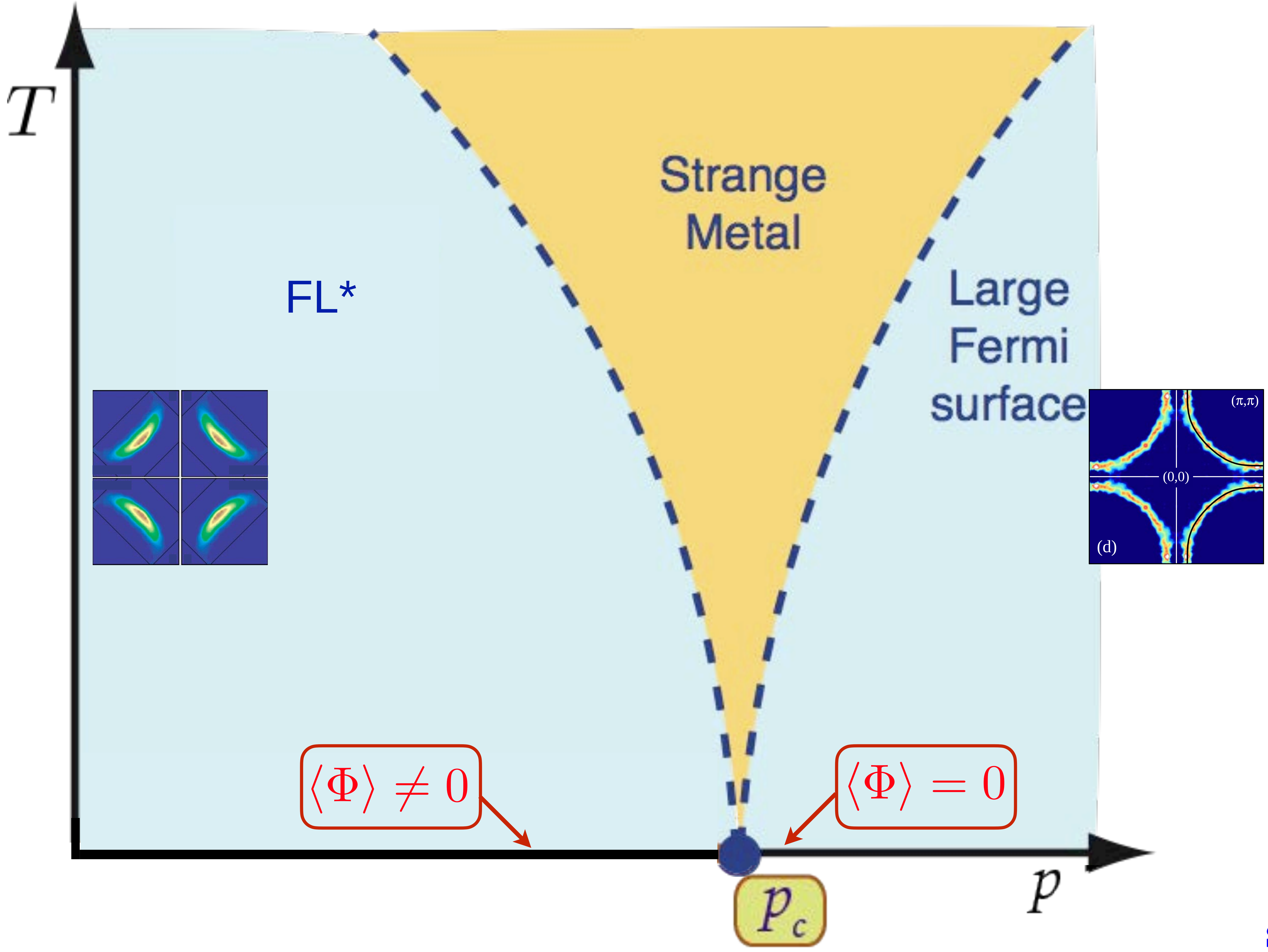
Strange metal is obtained from the $T > 0$ quantum criticality of the FL-FL* transition, *provided* there is momentum relaxation.

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)



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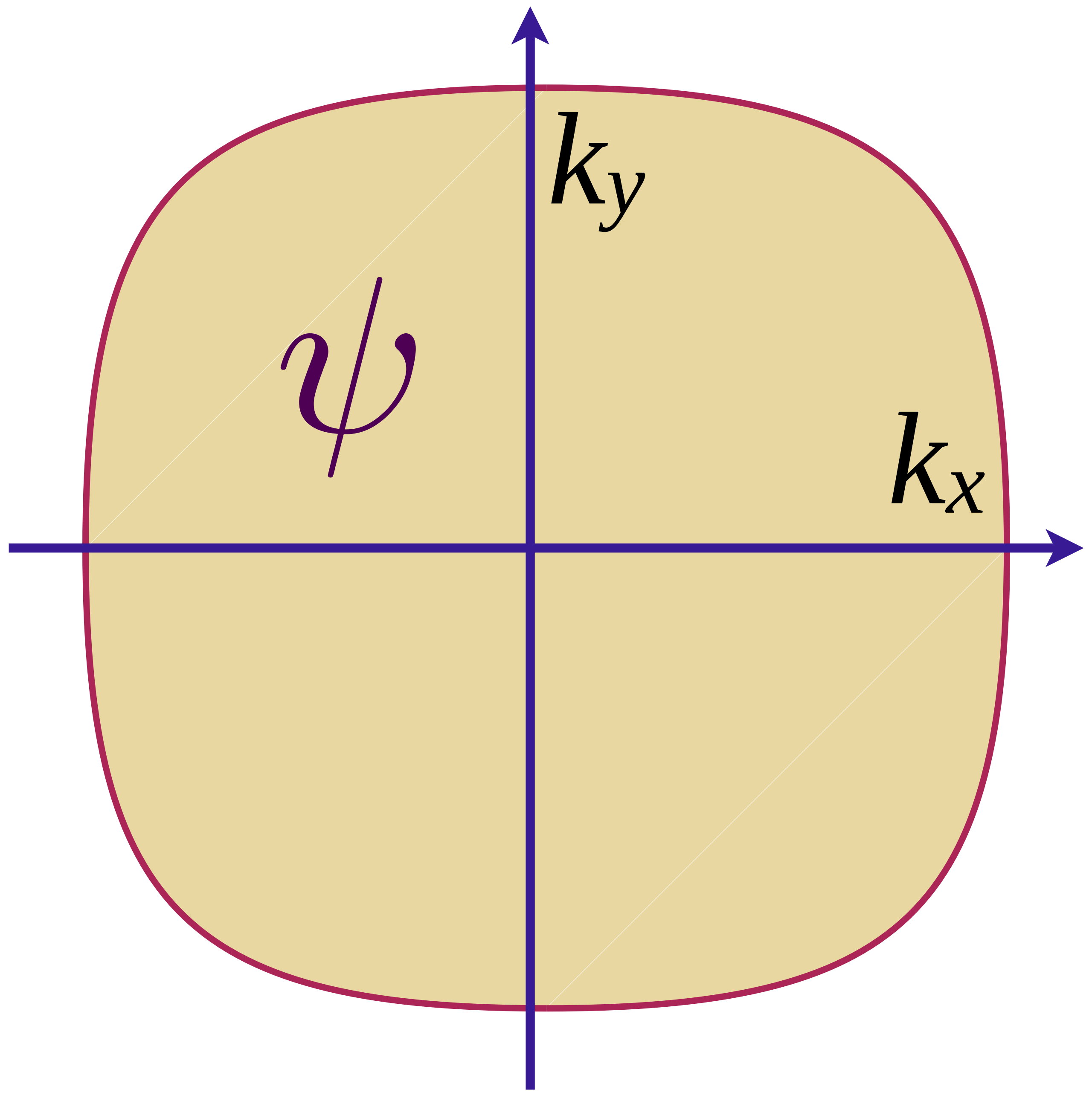
Strange metal is obtained from the $T > 0$ quantum criticality of the FL-FL* transition, *provided* there is momentum relaxation.
At low T this requires spatial disorder.
Most relevant is Harris disorder: spatial variation in the value of p_c.

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

2D-YSYK model: Fermi surface + Higgs boson with interaction disorder

$$\mathcal{L} = c^\dagger_{\boldsymbol{k}\alpha}\left(\frac{\partial}{\partial\tau} + \varepsilon(\boldsymbol{k})\right) c_{\boldsymbol{k}\alpha} + f^\dagger_{1\boldsymbol{k}\alpha}\left(\frac{\partial}{\partial\tau} + \widetilde{\varepsilon}(\boldsymbol{k})\right) f_{1\boldsymbol{k}\alpha}$$



$$+ [\nabla\Phi(\boldsymbol{r})]^2 + s\,[\Phi(\boldsymbol{r})]^2 + u\,[\Phi(\boldsymbol{r})]^4$$

$$+ [g + g'(\boldsymbol{r})]\, c^\dagger_\alpha(\boldsymbol{r}) f_{1\alpha}(\boldsymbol{r})\,\Phi(\boldsymbol{r}) + \text{H.c.}$$

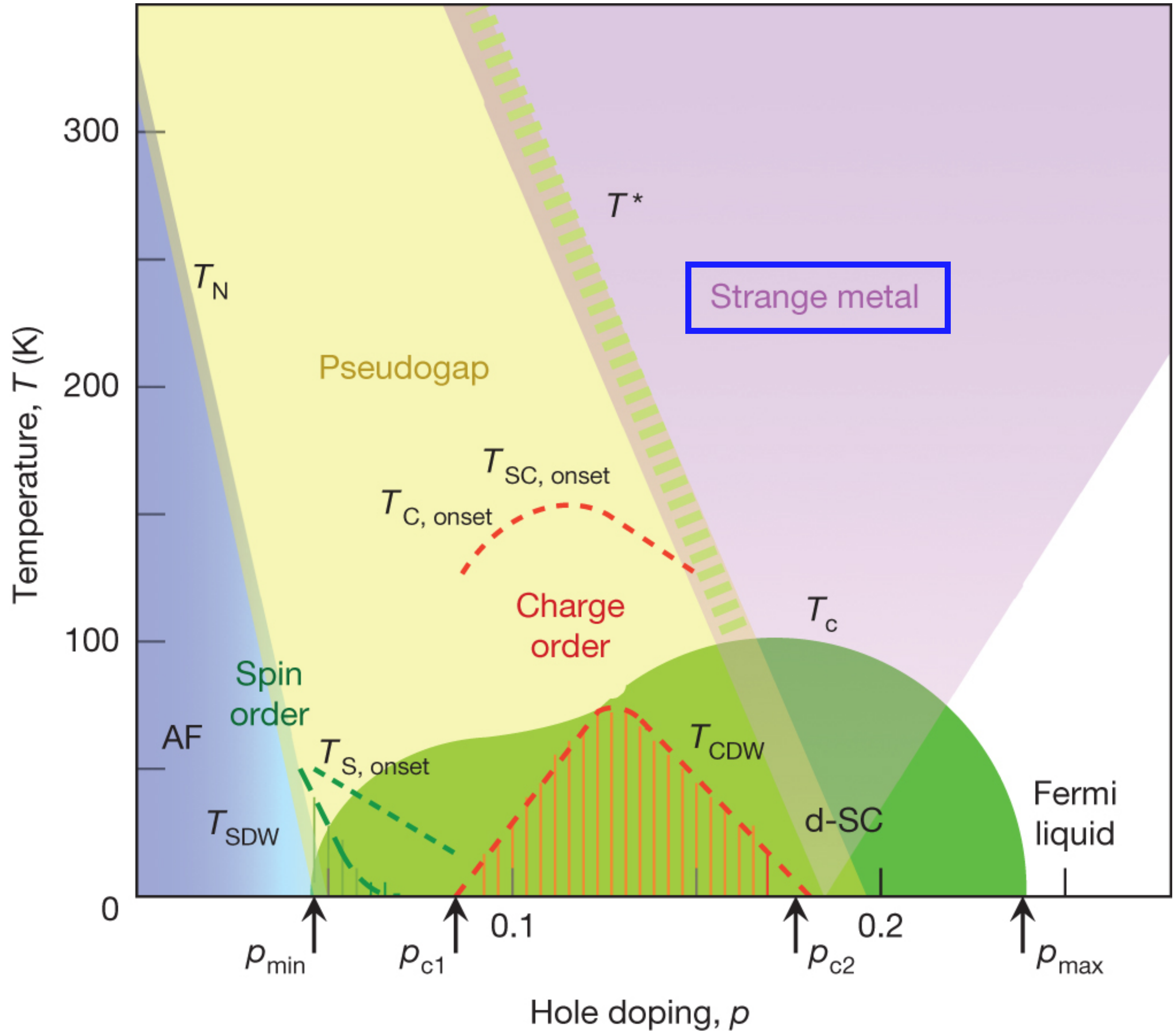
$$+ v(\boldsymbol{r}) c^\dagger_\alpha(\boldsymbol{r}) c_\alpha(\boldsymbol{r})$$

Φ^2 "mass" disorder $s \to s + \delta s(\boldsymbol{r})$ is strongly relevant; rescale Φ to move disorder to the Yukawa coupling.

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

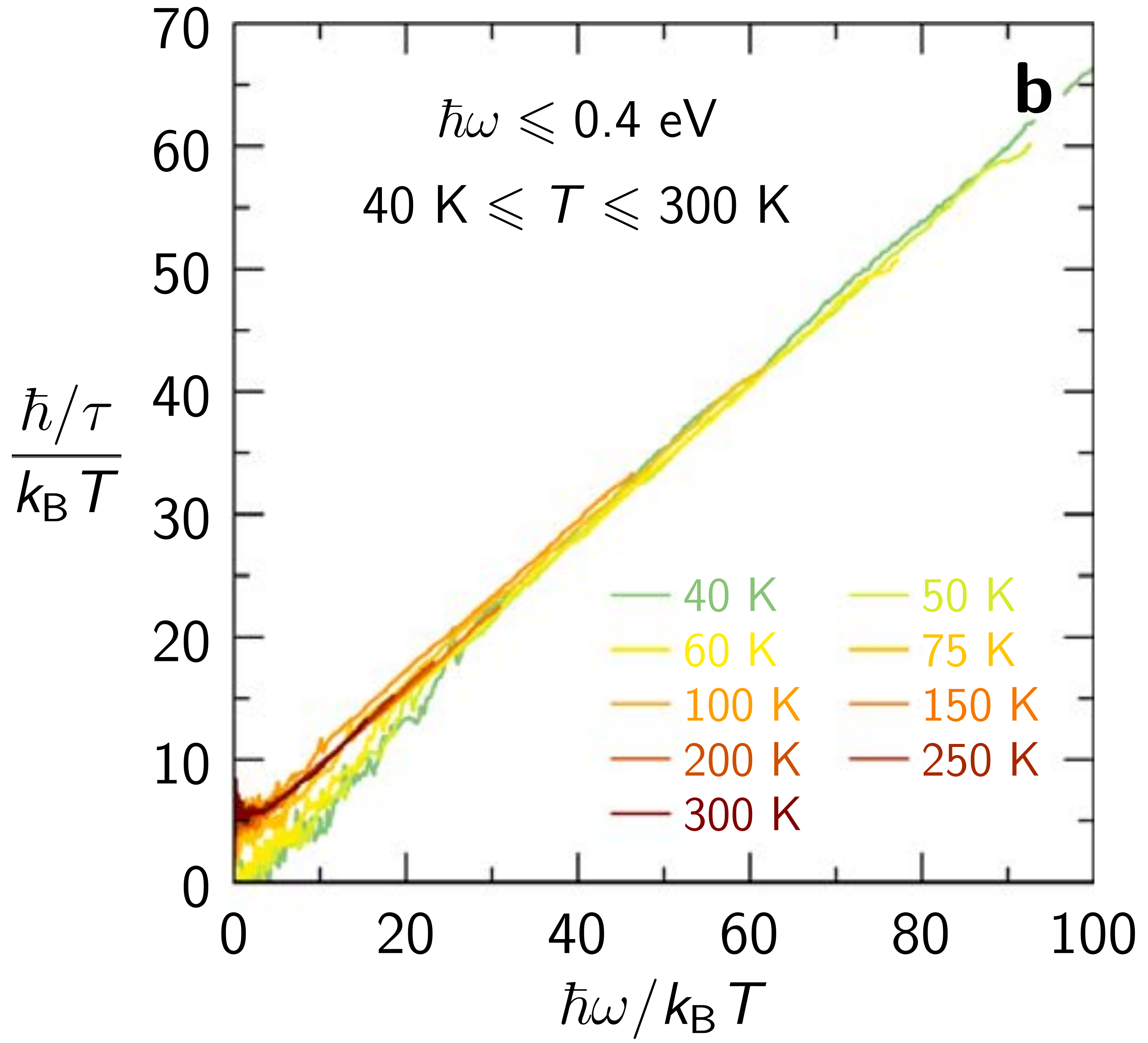
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



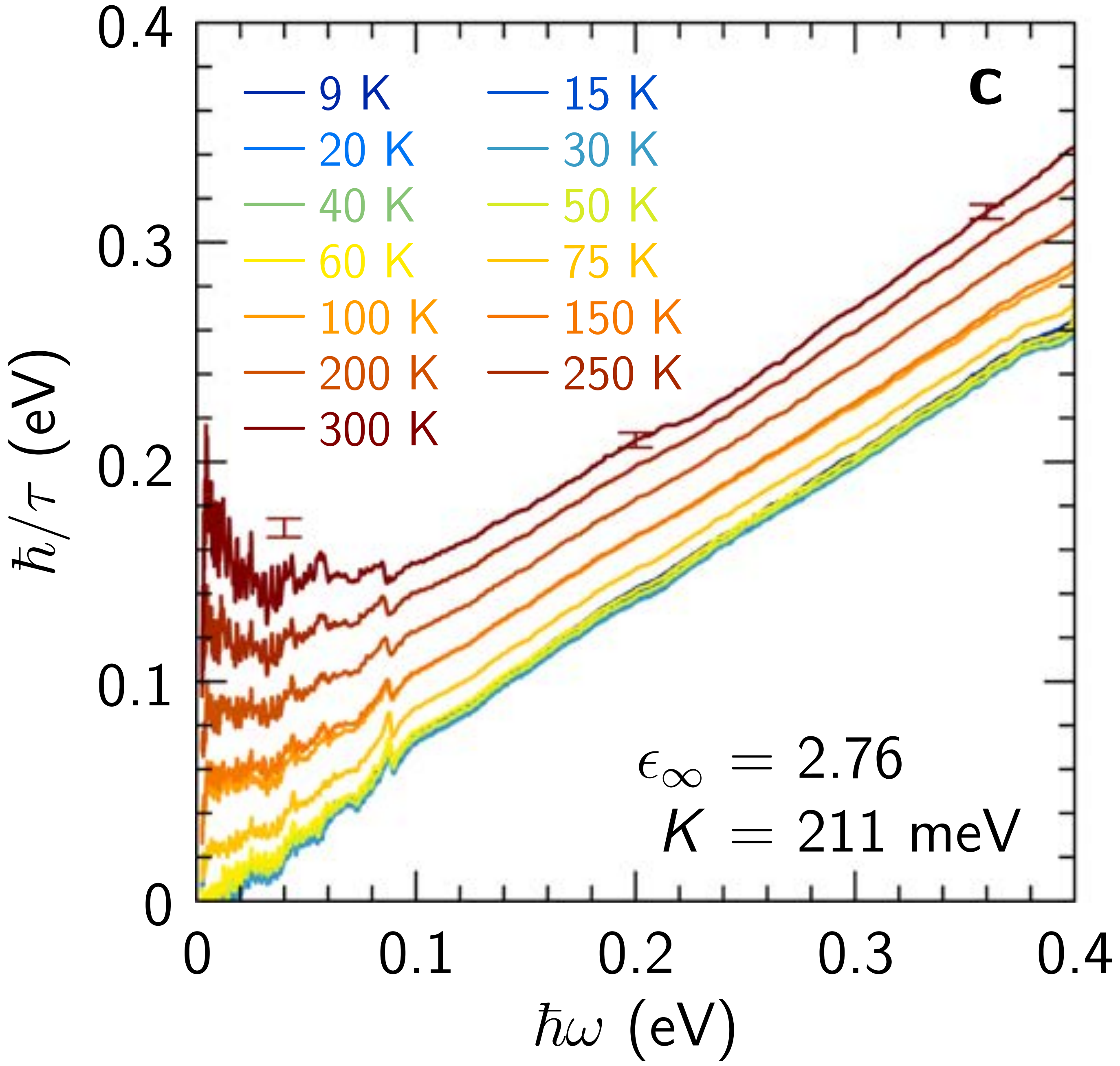
Non-Boltzmann
Planckian dynamics
of large Fermi surface

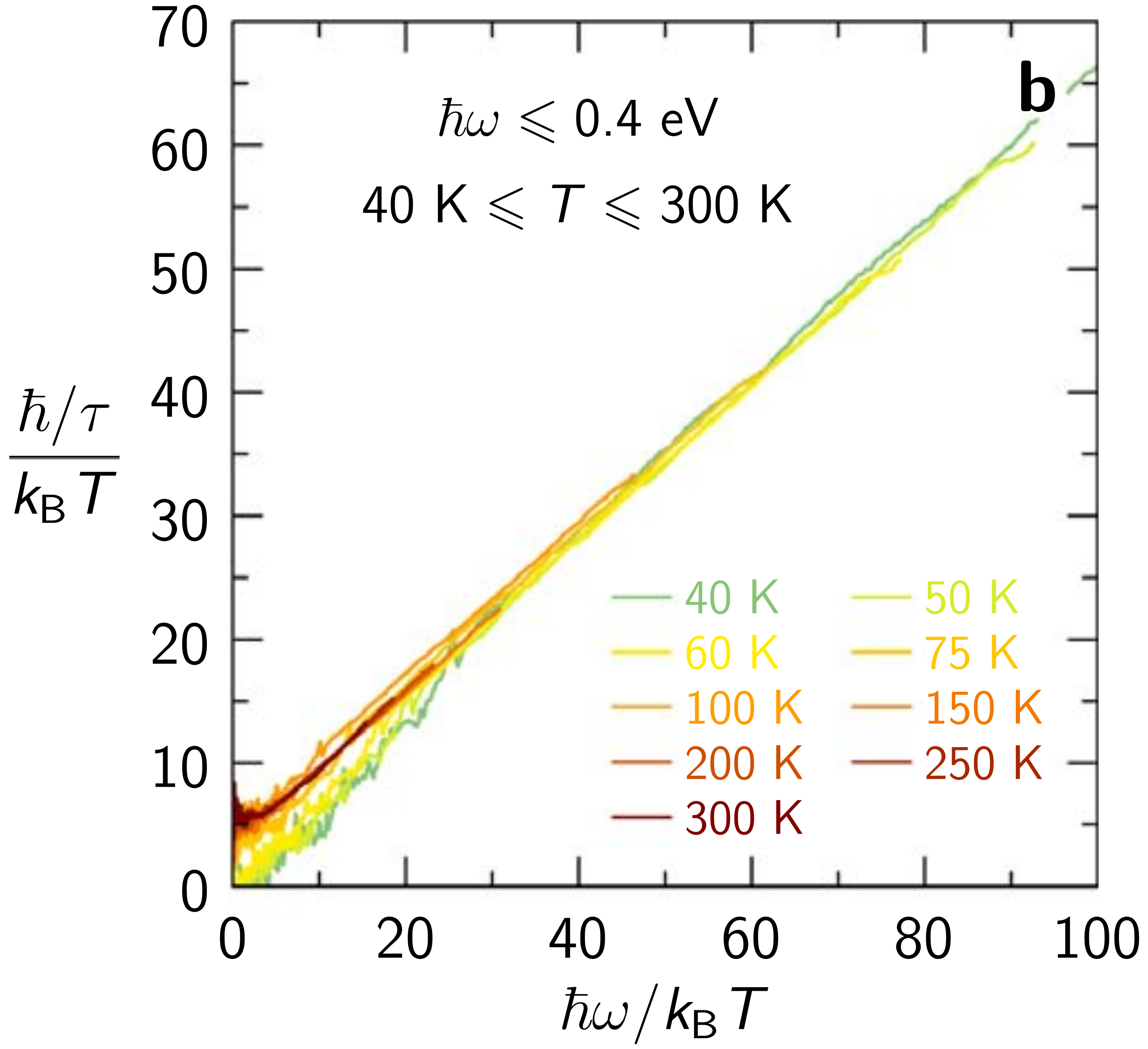
Electron scattering time τ from optical conductivity

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$



B. Michon, C. Berthod, C.W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges, Nature Comm. **14**, 3033 (2023)

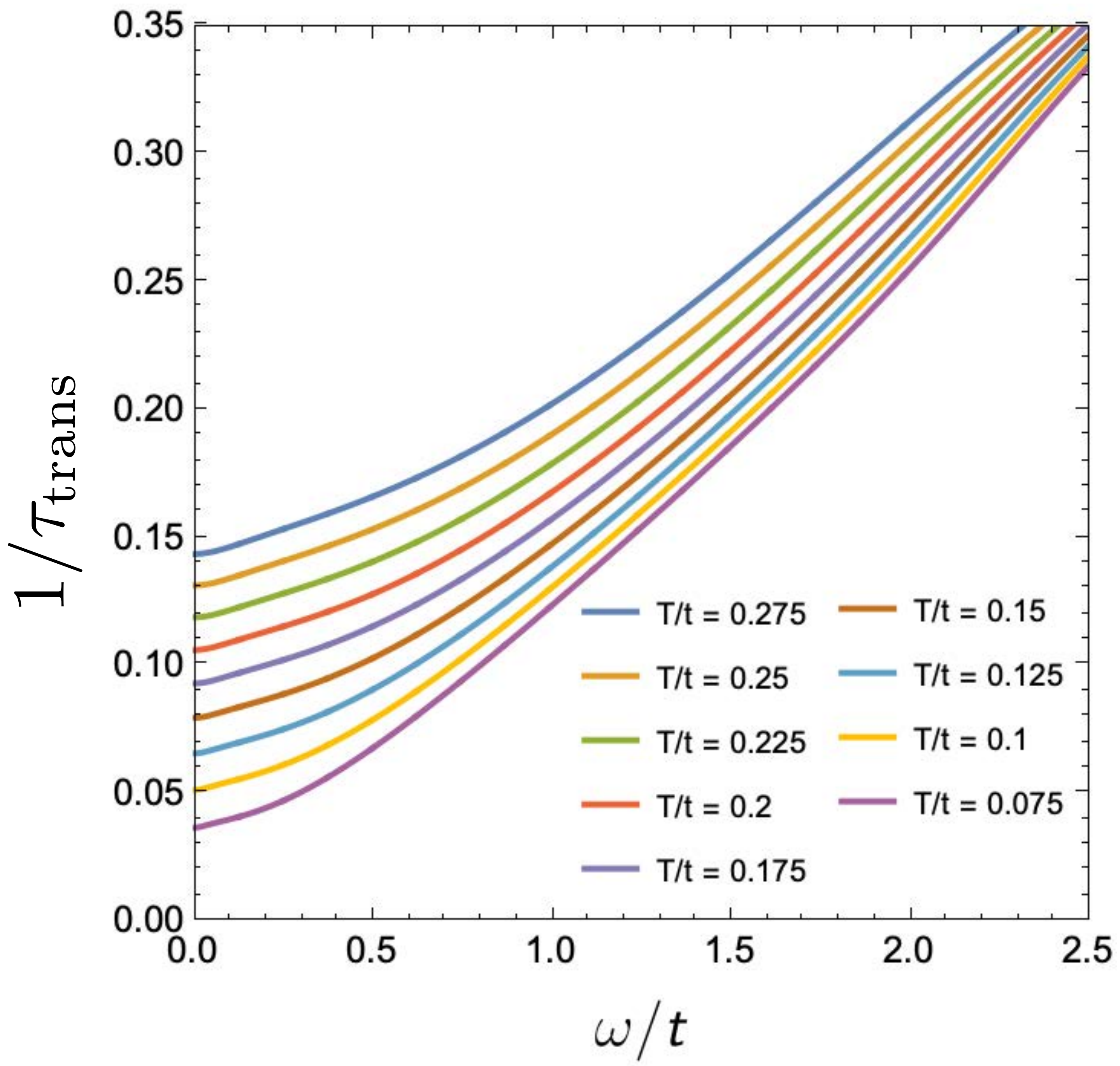


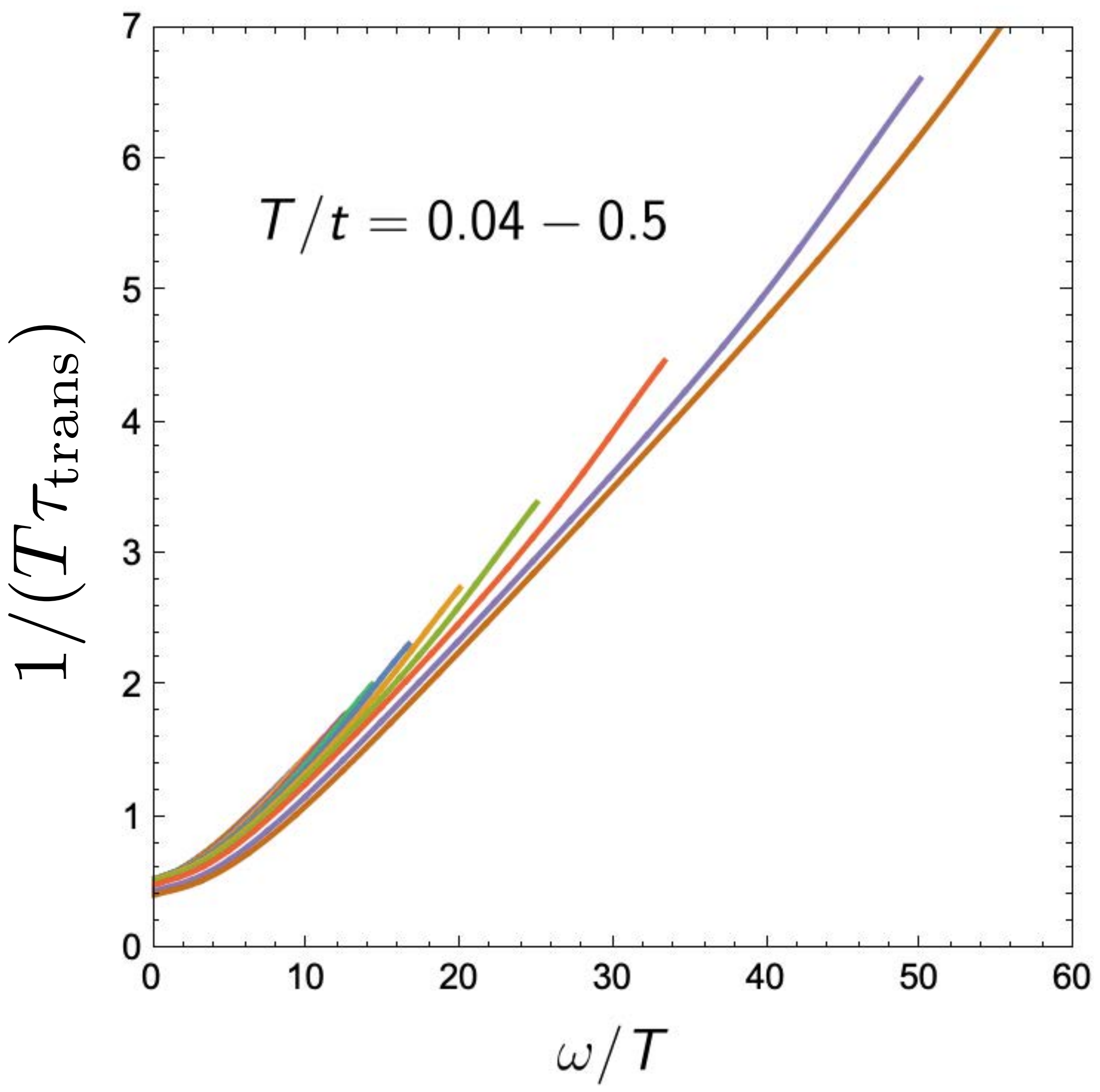


$$\sigma(\omega) = i \frac{e^2 K/(\hbar d_c)}{\hbar\omega \dfrac{m^*(\omega)}{m} + i \dfrac{\hbar}{\tau(\omega)}}$$

From optical conductivity data of Michon et al. (2023)

$$\frac{\hbar}{\tau(\omega)} = k_B T \, \Phi_\tau \left(\frac{\hbar\omega}{k_B T} \right)$$



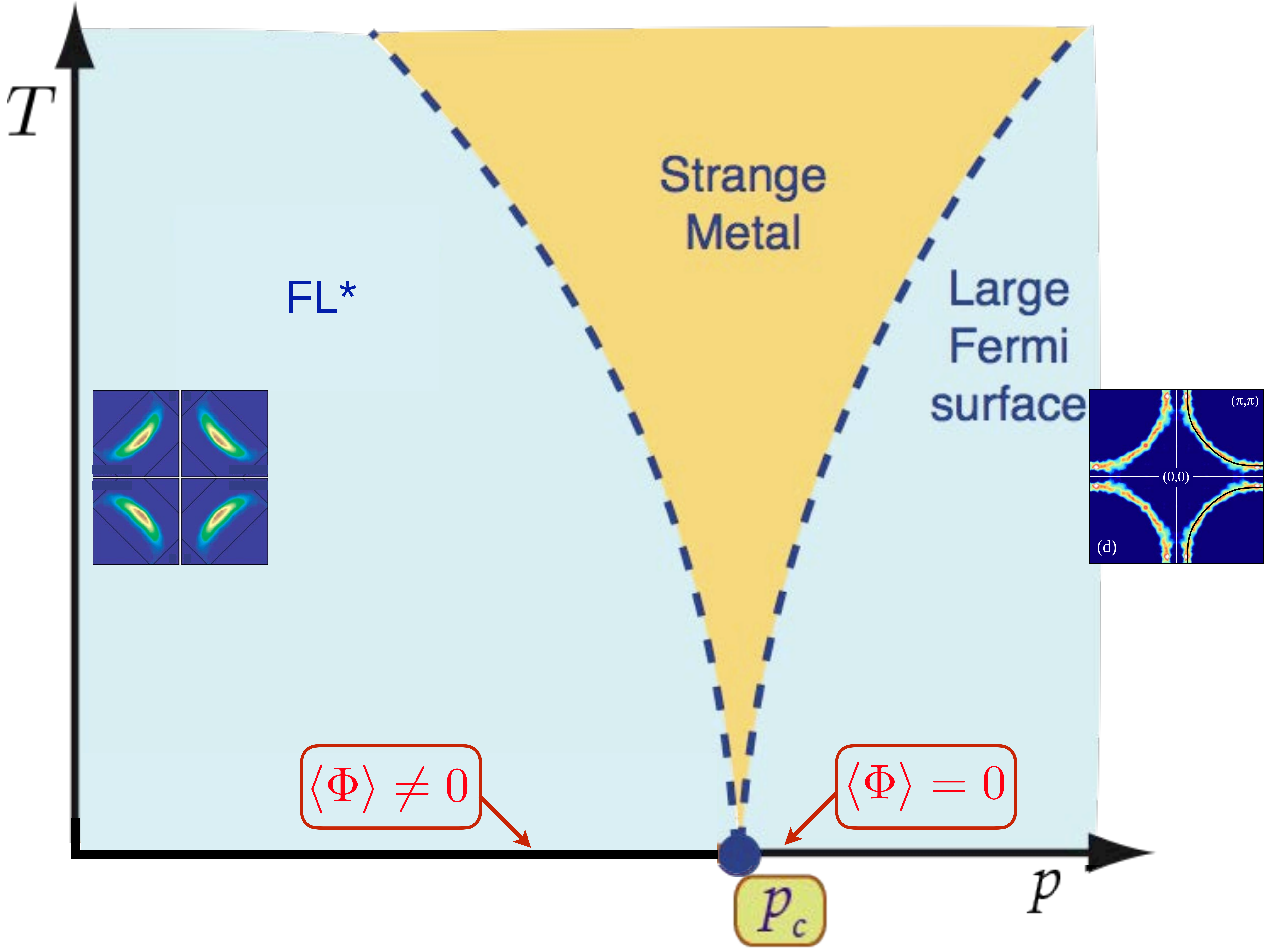


2d-YSYK theory

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. S., *Science* **381**, 790 (2023)

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)

From FL* and FL
to the
d-wave superconductor

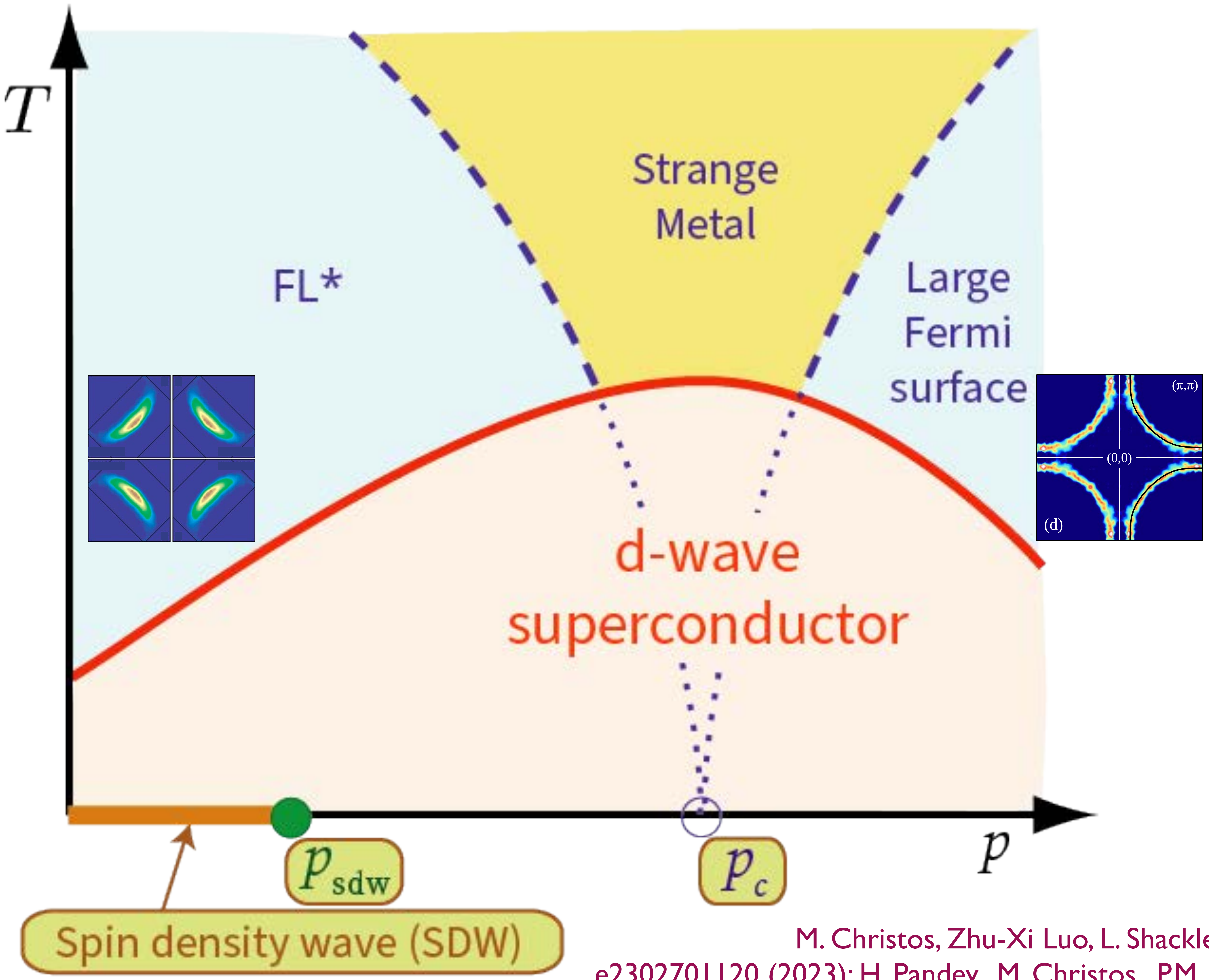


Quantum phase transition between two metals (FL* and FL) at $p = p_c$, with <u>no symmetry breaking</u>.

Described by the condensation of a <u>*Higgs*</u> field Φ.

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

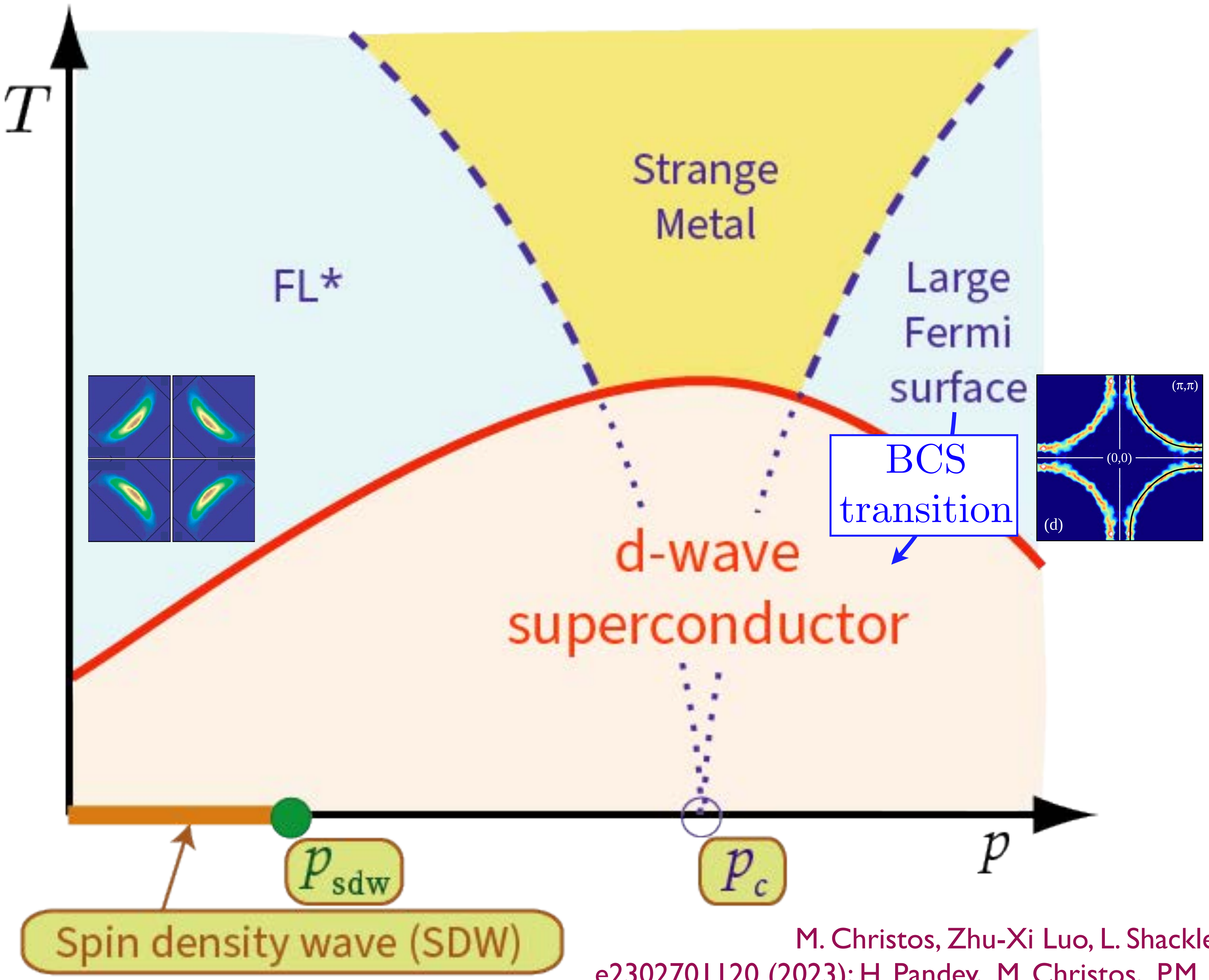
S. S., Physica Status Solidi B Basic Research **247**, 537 (2010)



Both metals lead to the same d-wave superconductor at lower temperatures, and so there is no transition at $p = p_c$ within the superconducting state.

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023); H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336

S. S., Physica Status Solidi B Basic Research **247**, 537 (2010)



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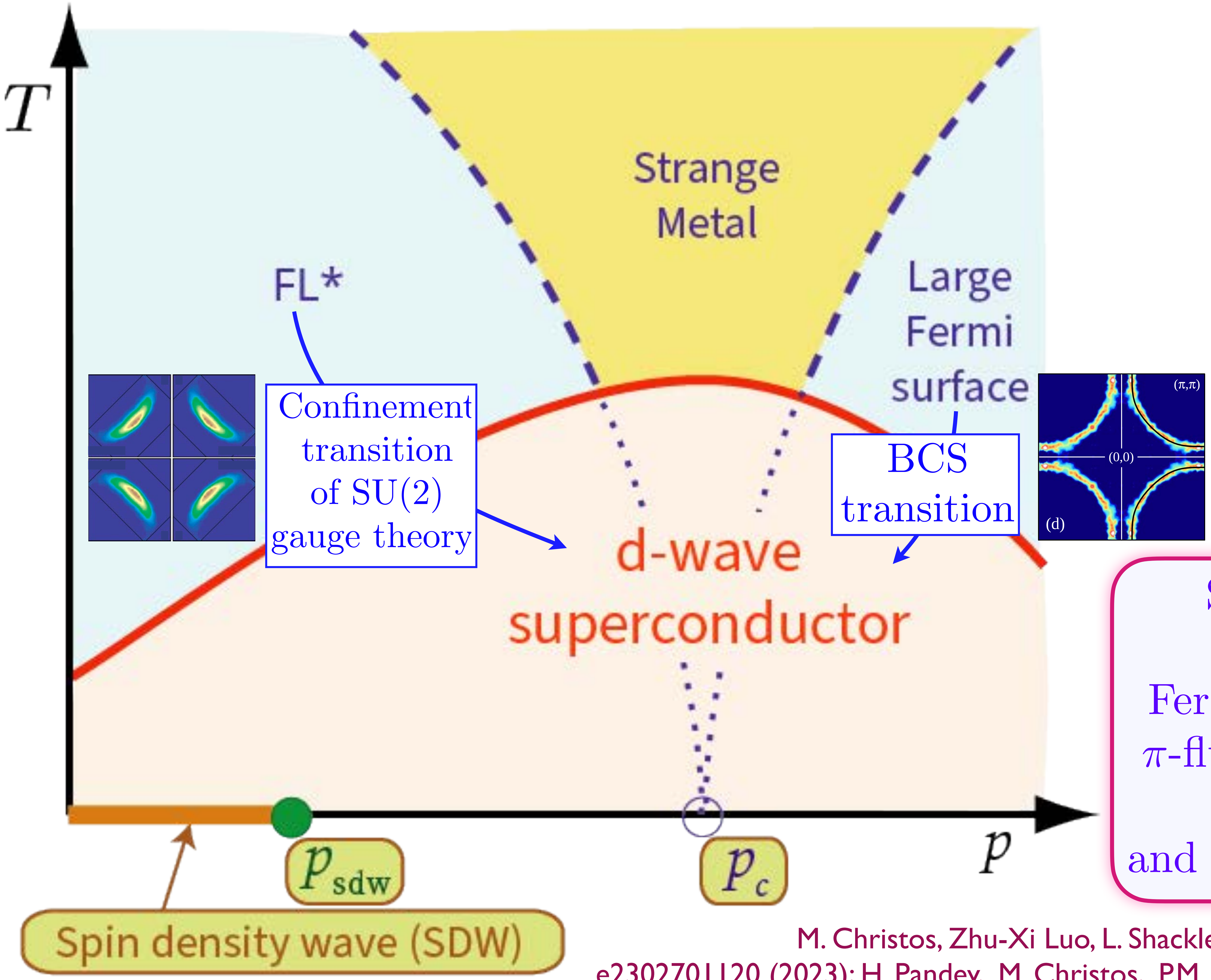
S. S., Physica Status Solidi B Basic Research **247**, 537 (2010)



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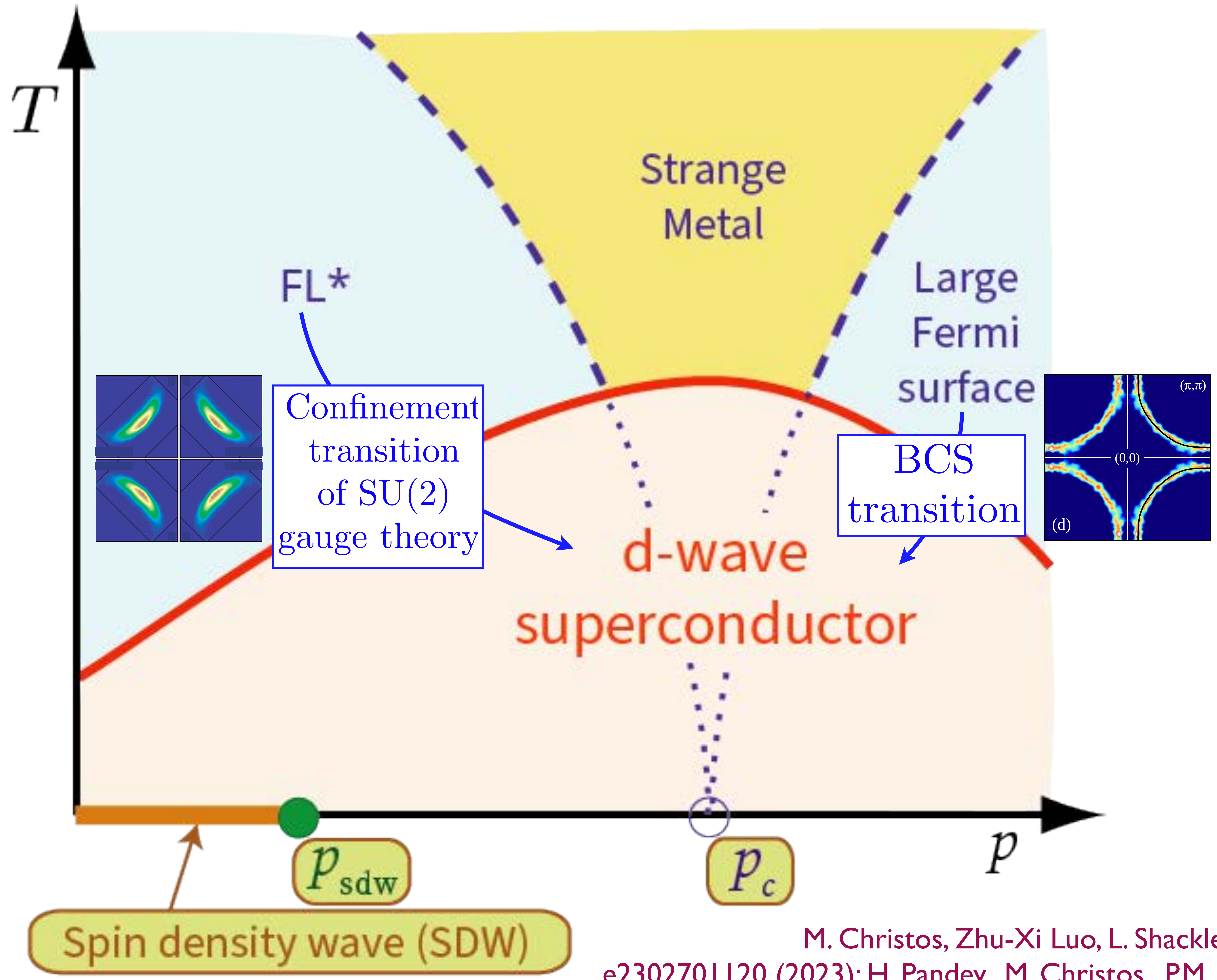
SU(2) gauge theory similar to Weinberg-Salam theory:
Fermionic spinons (*cf.* neutrinos) of π-flux state (Affleck-Marston, 1988),
electrons,
and SU(2) fundamental Higgs field B.

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120**, e2302701120 (2023); H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336

S. S., Physica Status Solidi B Basic Research **247**, 537 (2010)



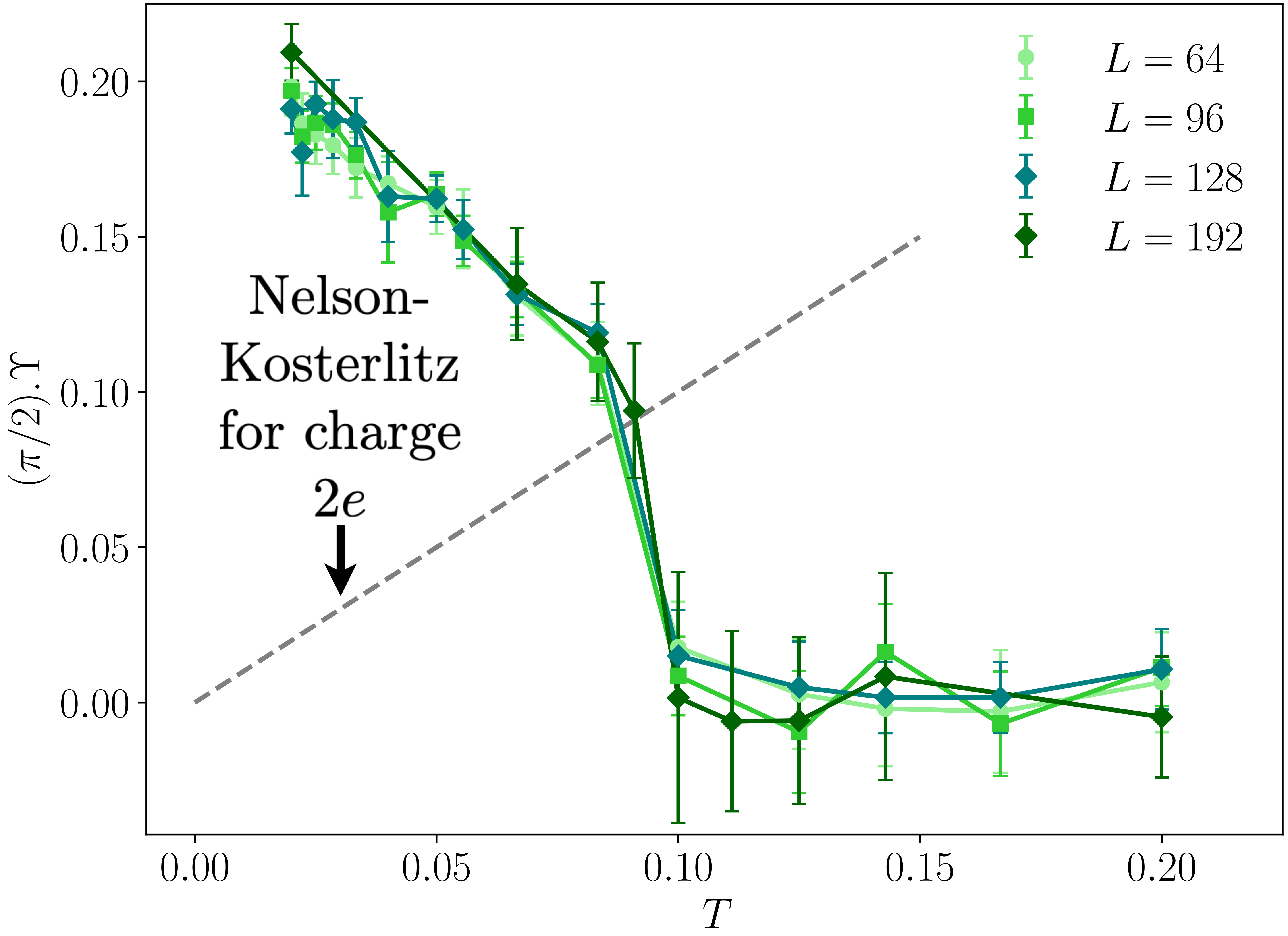
Both metals lead to the same d-wave superconductor at lower temperatures, and so there is no transition at $p = p_c$ within the superconducting state.

But electron spectra in the normal state, and vortex core structures in the superconducting state are distinct!

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120**, e2302701120 (2023); H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336

- Classical thermal ensemble of charge e Higgs boson B and SU(2) gauge field U.

$\Upsilon =$ Helicity Modulus



H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, A. Nikolaenko, S. Sharma, S.S., arXiv:2507.05336

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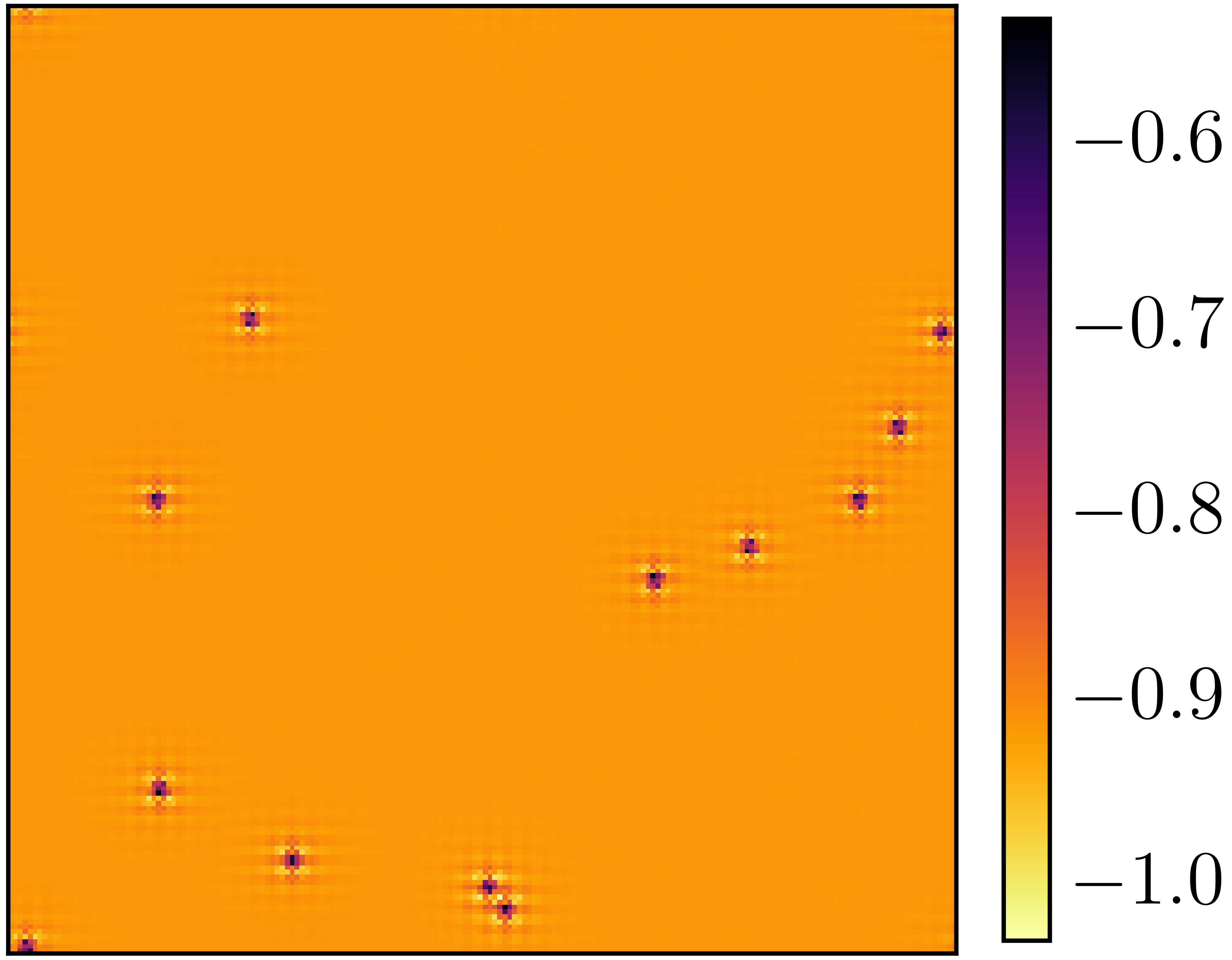
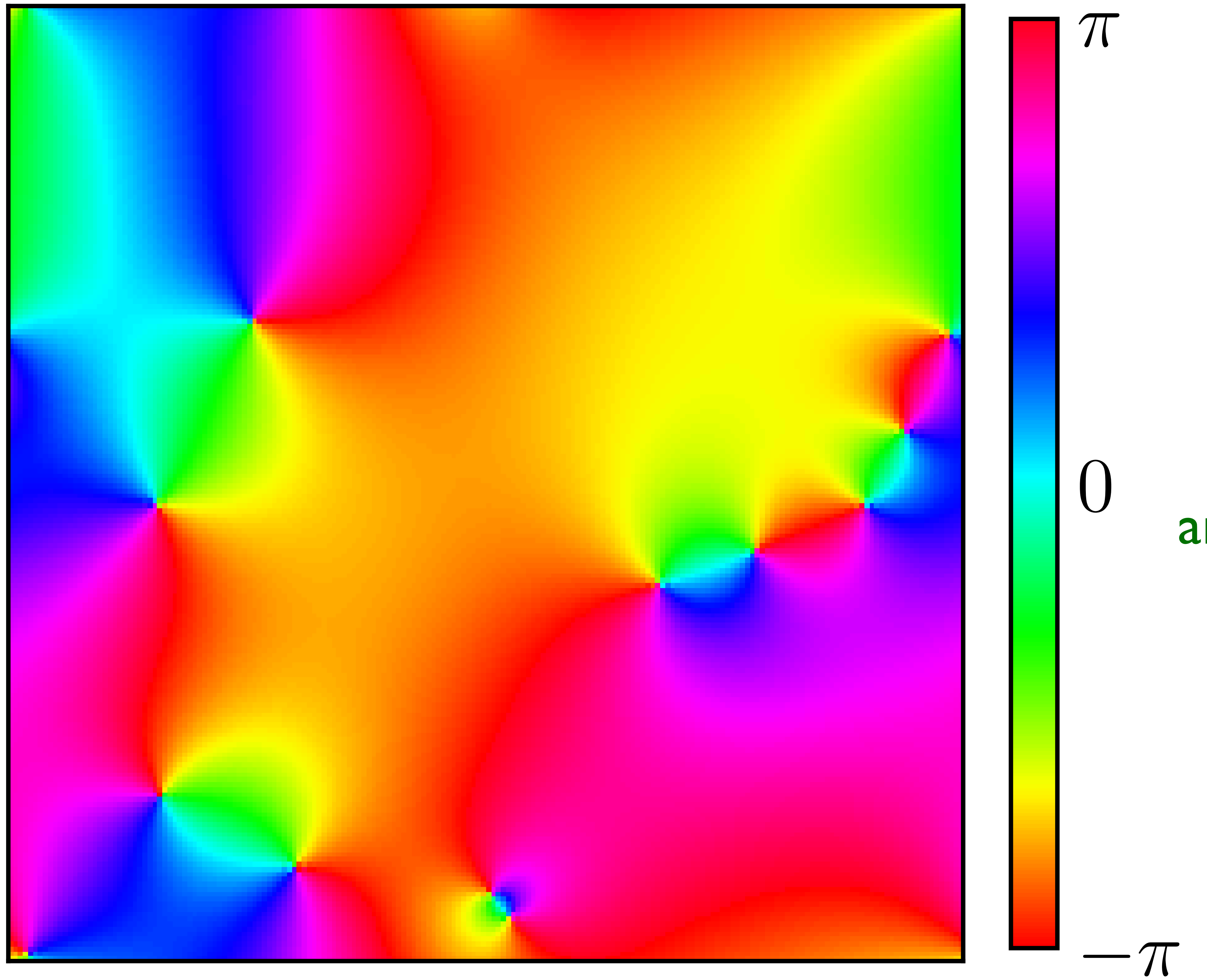


Bond density

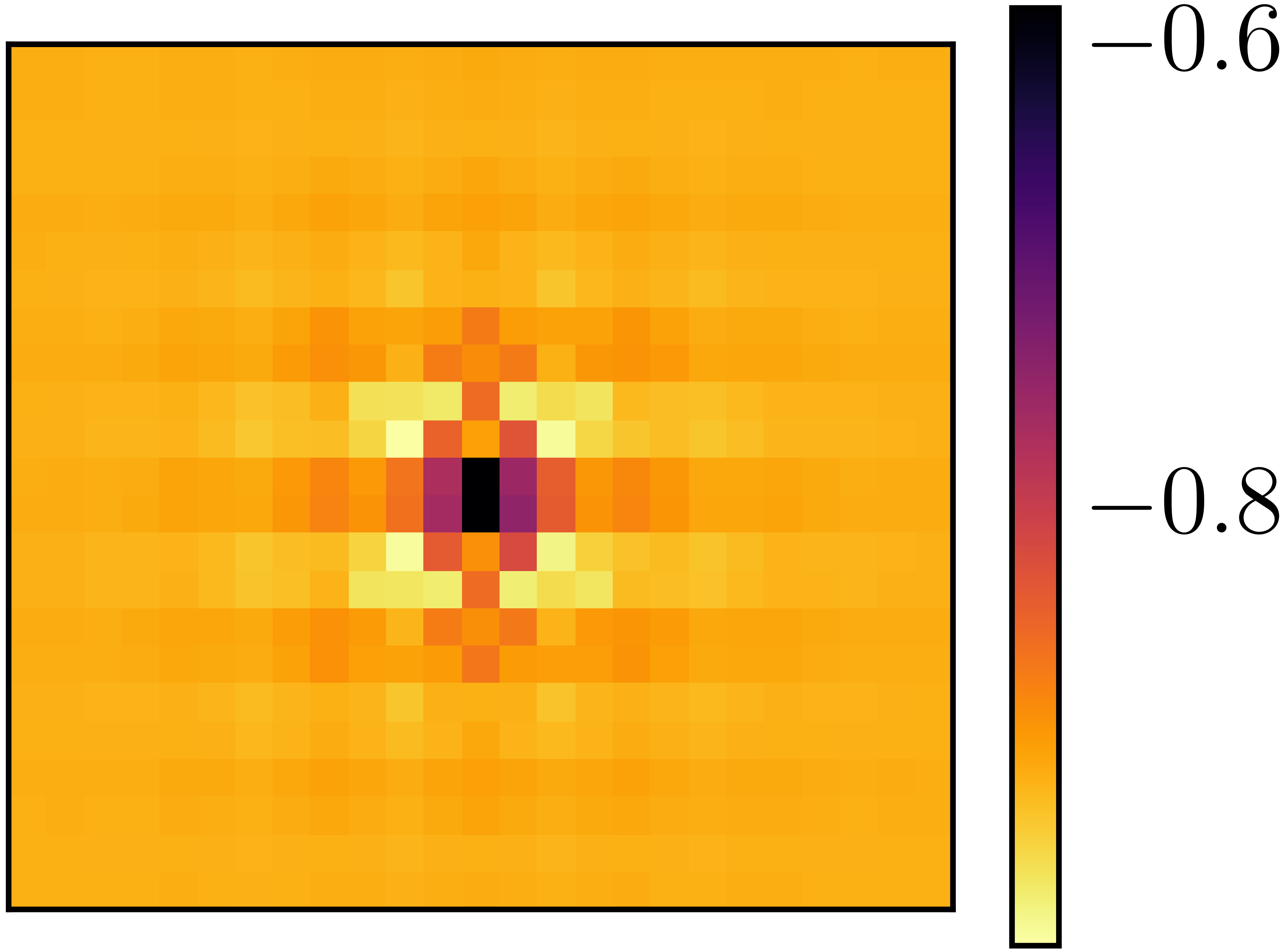


Phase of pairing amplitude

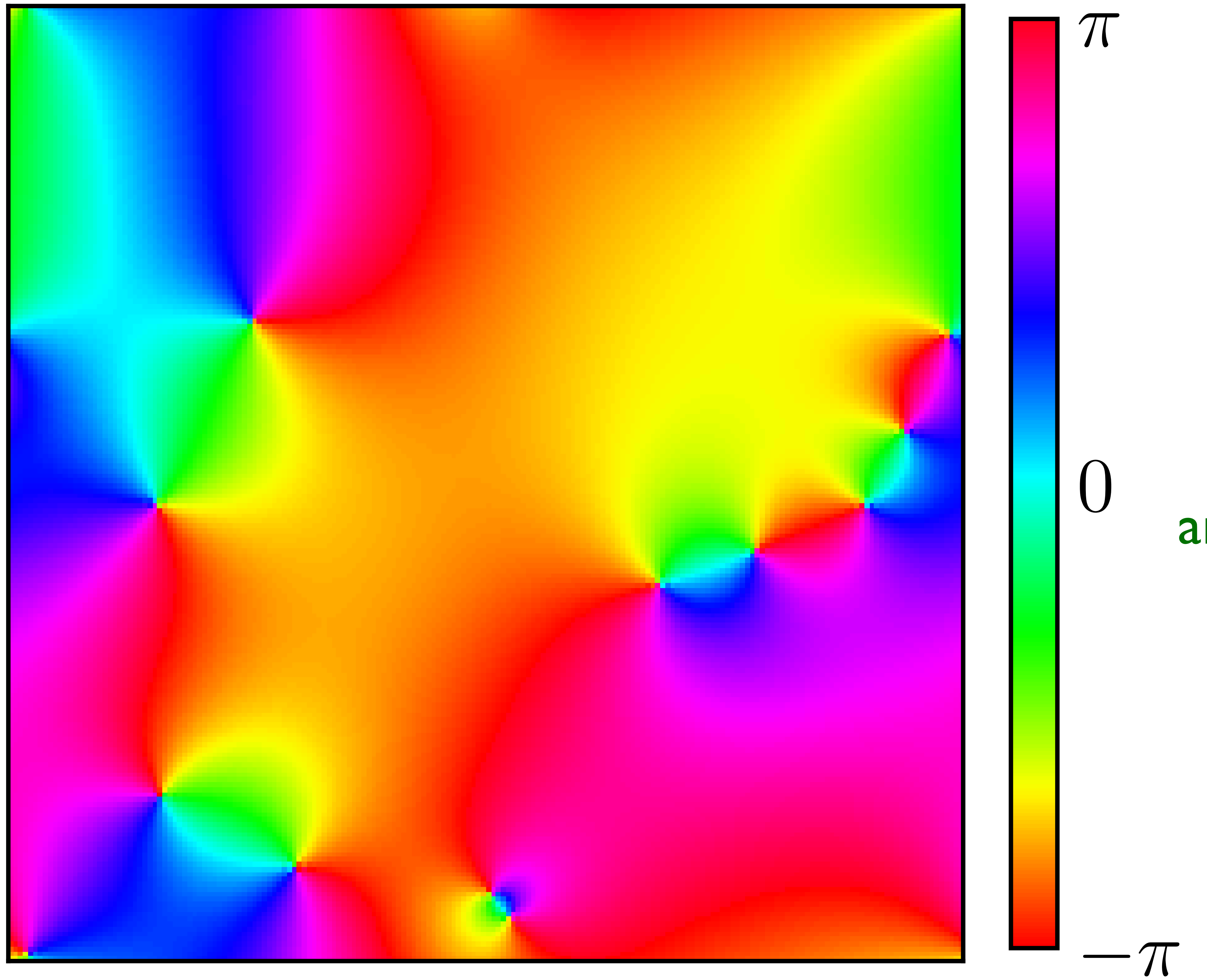
H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, A. Nikolaenko, S. Sharma, S.S., arXiv:2507.05336

See also Jia-Xin Zhang and S. S., PRB **110**, 235120 (2024)

- Classical thermal ensemble of charge e Higgs boson B and SU(2) gauge field U.



Bond density



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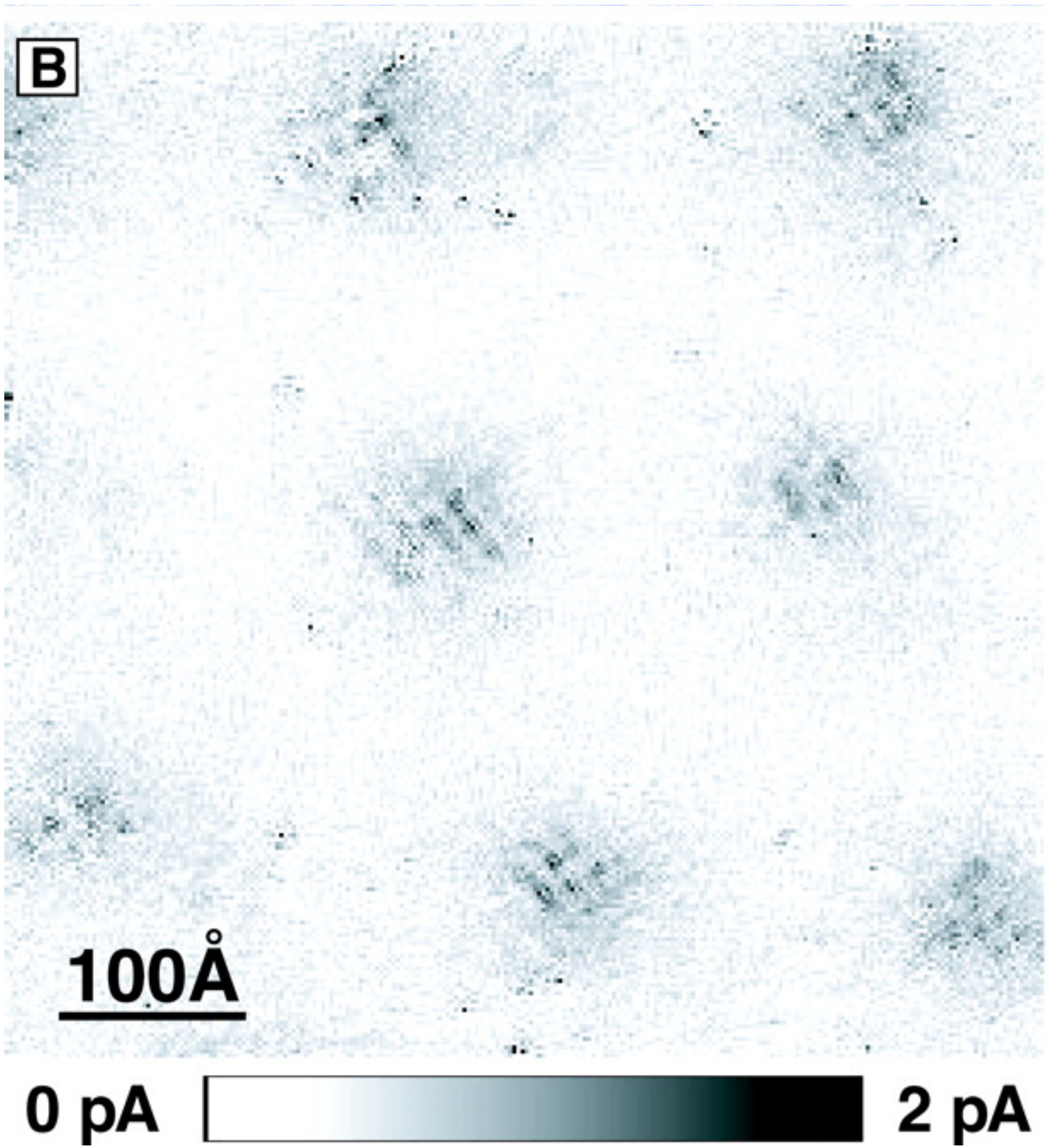
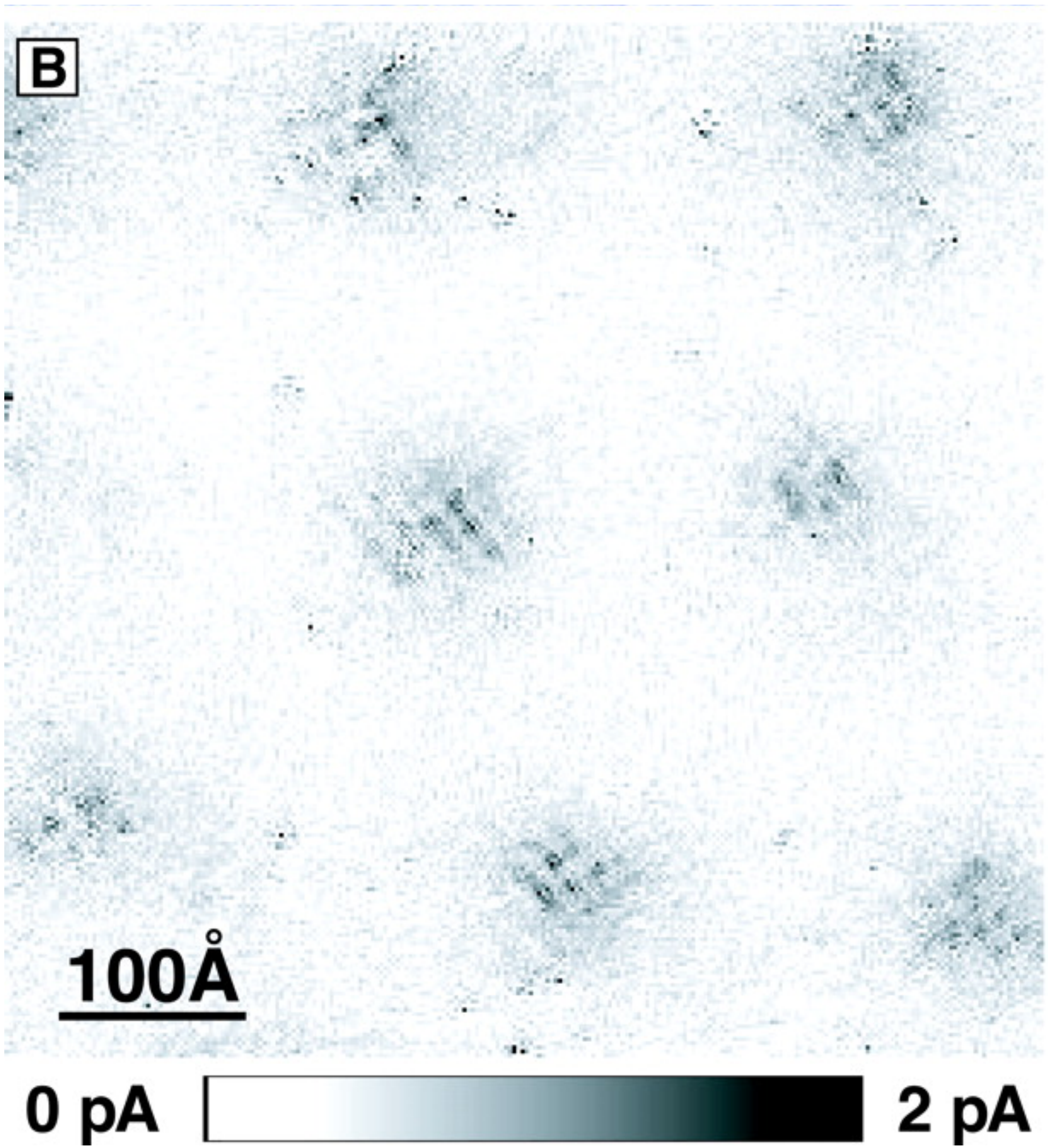
See also Jia-Xin Zhang and S. S., PRB **110**, 235120 (2024)



A Four Unit Cell Periodic Pattern of Quasi-Particle States Surrounding Vortex Cores in $Bi_2Sr_2CaCu_2O_{8+\delta}$

J. E. Hoffman, E. W. Hudson, K. M. Lang,
V. Madhavan, H. Eisaki, S. Uchida, J.C. Davis
Science **295**, 466 (2002)



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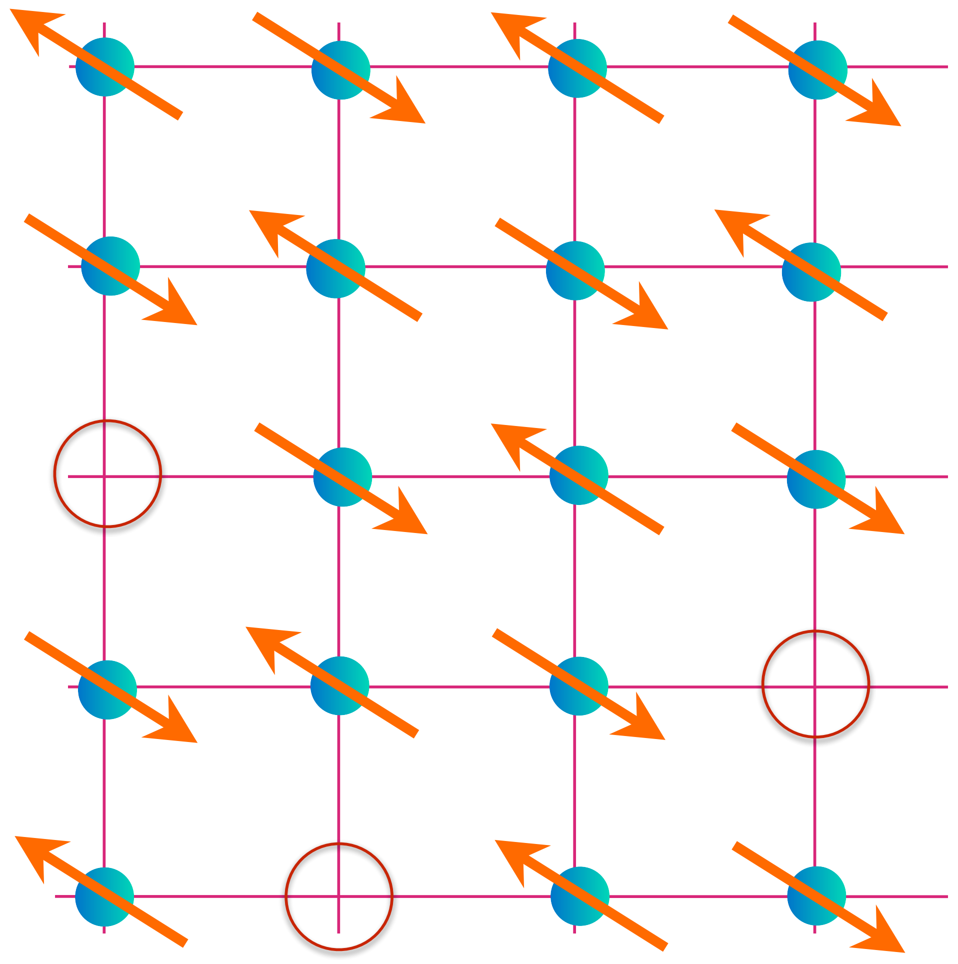
Science **295**, 466 (2002)

The underlying FL*-FL transition in the normal state, is visible in the distinct vortex core structures of the superconducting state at small p and large p

At large p, STM shows the Wang-MacDonald peak of BCS theory.

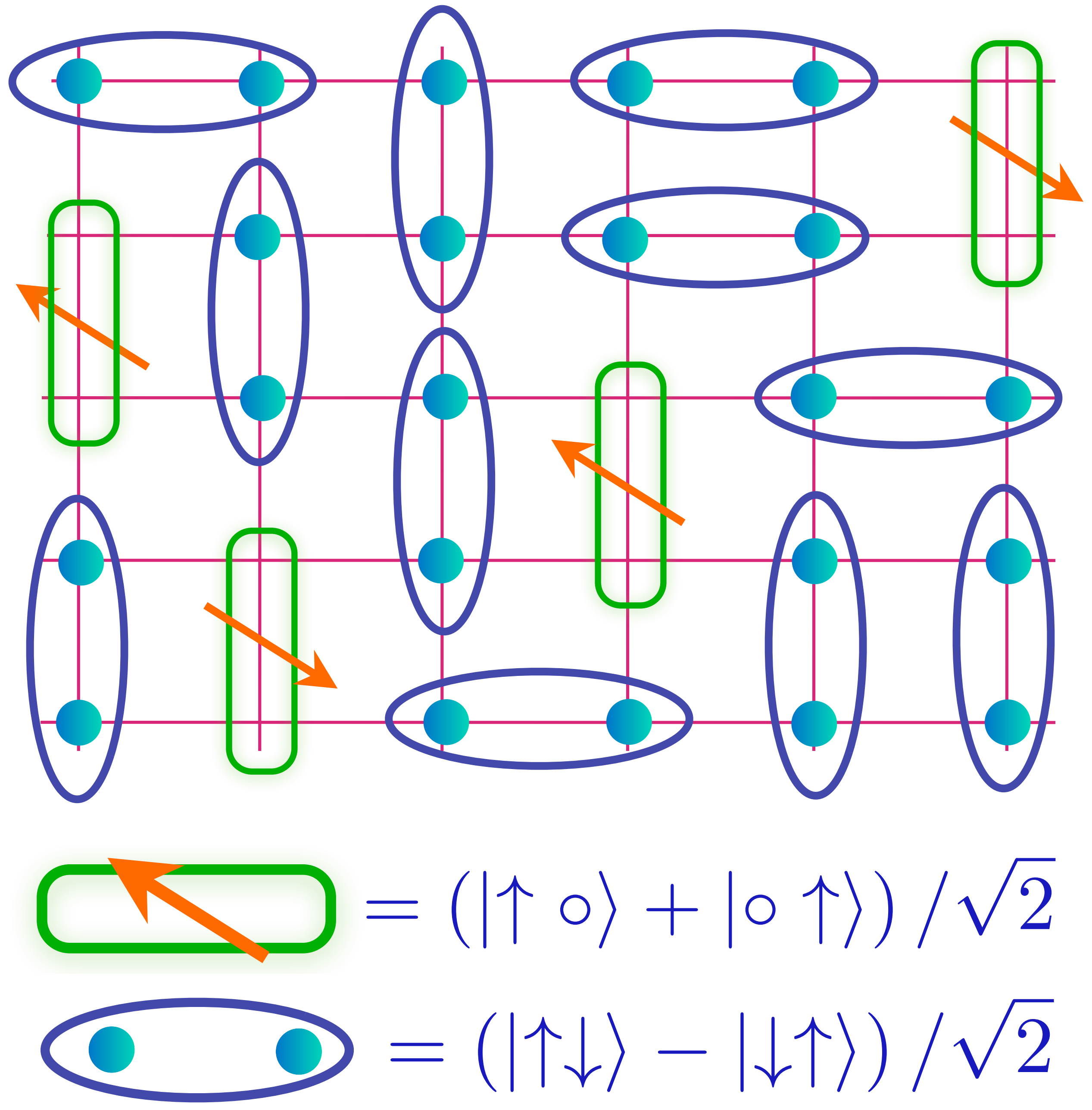
T. Gazdić, I. Maggio-Aprile, G. Gu and C. Renner, PRX **11**, 031040 (2021)

AF Metal



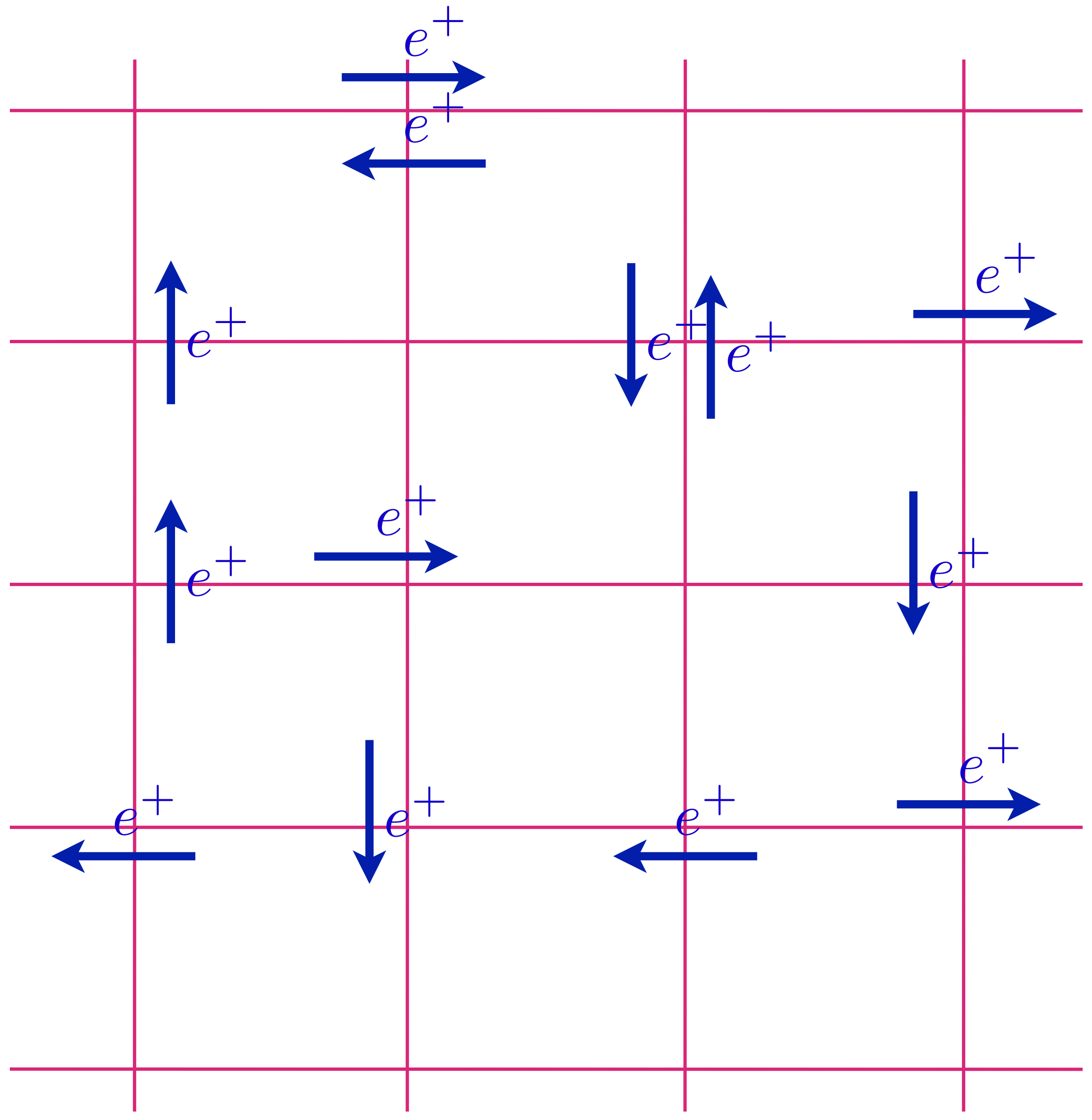
Carrier density p
Pocket area $p/4$

FL*



Carrier density p
Pocket area $p/8$

FL



Carrier density $1 + p$
Fermi area $(1 + p)/2$

p_{sdw} $\quad$ p_c $\quad$ p

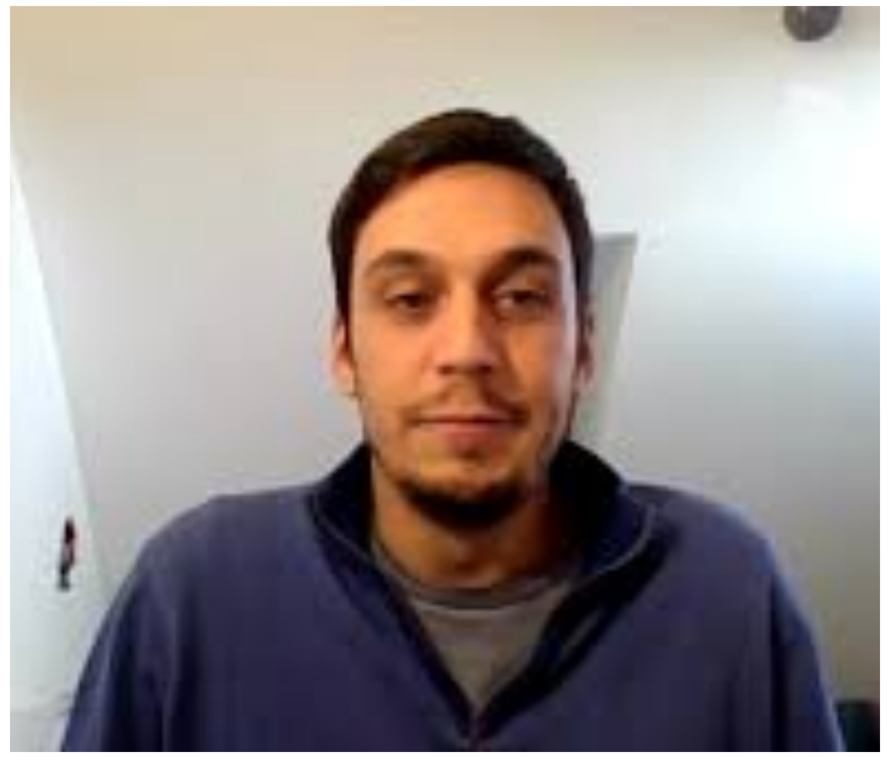
Quantum phase transition between two metals (FL* and FL) at $p = p_c$, with no symmetry breaking.

Forms the basis for understanding the complex phase diagram of the cuprates

S. Sachdev, M.A. Metlitski and M. Punk, Journal of Physics Condensed Matter **24**, 294205 (2012)

- Fractionalized Fermi liquids (FL*)
  - Observation of the Yamaji effect in the cuprate **pseudogap**
  - Wavefunction for FL* and ultracold atom observations
- Sachdev-Ye-Kitaev (SYK) liquids
  - The Yukawa-SYK model
  - From FL* to FL via the **strange metal** using the 2D-YSYK model
- From FL and FL* to the d-wave superconductor

Maine Christos
Caltech



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Chennai

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Ravi Shanker

Sayantan Sharma

- *Lectures on insulating and conducting quantum spin liquids*, S. Sachdev, arXiv:2512.23962
- *Fractionalized Fermi liquids and the cuprate phase diagram*, P. M. Bonetti, M. Christos, A. Nikolaenko, A.A. Patel, and S. Sachdev, arXiv:2508.20164
- *Thermal SU(2) lattice gauge theory of the cuprate pseudogap: reconciling Fermi arcs and hole pockets*, H. Pandey, M. Christos, P. M. Bonetti, R. Shanker, A. Nikolaenko, S. Sharma, and S. Sachdev, arXiv:2507.05336