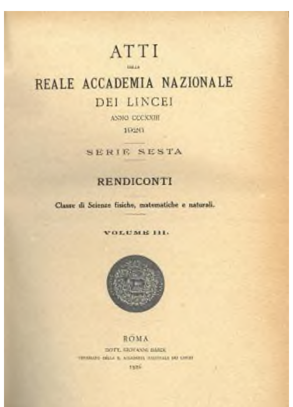
Center for Electronic Correlations and Magnetism
University of Augsburg

Superfluid helium-3:

A fermionic test system for universal concepts in physics

Dieter Vollhardt



ATTI
DELLA
REALE ACCADEMIA NAZIONALE
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1926
SERIE SESTA
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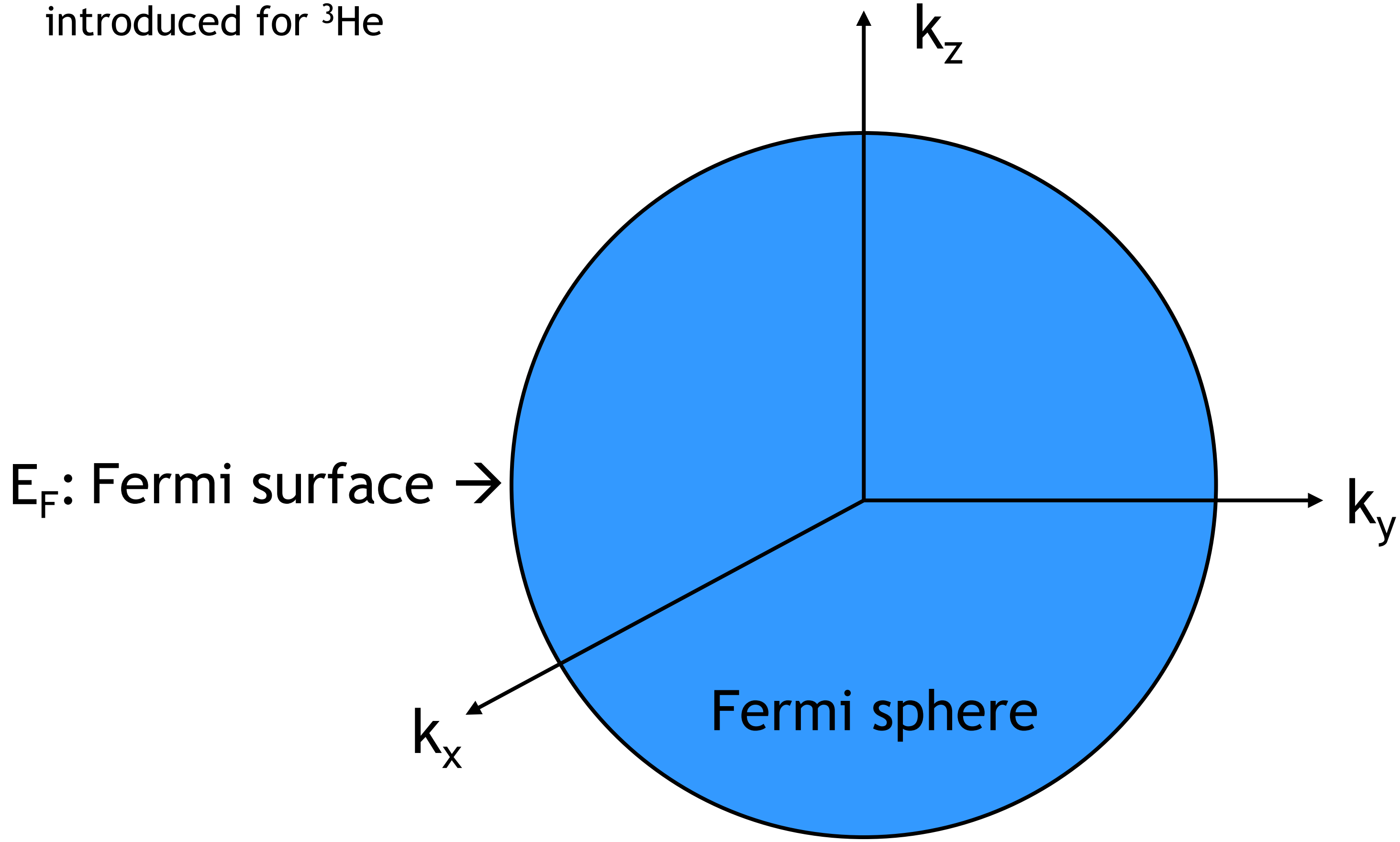
Classe di Scienze fisiche, matematiche e naturali.
VOLUME III.
ROMA

Fisica. — *Sulla quantizzazione del gas perfetto monoatomico.* Nota di Enrico Fermi, presentata dal Socio Garbasso.

Interacting Fermions: Fermi liquid

Landau (1956/57)

“Standard model of
condensed matter physics“
introduced for ^3He

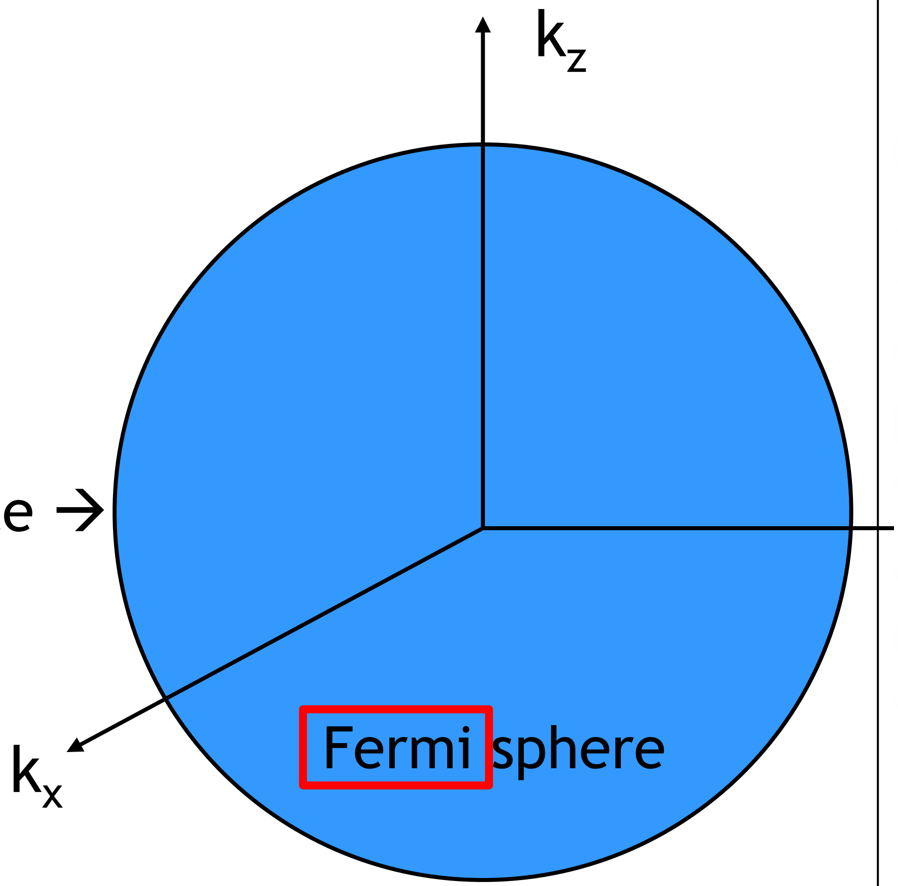


Interacting Fermions: Fermi liquid

Landau (1956/57)

“Standard model of condensed matter physics“ introduced for ^{3}He

E_F: Fermi surface →



Fermi: most frequently occurring name in condensed matter physics

- Fermi–Dirac statistics
- Fermion
- Fermi gas
- Fermi liquid
- Fermi energy E_F
- Fermi level
- Fermi temperature T_F
- Fermi momentum p_F
- Fermi velocity v_F
- Fermi wave vector k_F
- Fermi surface
- Fermi sphere
- Fermi sea
- Fermi's golden rule

...

Interacting Fermions: Fermi liquid

Landau (1956/57)

“Standard model of
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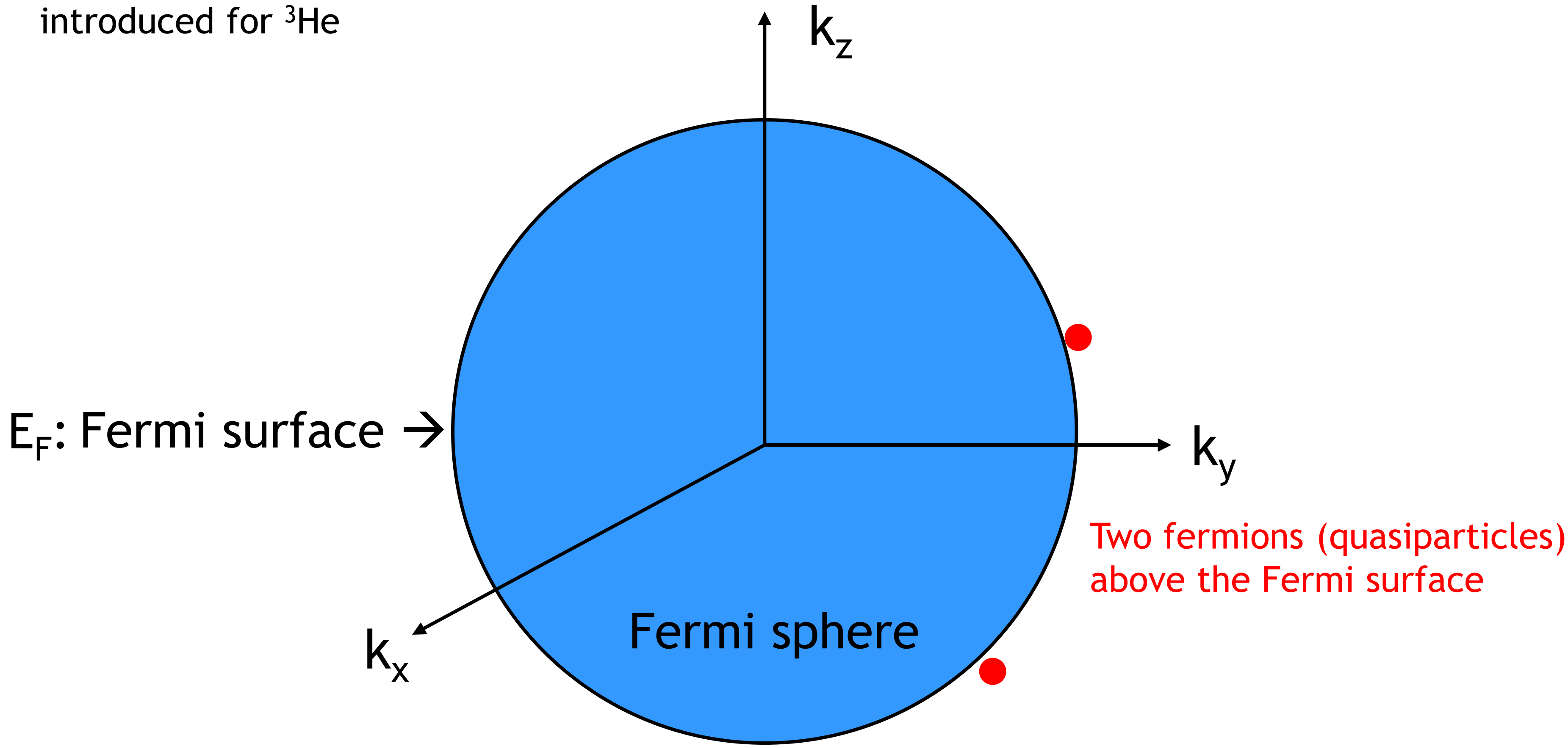
introduced for ^{3}He

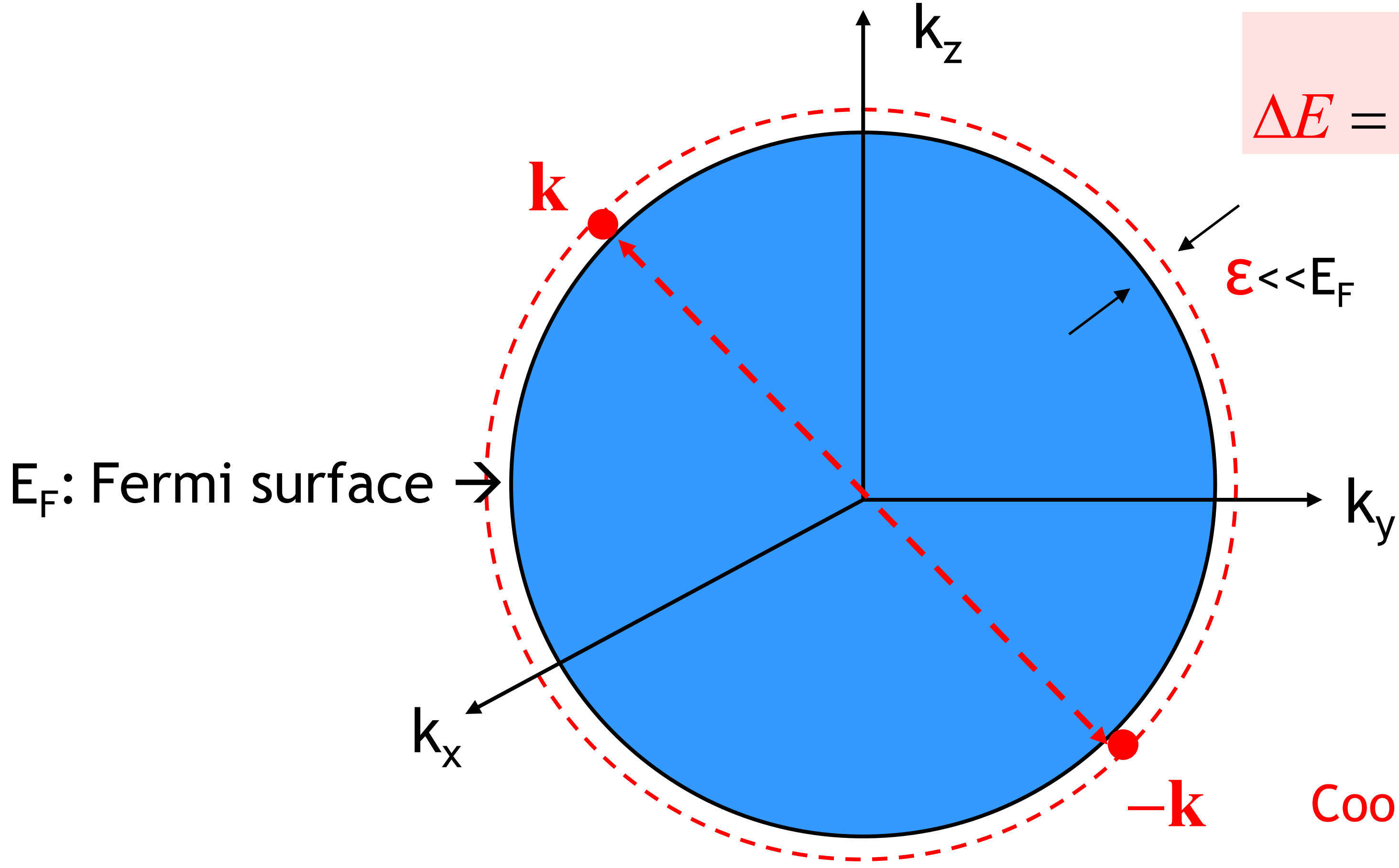


Interacting Fermions: Fermi liquid

Arbitrarily weak attraction $\Rightarrow$ instability

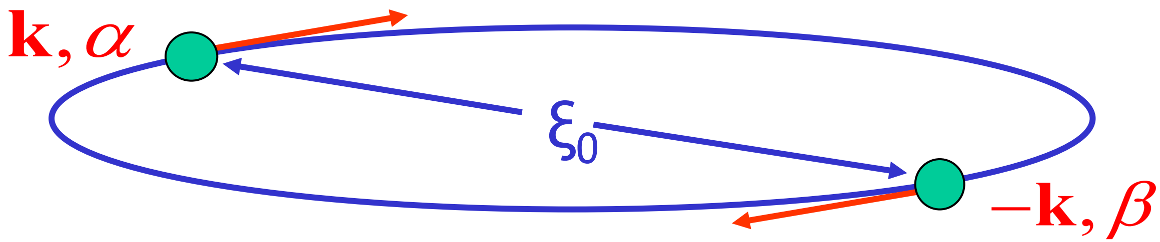
Cooper (1956)

$$\Delta E = -2\,\varepsilon\, e^{-\frac{2}{N(0)|V|}}$$



Universal property of Fermi systems

Cooper pair $(\mathbf{k},\alpha;-\mathbf{k},\beta)$



$$\boxed{\Psi_{L=0,2,4,\ldots} = \psi(\mathbf{r})\left|\uparrow\downarrow - \downarrow\uparrow\right\rangle}$$

S=0 (singlet)

$$\boxed{\begin{aligned}\Psi_{L=1,3,5,\ldots} &= \psi_+(\mathbf{r})\left|\uparrow\uparrow\right\rangle \\ &+\psi_0(\mathbf{r})\left|\uparrow\downarrow + \downarrow\uparrow\right\rangle \\ &+\psi_-(\mathbf{r})\left|\downarrow\downarrow\right\rangle\end{aligned}}$$

S=1 (triplet)

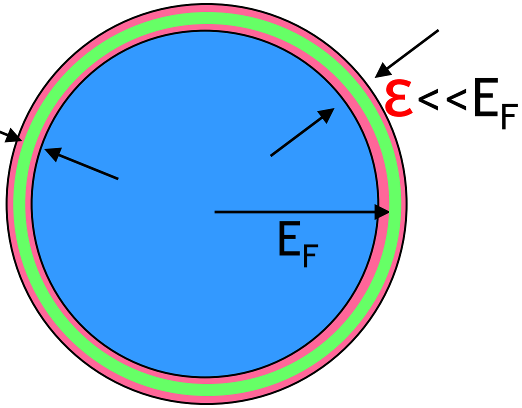
^{3}He: Strong short-range repulsion → L > 0 expected   Pitaevskii (1959)

BCS theory

Bardeen, Cooper, Schrieffer (1957)

Pair condensate

Energy gap Δ(T)
here: L=0 (s-wave)



transition temperature
(weak coupling theory)

$$T_c = 1.13\,\varepsilon\,\mathrm{e}^{-\frac{1}{N(0)|V_L|}}$$

^{3}He: ε, V_L: Origin? Magnitude? → T_c ?

Long experimental search

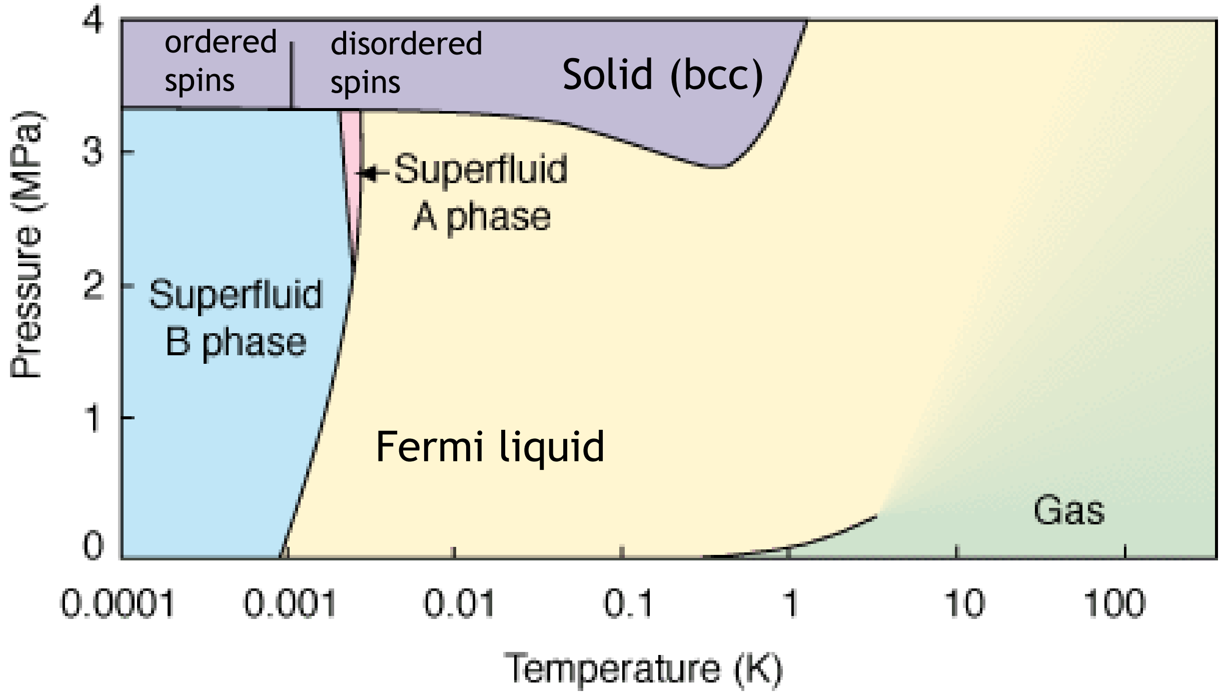
Thanksgiving 1971: Transition in ^{3}He discovered at T_c = 0.0026 K

Osheroff, Richardson, Lee (1972)
Osheroff, Gully, Richardson, Lee (1972)

Nobel Prize in Physics 1996

The superfluid phases of ^3He

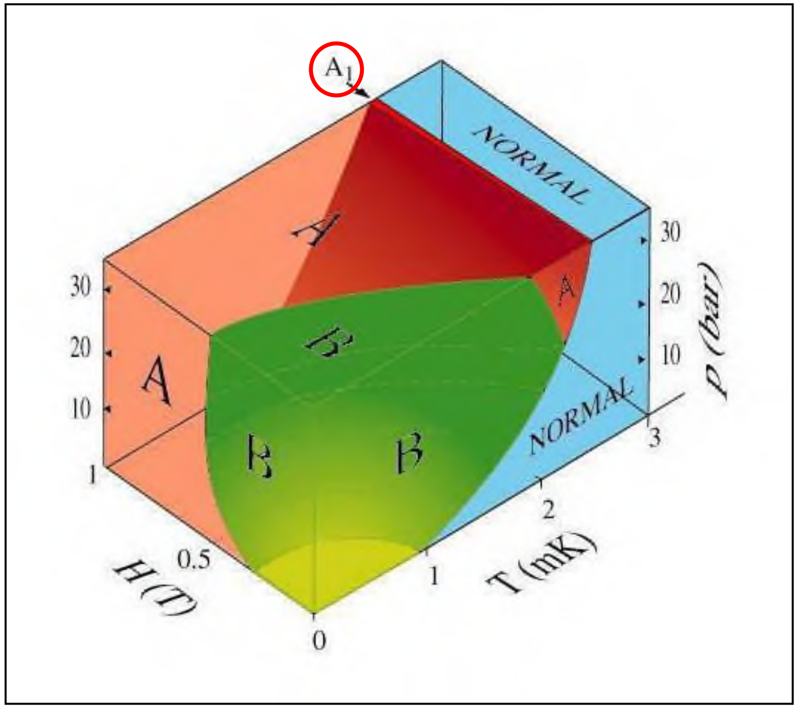
Phase diagram of helium-3



http://ltl.tkk.fi/research/theory/helium.html

Dense, simple liquid
- isotropic
- extremely pure
- nuclear spin $S=\frac{1}{2}\hbar$ → Fermi-Dirac statistics

Phase diagram of helium-3 in a magnetic field



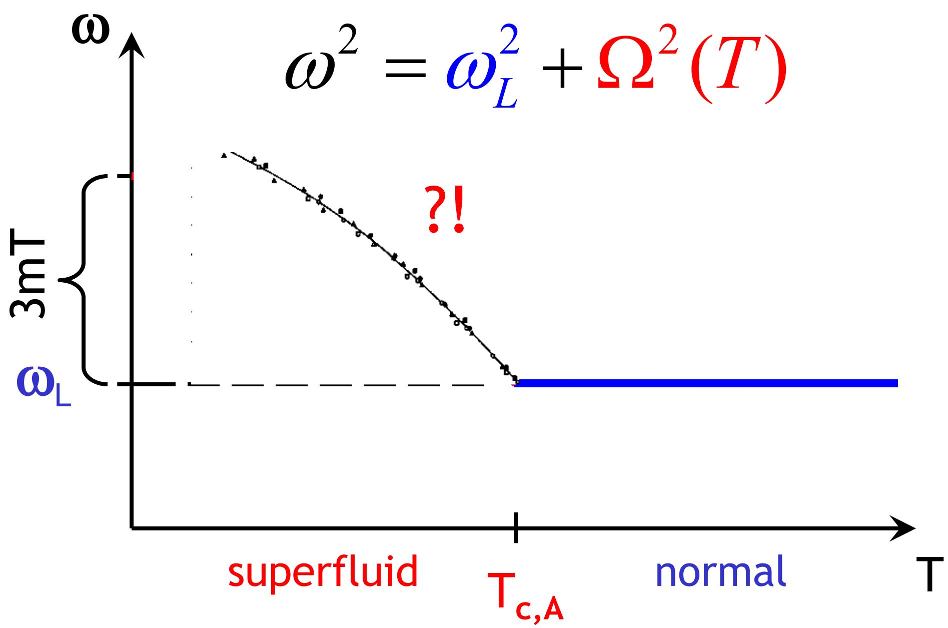
http://ltl.tkk.fi/images/archive/ab.jpg

Dense, simple liquid
- isotropic
- extremely pure
- nuclear spin S= $\frac{1}{2}\hbar$ → Fermi-Dirac statistics

… and a mystery!

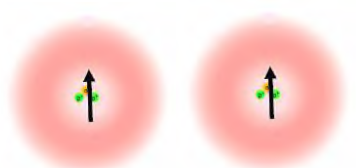
NMR experiment on nuclear spins $I = \frac{1}{2}\hbar$ Osheroff, Gully, Richardson, Lee (1972)

$$\omega^2 = \omega_L^2 + \Omega^2(T)$$



← Larmor frequency: $\omega_L = \gamma H$

Shift of ω_L ⟺ spin-nonconserving interactions

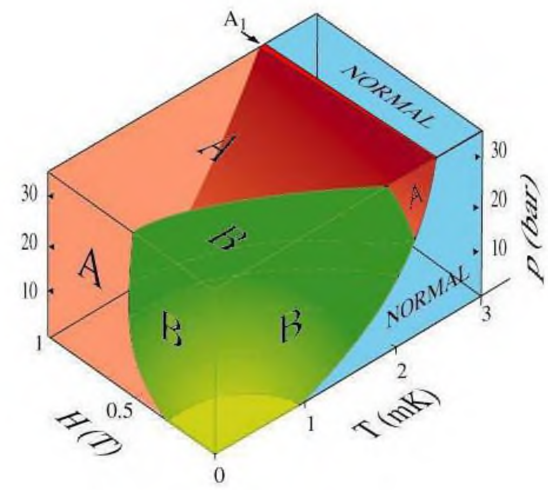


→ nuclear dipole interaction $g_D \sim 10^{-7} K \ll T_c$

Origin of frequency shift ?!

Leggett (1973)

Superfluid ^{3}He in the early 1970s



Experiment: Osheroff, Richardson, Lee, Wheatley, ...

Theory: Leggett, Anderson, Brinkman, Wölfle, Mermin, Varma, …

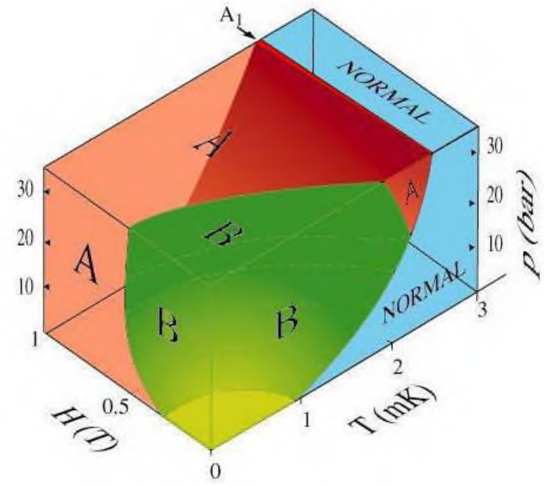
→ L=1, S=1 (p-wave, spin-triplet) Cooper pairs in all 3 phases

^{3}He: Attraction due to strong interaction-induced spin fluctuations

Anderson, Brinkman (1973)

B-phase

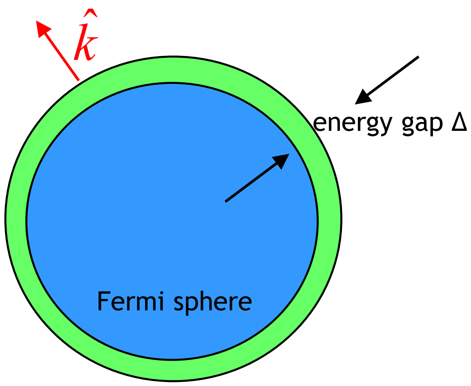
All spin states $|\uparrow\uparrow\rangle,\ |\uparrow\downarrow+\downarrow\uparrow\rangle,\ |\downarrow\downarrow\rangle$ occur equally



$$\boxed{\Delta(\hat{k}) = \Delta}$$

Balian, Werthamer (1963)
Vdovin (1963)

(pseudo-) isotropic state

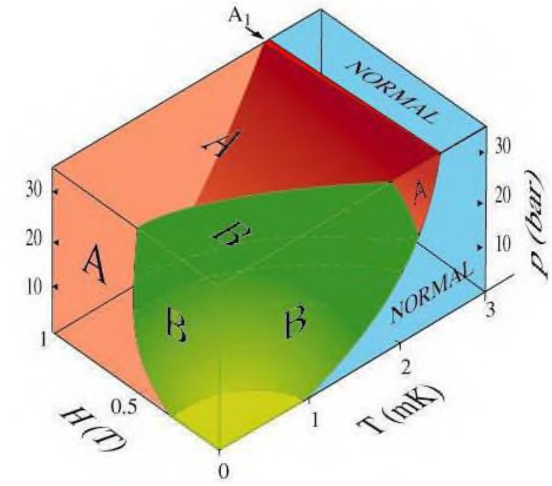


→ specific heat $c_V(T) \propto e^{-\Delta/T}$, $T \to 0$

BCS weak-coupling theory: only the B-phase is stable below T_c

A-phase

Spin states $|\uparrow\uparrow\rangle, \ |\downarrow\downarrow\rangle$ occur equally



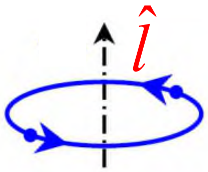
→ strong gap anisotropy

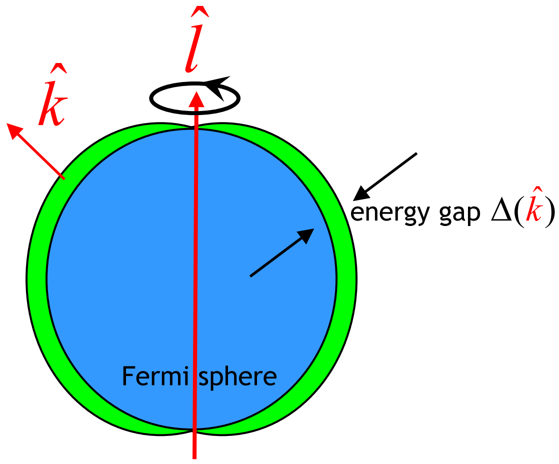
$$\boxed{\Delta(\hat{k}) = \Delta \sin(\hat{k}, \hat{l})}$$

Anderson, Morel (1961)

Energy gap with point nodes

$\hat{l}$: orbital angular momentum of Cooper pairs





"axial state"

→ specific heat $c_V(T) \propto T^3$

→ helped to understand pairing in

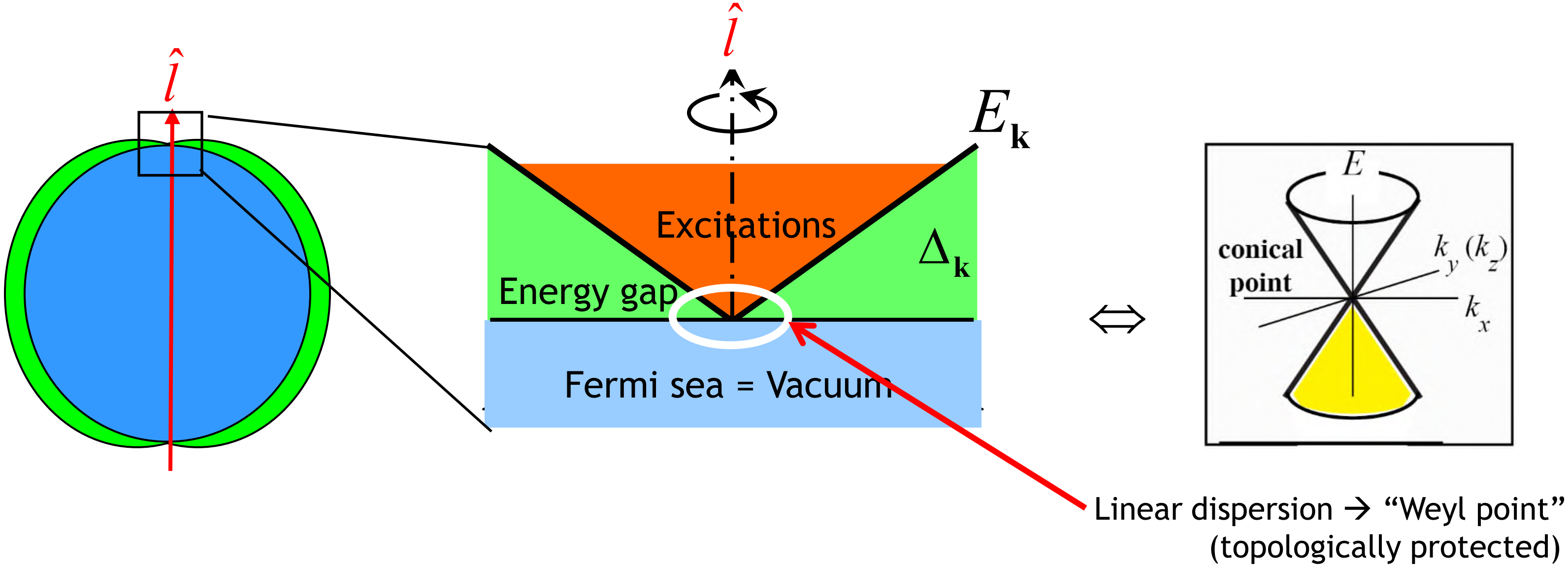
- heavy fermion superconductors ($CeCu_2Si_2$, UPt_3, ...)
- high-T_c (cuprate) superconductors

Stabilized by strong-coupling effects

Anderson, Brinkman (1973)

^{3}He-A: Spectrum near point nodes

Volovik (1987, 2003)



$$E_{\mathbf{k}}^2 = \mathrm{v}_F^2 \left(k - k_F\right)^2 + \Delta_0^2 \sin^2(\hat{k},\hat{l}) = \mathrm{g}^{ij} p_i p_j$$

covariant → Lorentz invariant

$$e = \begin{cases} +1 & \hat{k} \parallel +\hat{l} \quad \text{chirality "up"} \\ -1 & \hat{k} \parallel -\hat{l} \quad \text{chirality "down"} \end{cases}$$

$\boldsymbol{A} = k_F \hat{l}$

$\boldsymbol{p} = \boldsymbol{k} - e\boldsymbol{A}$

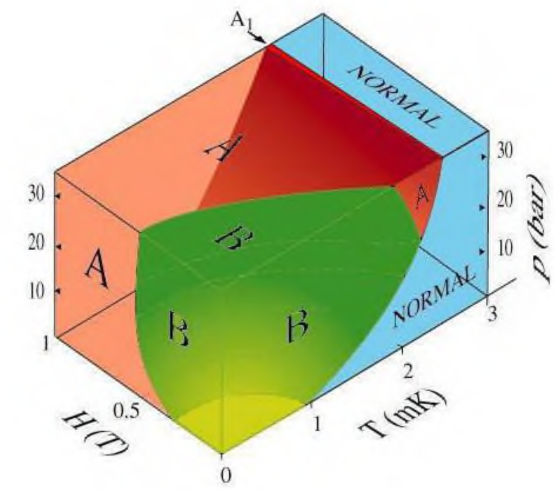
- ^{3}He-A: chiral superfluid
- quasiparticle excitations are Weyl fermions ("massless, chiral leptons" with $E(\mathbf{p}) = cp$)
- at Weyl point: spectral flow of fermionic charge

  → chiral (Adler) anomaly

  observed in ^{3}He-A Bevan *et al.* (1997)

^{3}He-A: example of an emergent quantum field theory

A_1-phase in finite magnetic field

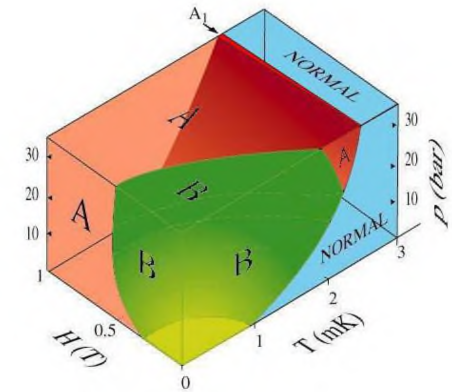


Spin state $|\uparrow\uparrow\rangle$ only

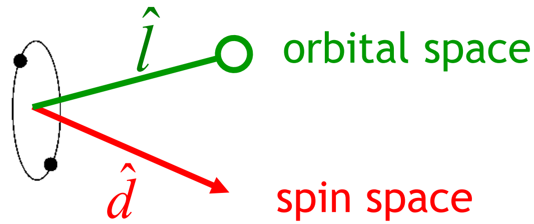
Long-range ordered magnetic ***liquid***

Broken symmetries and long-range order in superfluid ^3He

Cooper pairs with L=1, S=1 in all 3 phases



L=1, S=1: 3 projections each,
describable by 3-dim. vectors $\hat{l}$, $\hat{d}$
("characteristic directions")



Macroscopic quantum coherence + LRO of the	pair amplitude	→ superfluid
	characteristic direction in orbital space	→ liquid crystal
	characteristic direction in spin space	→ magnet

Order characterized by $2\times(2L+1)\times(2S+1)$ = 18 real numbers

Order parameter: complex 3x3 matrix $A_{i\mu}$

$SO(3)_S \times SO(3)_L \times U(1)_\varphi$ symmetry spontaneously broken

Leggett (1975)

→ high-dimensional / topologically rich order-parameter space

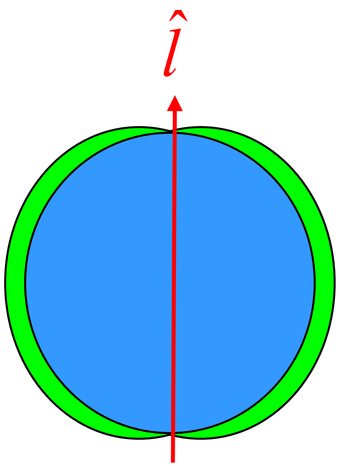
^{3}He-A

$$SO(3)_S \times SO(3)_L \times U(1)_\varphi$$

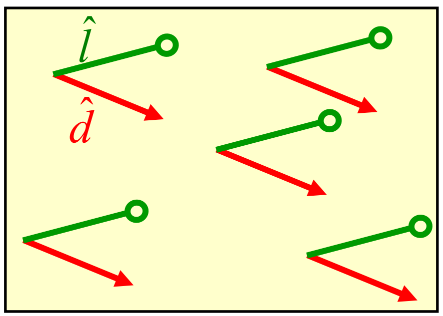
$$U(1)_{S_z} \times U(1)_{L_z-\varphi}$$

„Unconventional" pairing

Mineev (1980)
Bruder, Vollhardt (1986)



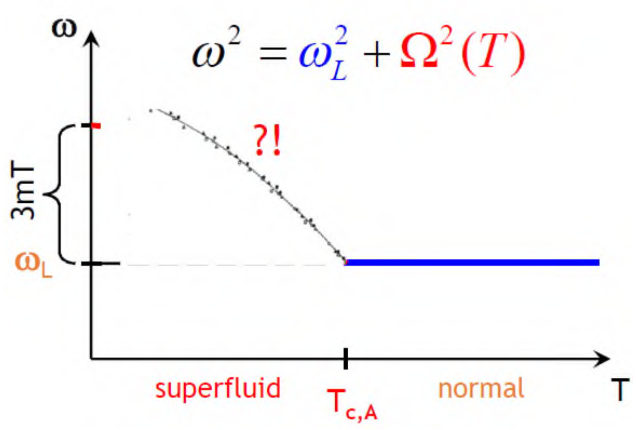
Cooper pairs



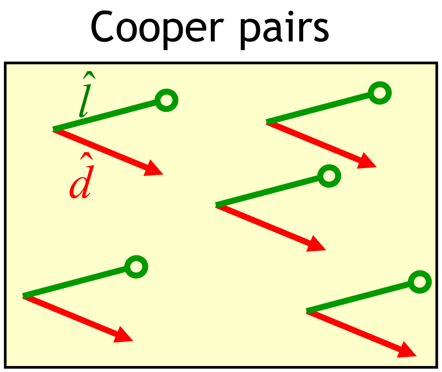
Long-range order of $\hat{d}, \hat{l}$

"Spontaneously broken spin-orbit symmetry" Leggett (1972-1974)

… solves the NMR mystery



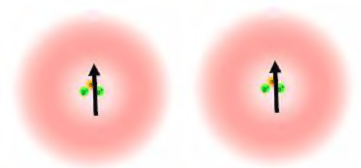
NMR resonance in superfluid ^{3}He-A



- Long-range order of $\hat{d}, \hat{l}$
- What fixes the *relative* orientation of $\hat{d}, \hat{l}$?

requires spin-orbit coupling:

→ Interaction of nuclear dipoles:

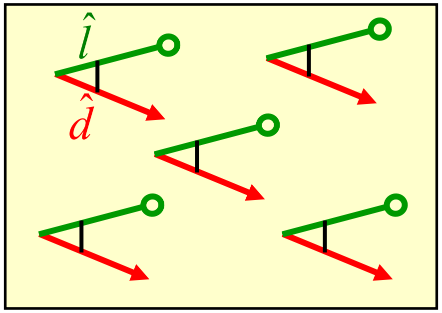


Dipole-dipole coupling of ^{3}He nuclei: $g_D \sim 10^{-7} K \ll T_c$

Unimportant ?!

NMR resonance in superfluid ^{3}He-A

Cooper pairs



- Long-range order of $\hat{d}, \hat{l}$
- What fixes the *relative* orientation of $\hat{d}, \hat{l}$?

requires spin-orbit coupling:

$g_D \sim 10^{-7} K \ll T_c$: tiny, but lifts degeneracy of relative orientation

$\Downarrow$ macroscopic quantum coherence

Fixes relative orientation of $\hat{d}, \hat{l}$ in all Cooper pairs

$\Downarrow$

NMR frequency increases: $\boxed{\omega^2 = (\gamma H)^2 + g_D \Delta^2(T)}$

Leggett (1973)

Nobel Prize in Physics 2003

→ Nuclear dipole interaction is macroscopically measurable

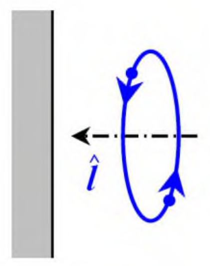
Orientation of the order parameter:

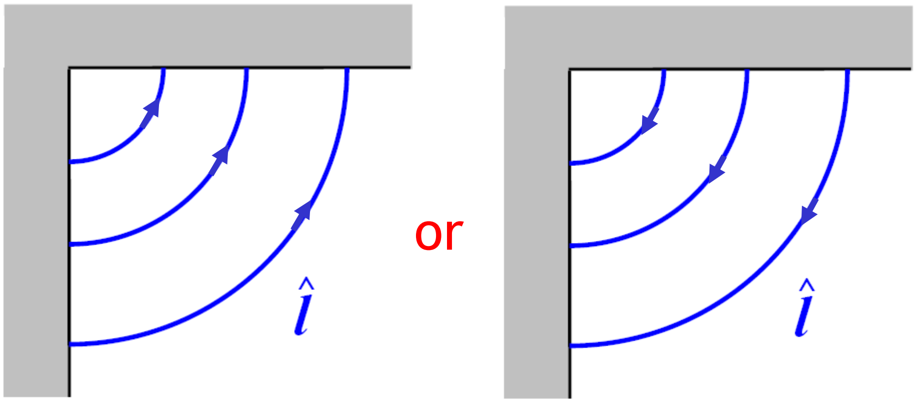
Textures and topological defects

Orientation of the order-parameter in ^{3}He-A

Macroscopic anisotropy directions $\hat{d}, \hat{l}$ are oriented by:

1) Walls → $\hat{l}$





Torsional oscillator exp.: Chirality confirmed
Walmsley, Golov (2012)

2) Magnetic field → $\hat{d}$

→ $\hat{d}, \hat{l}$ configurations generally non-uniform

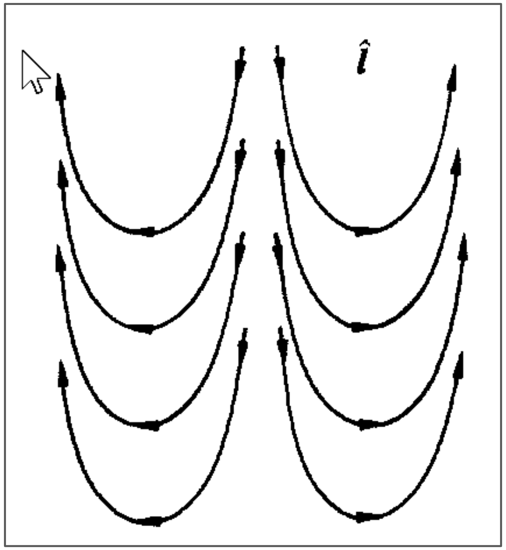
"Textures" in $\hat{d}, \hat{l}$ ↔ liquid crystals

De Gennes (1974)

Orientation of the order-parameter in ^{3}He-A: Topological defects

Generated by rotation

Vortices (d=1)



Vollhardt, Wölfle (1990)

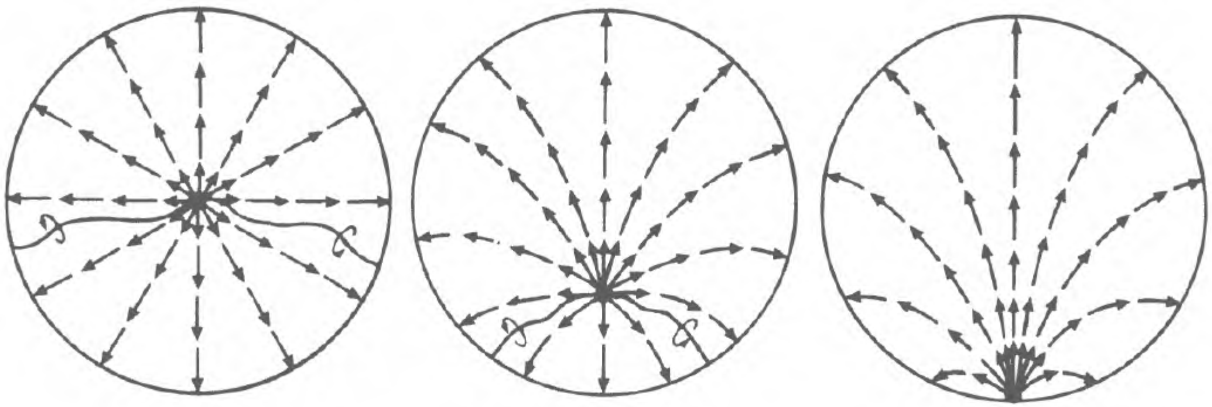
e.g., Anderson-Toulouse vortex
(non-singular, two quanta of circulation)

- At least 6 different types of vortices predicted
- 4 vortices experimentally observed (NMR)

Orientation of the order-parameter in ^{3}He-A: Topological defects

Generated by geometric constraints

Monopoles (d=0)



Vollhardt, Wölfle (1990)

Nanoscale confinement $(\sim \xi_0)$
→ Novel edge states: e.g., gapless (E=0) "Majorana" modes (fermionic quasiparticles)

Defect formation by	• rotation • geometric constraints • rapid crossing through a symmetry breaking phase transition

Universality in symmetry breaking phase transitions

Big Bang

High symmetry,
short-range order

$T>T_c$

Spins: para-magnetic

Helium: normal liquid

Universe: Unified forces and fields

$T=T_c$ Phase transition

Broken symmetry,
long-range order

ferromagnetic

superfluid

elementary particles, fundamental interactions

Defects:

domain walls

vortices, etc.

cosmic strings, etc. Kibble (1976)

$T<T_c$

nucleation of galaxies?

Laboratory simulation of cosmic string formation in the early universe

System at $T < T_c$

Bäuerle *et al.* (1996)

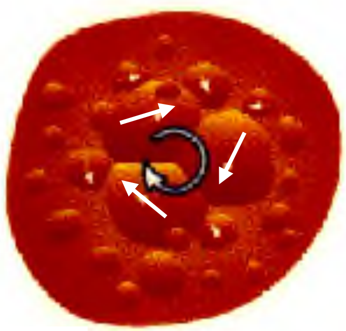
1. Local temperature $T \gg T_c$

   → Expansion of hot bubble, then rapid cooling through T_c

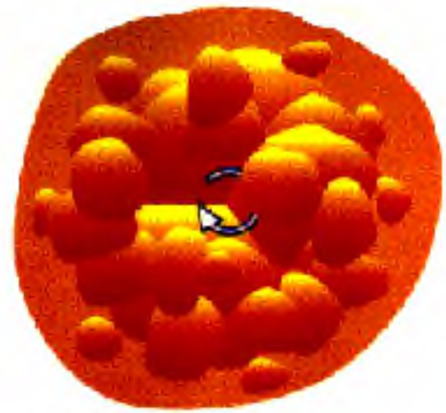
2. Nucleation of independently ordered regions

Clustering of ordered regions

→ Defects form

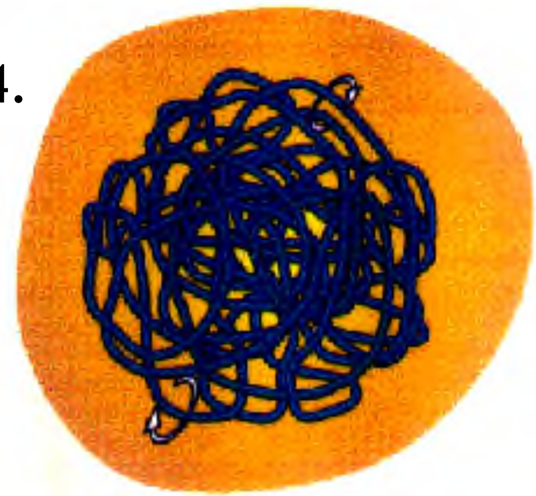


3.



Defects overlap

4.



Vortex tangle

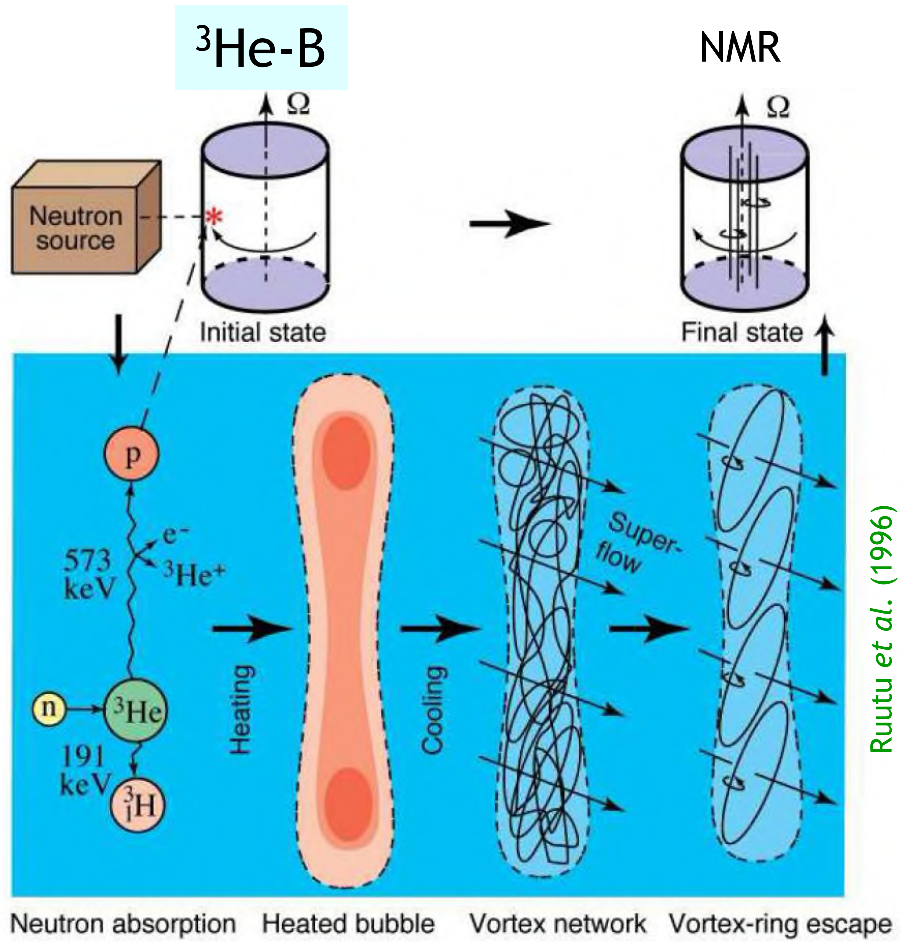
Estimate of density of defects: Zurek (1985)

"Kibble-Zurek mechanism" of defect formation: How to test?

Laboratory simulation of cosmic string formation in the early universe

Grenoble: Bäuerle *et al.* (1996)

Helsinki: Ruutu *et al.* (1996)



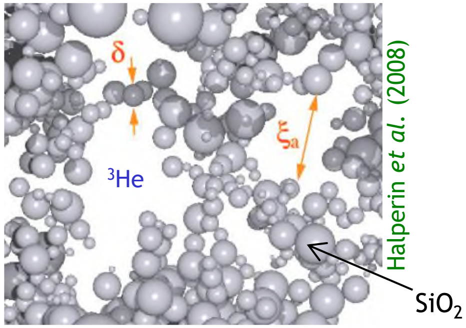
Measured vortex-tangle density:

Quantitative support for Kibble-Zurek mechanism

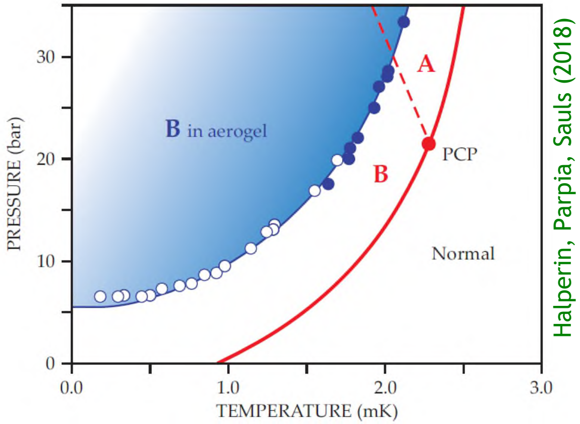
Recent developments

Influence of **disorder** on quasiparticle scattering in superfluid ^3He

^3He in 98% open silica aerogel

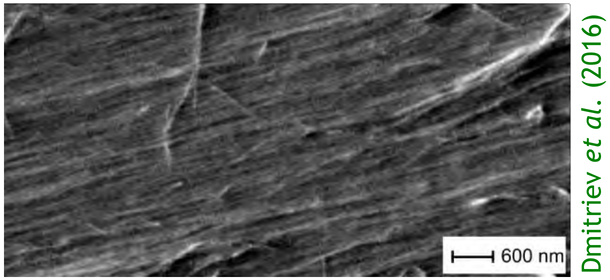


isotropic aerogel

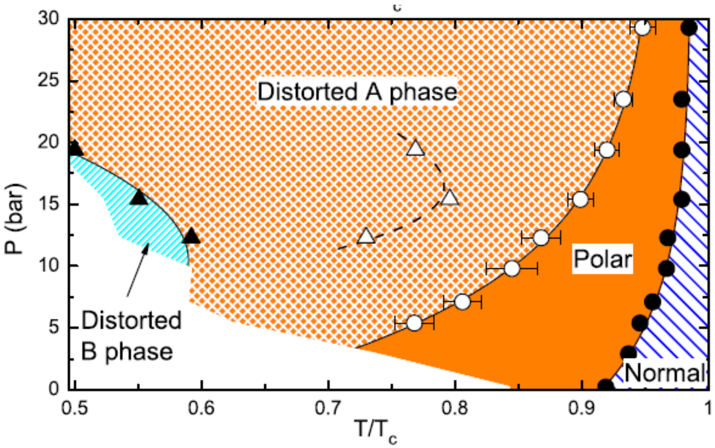


Porto, Parpia (1995)
Sprague *et al.* (1995)

97.8% porosity Al_2O_3 strands ("nafen-90")



anisotropic aerogel

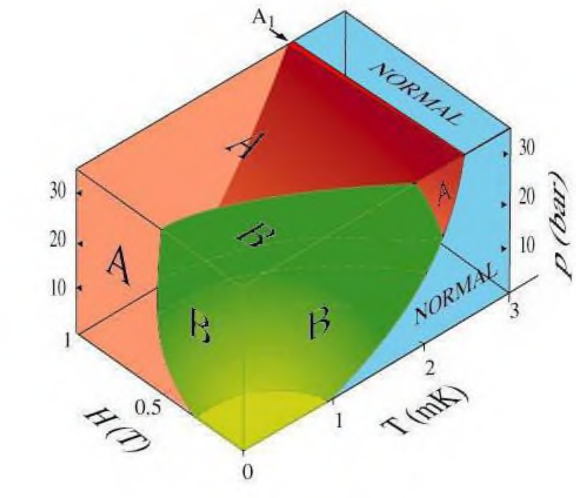


Dmitriev, Senin, Soldatov, Yudin (2015)

polar phase

Polar phase: Cooper pairs align along strands of aerogel

Conclusion



http://ltl.tkk.fi/images/archive/ab.jpg

Superfluid helium-3:

- Anisotropic fermionic superfluid (p-wave, spin-triplet pairing)
  - 4 bulk phases known (A, B, A_1, polar)
- ^{3}He-A examplifies emergence of a quantum field theory
- Large symmetry-group spontaneously broken
  - Zoo of topological defects