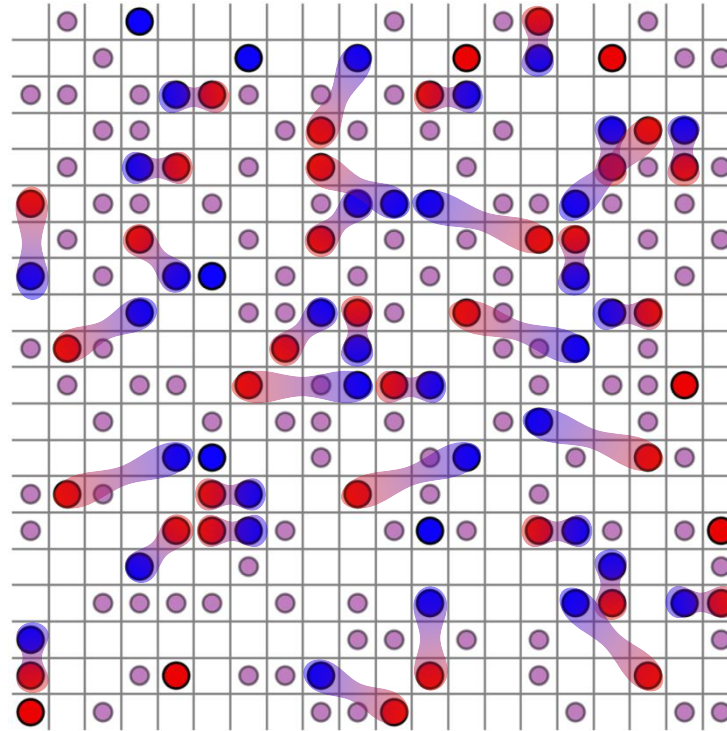


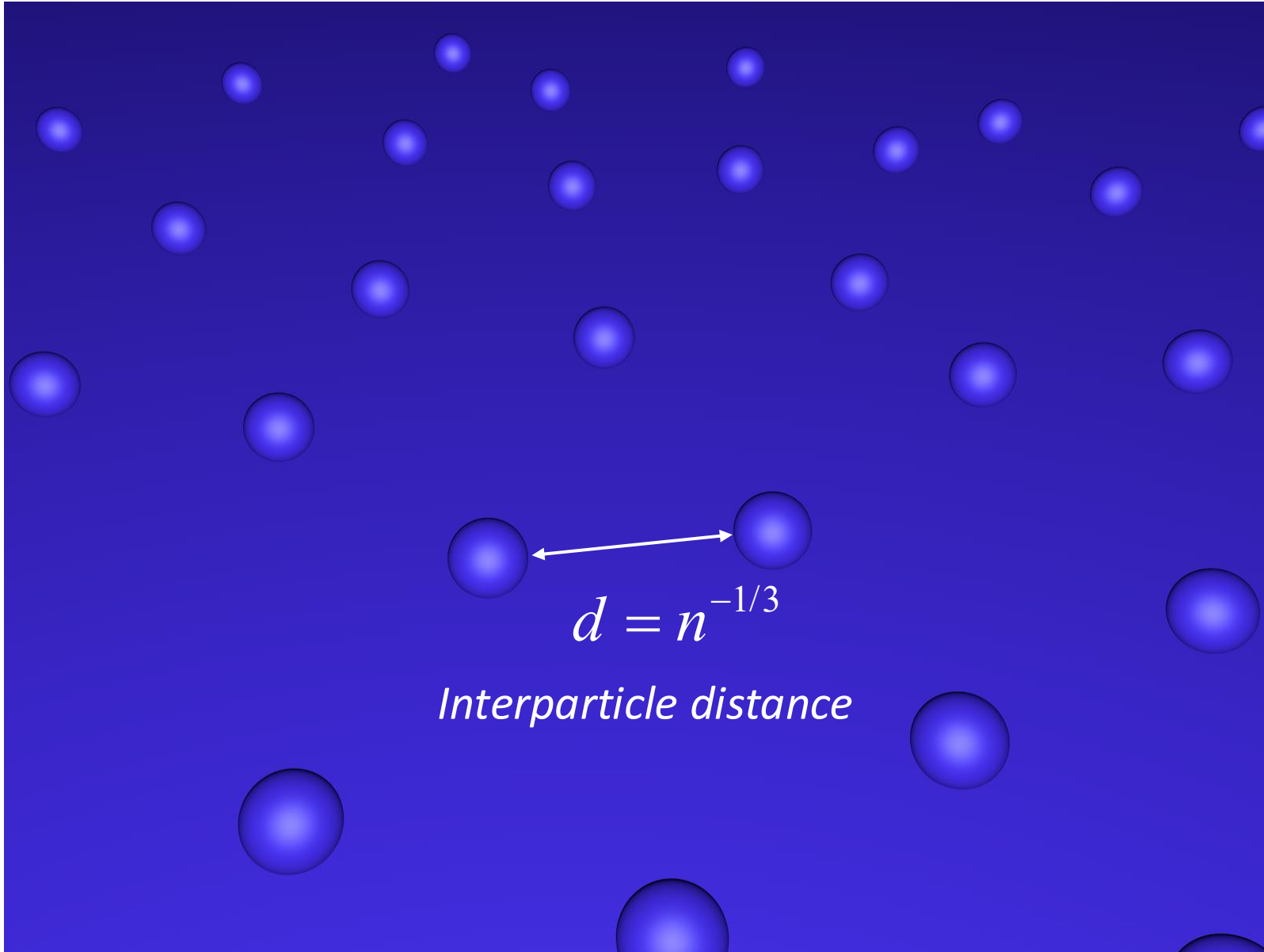
Pairing and Superfluidity in Strongly Interacting Fermi Gases

Martin Zwierlein

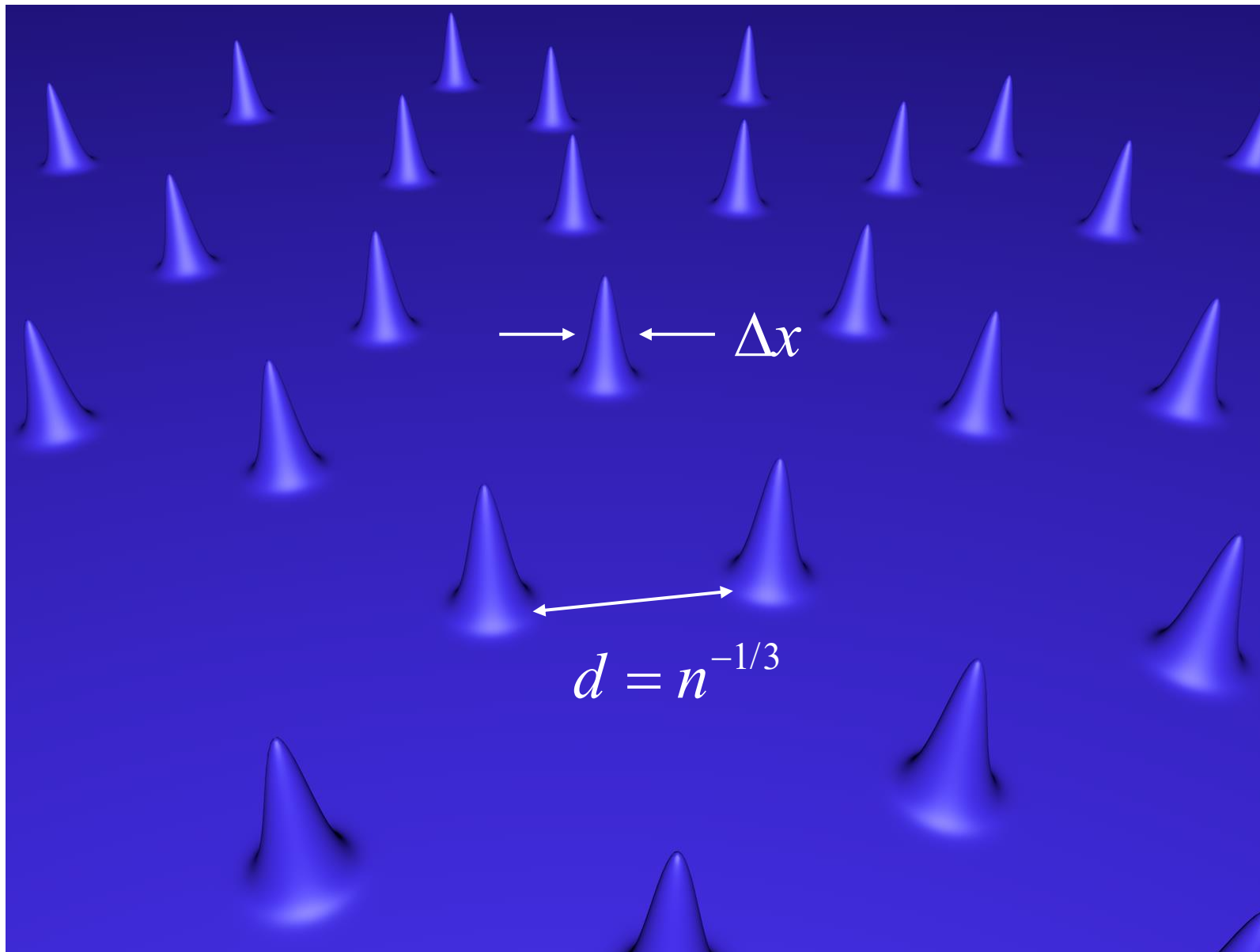
MIT, Center for Ultracold Atoms



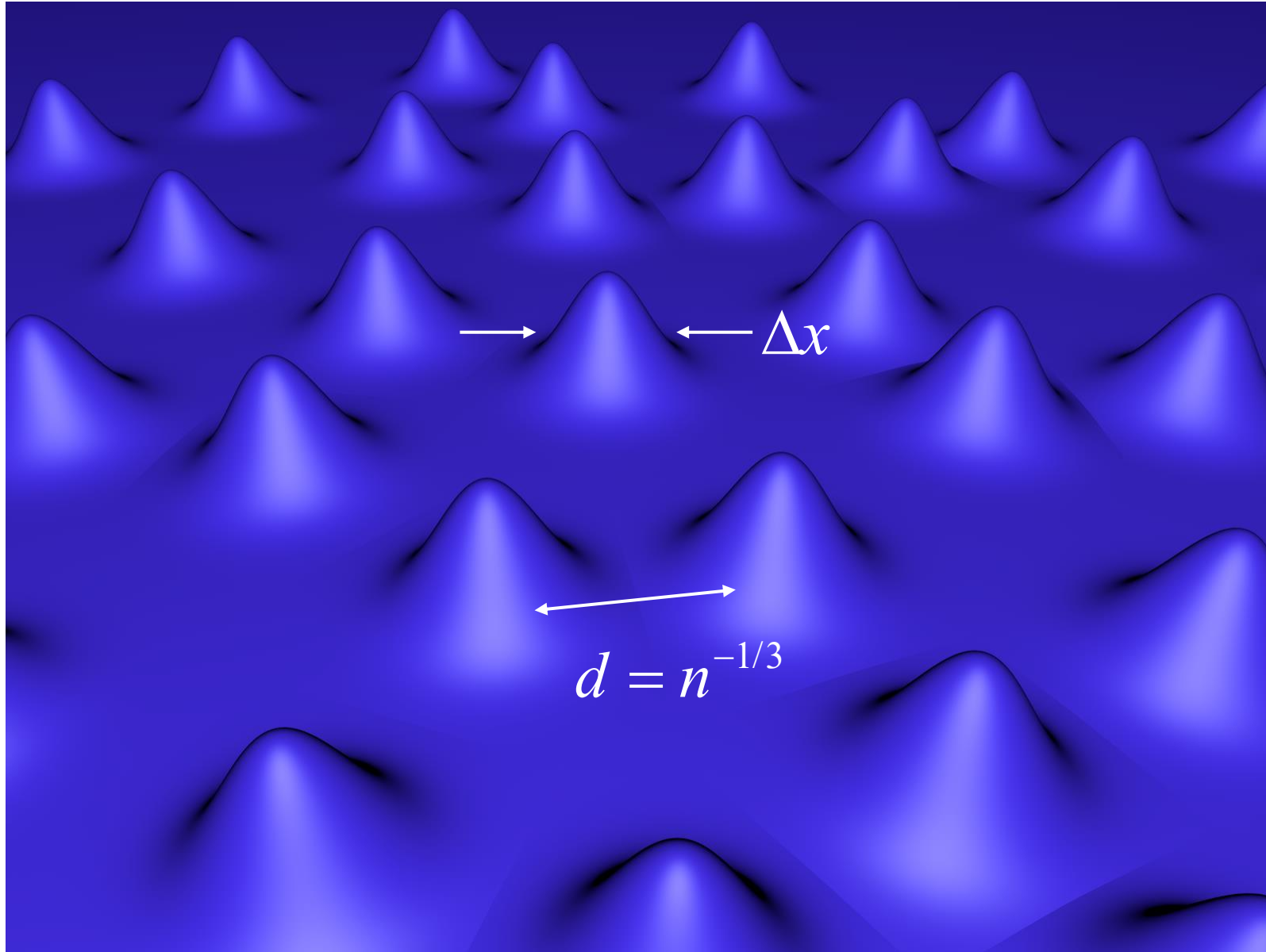
Particles...



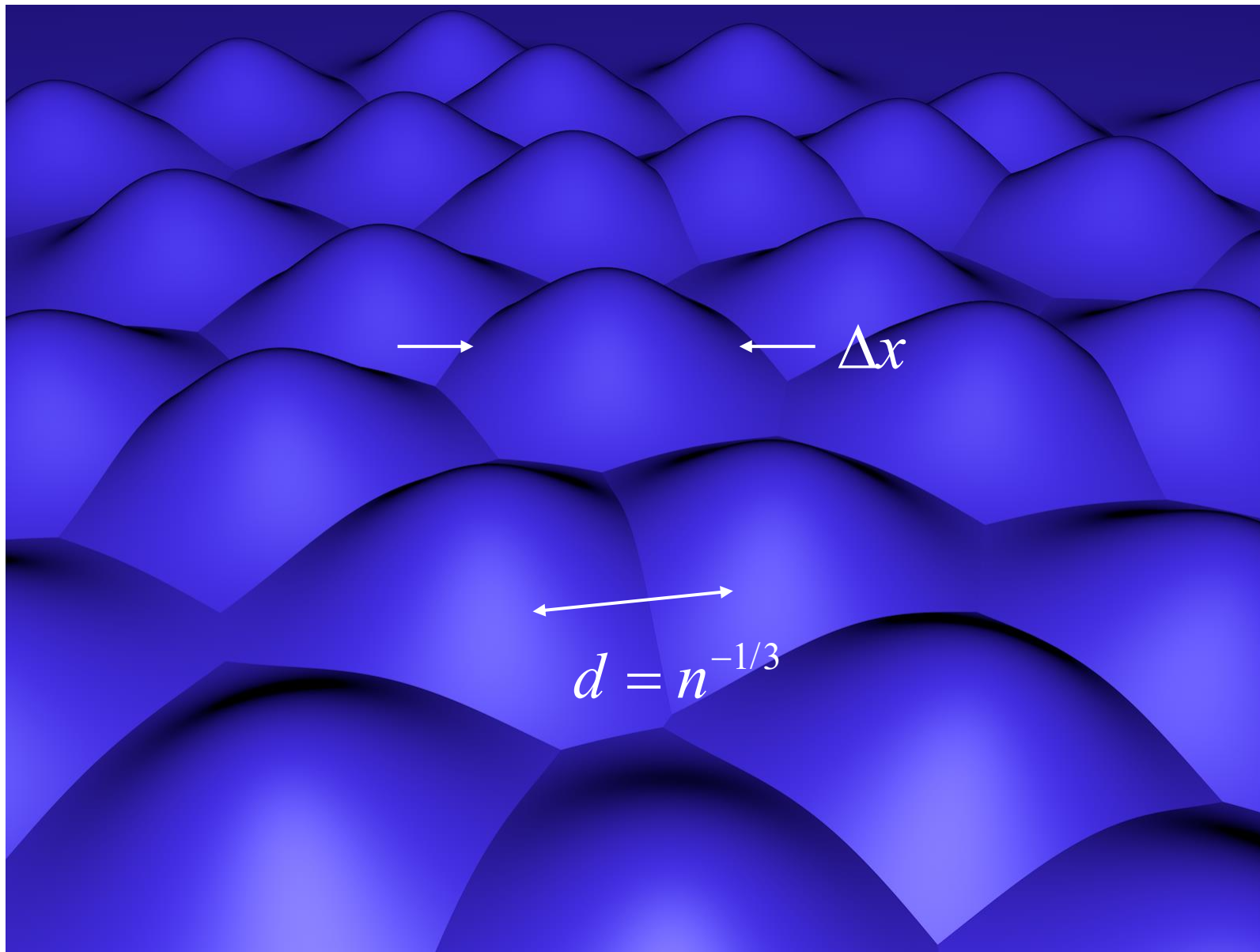
...behave as waves



When does wave mechanics matter?



When does wave mechanics matter?



Fermi's Paper

SULLA QUANTIZZAZIONE DEL GAS PERFETTO MONOATOMICO

« Rend. Lincei », 3, 145–149 (1926) (*).

- Applies Pauli's principle to motional states!
- Obtains the equation of state of the ideal gas under this constraint

$$p = \frac{2}{3} n \bar{\epsilon} = \frac{h^2 n^{5/3}}{2 \pi m} P \left(\frac{2 \pi m k T}{h^2 n^{2/3}} \right)$$

So that is

$$p = \frac{2}{5} n E_F P \left(\frac{T}{T_F} \right)$$

with Fermi Energy

$$E_F = k_B T_F$$



Bosons versus Fermions

Fermions (unsociable):

Half-Integer Spin

Pauli blocking → Form Fermi sea

No phase transition at low Temperature



Fermi Feb. 7, 1926

Dirac Oct. 1926

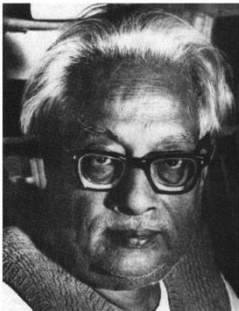
Bosons (sociable):

Integer Spin

Can share quantum states

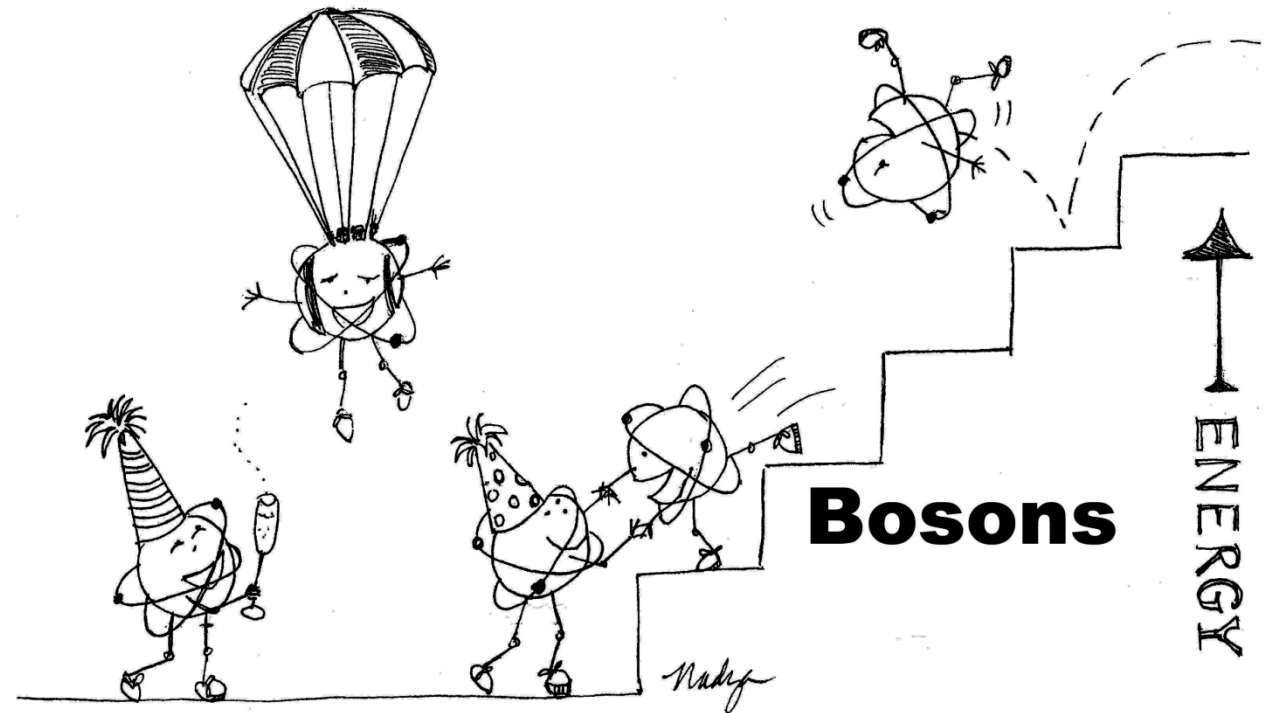
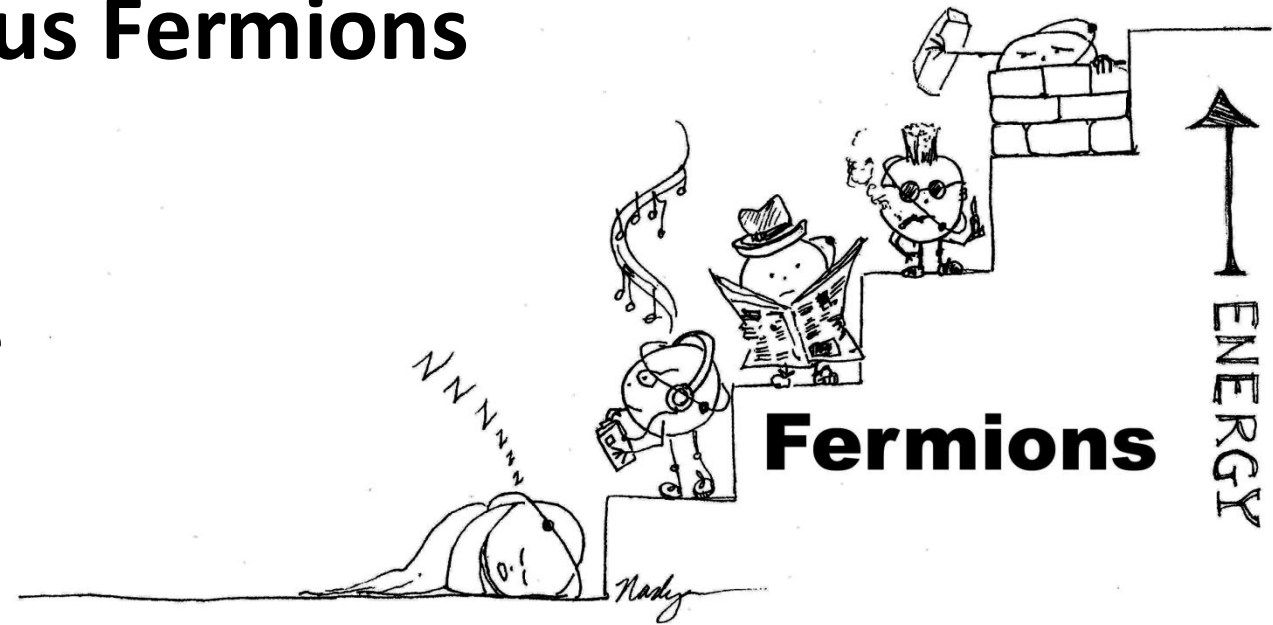
At low temperatures:

Bose-Einstein condensation

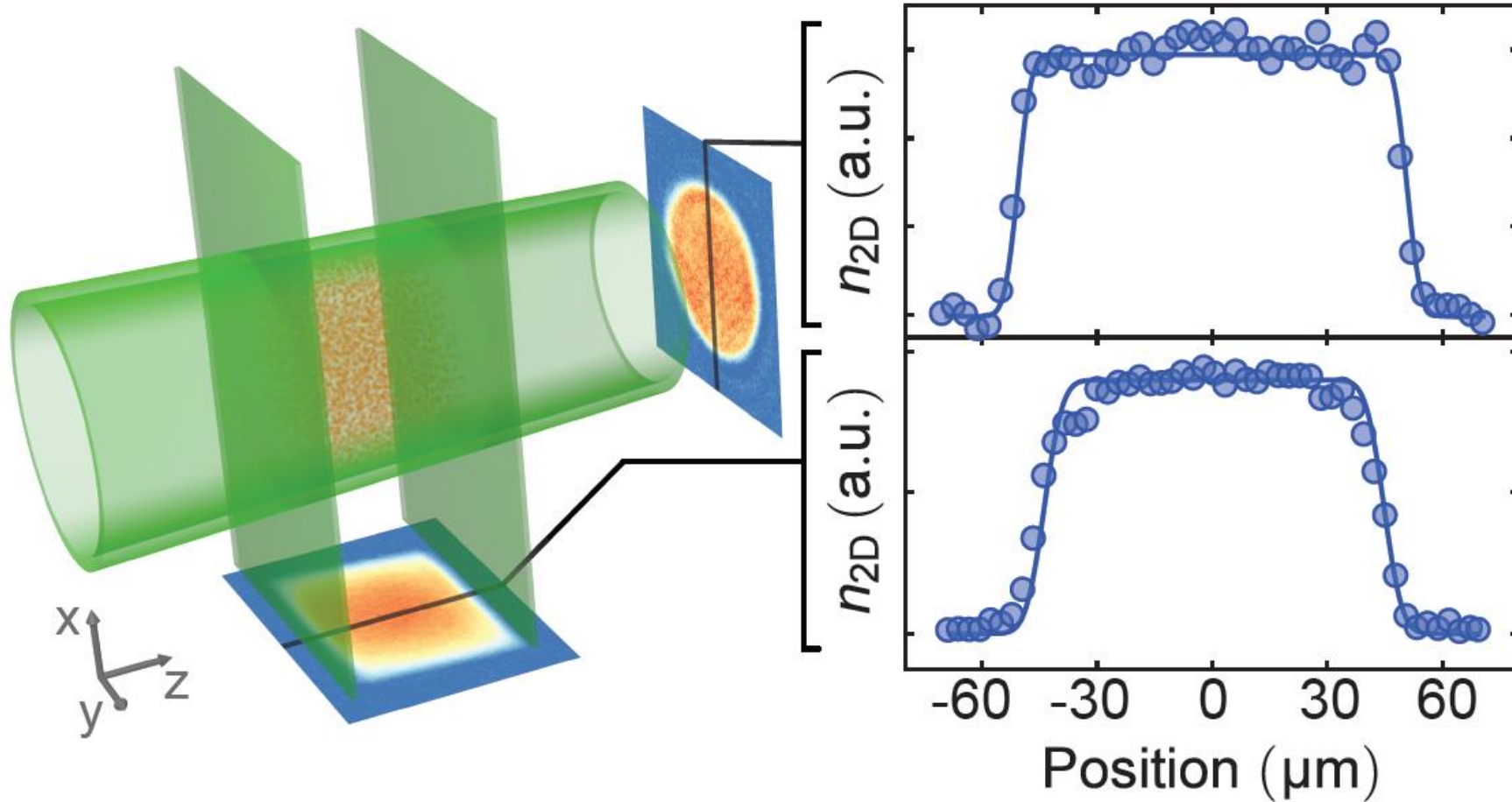


Bose 1924

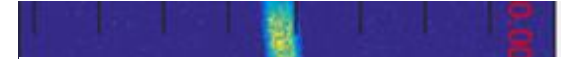
Einstein 1924/25



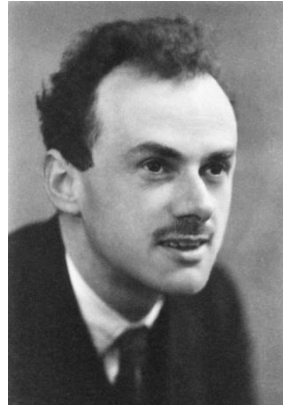
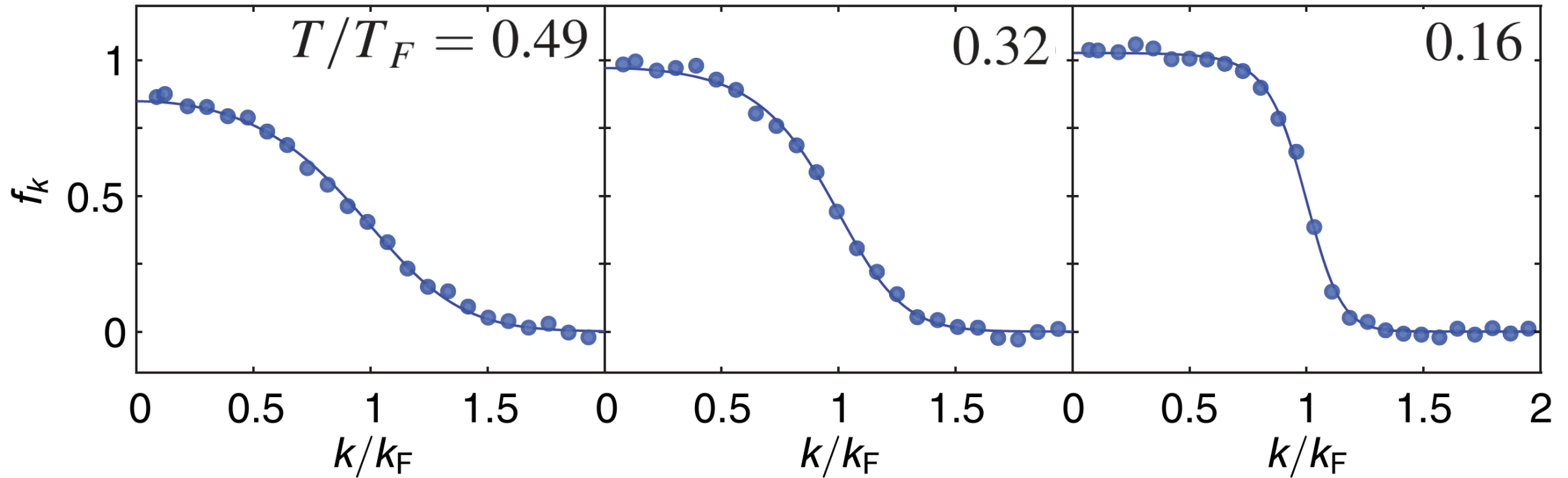
Fermions in a Box



Time of Flight:



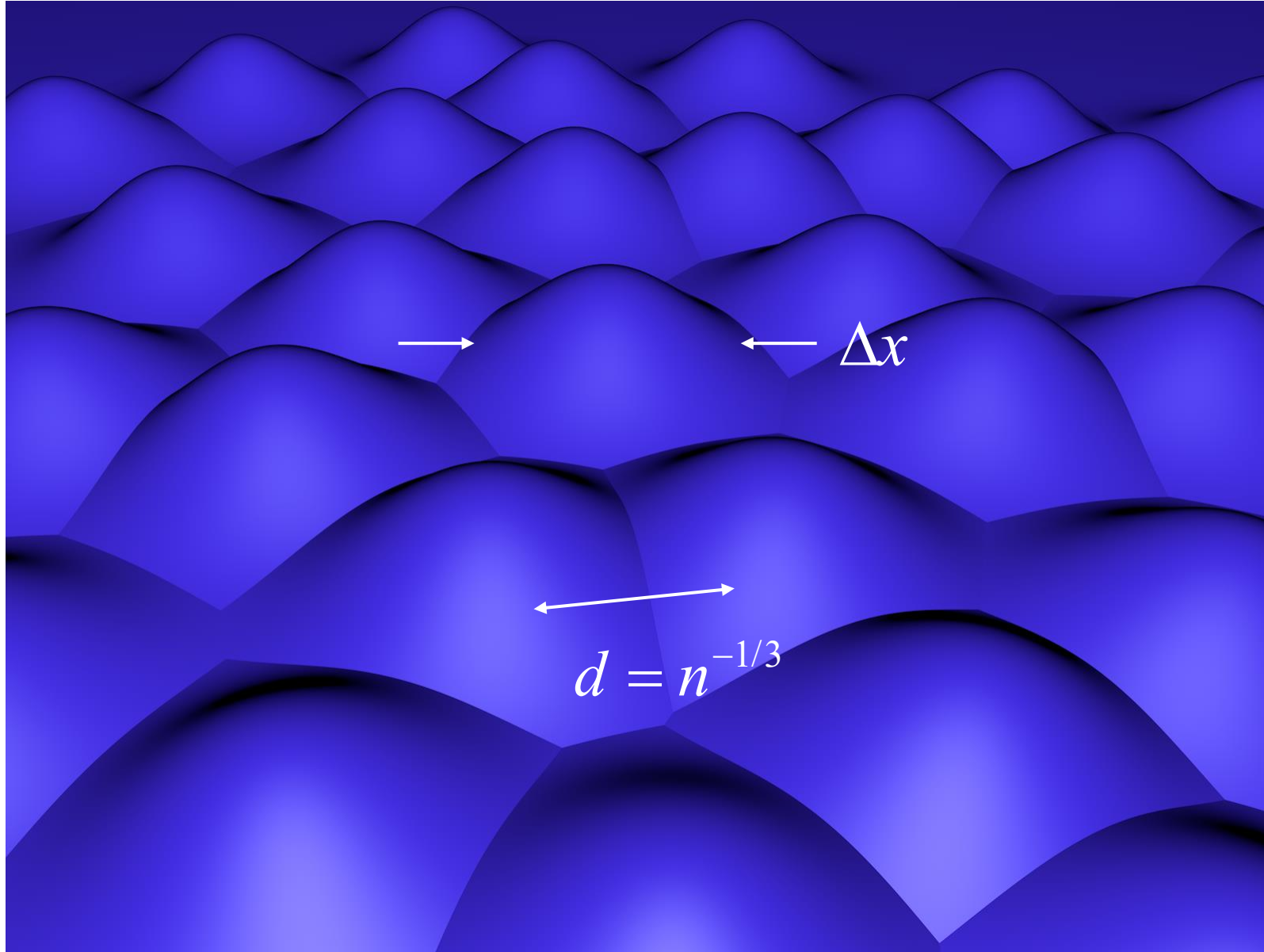
Measuring the Fermi-Dirac distribution



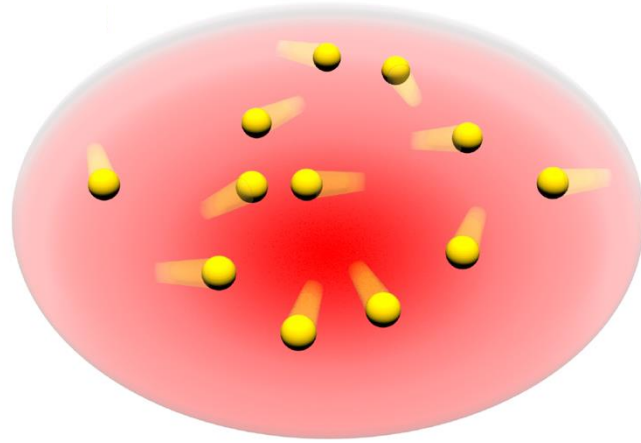
$$f_k = \frac{1}{e^{\beta(\epsilon_k - \mu)} + 1}$$

**100th
anniversary**

Can one “see” the de Broglie waves in situ?

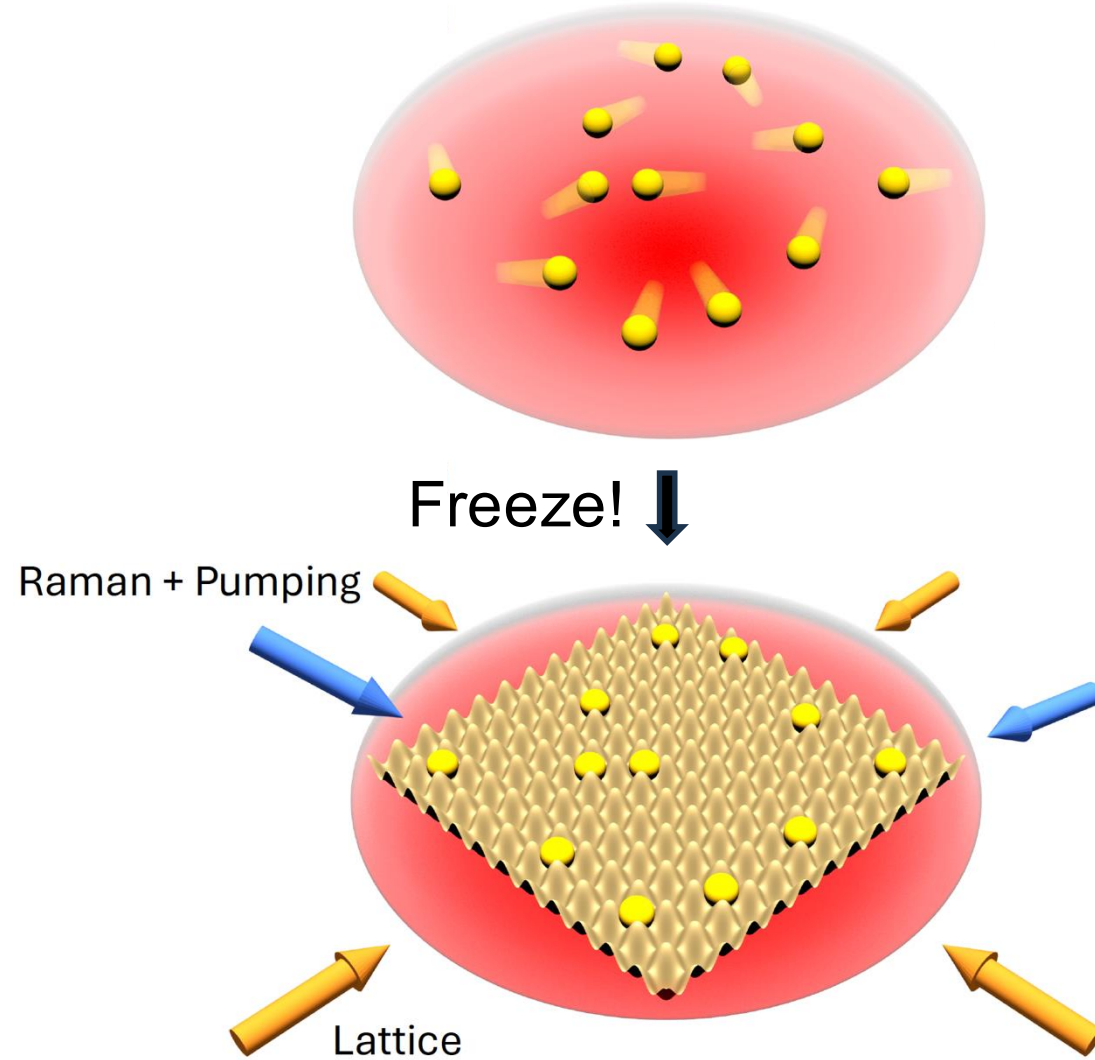


Build a “continuum” quantum gas microscope



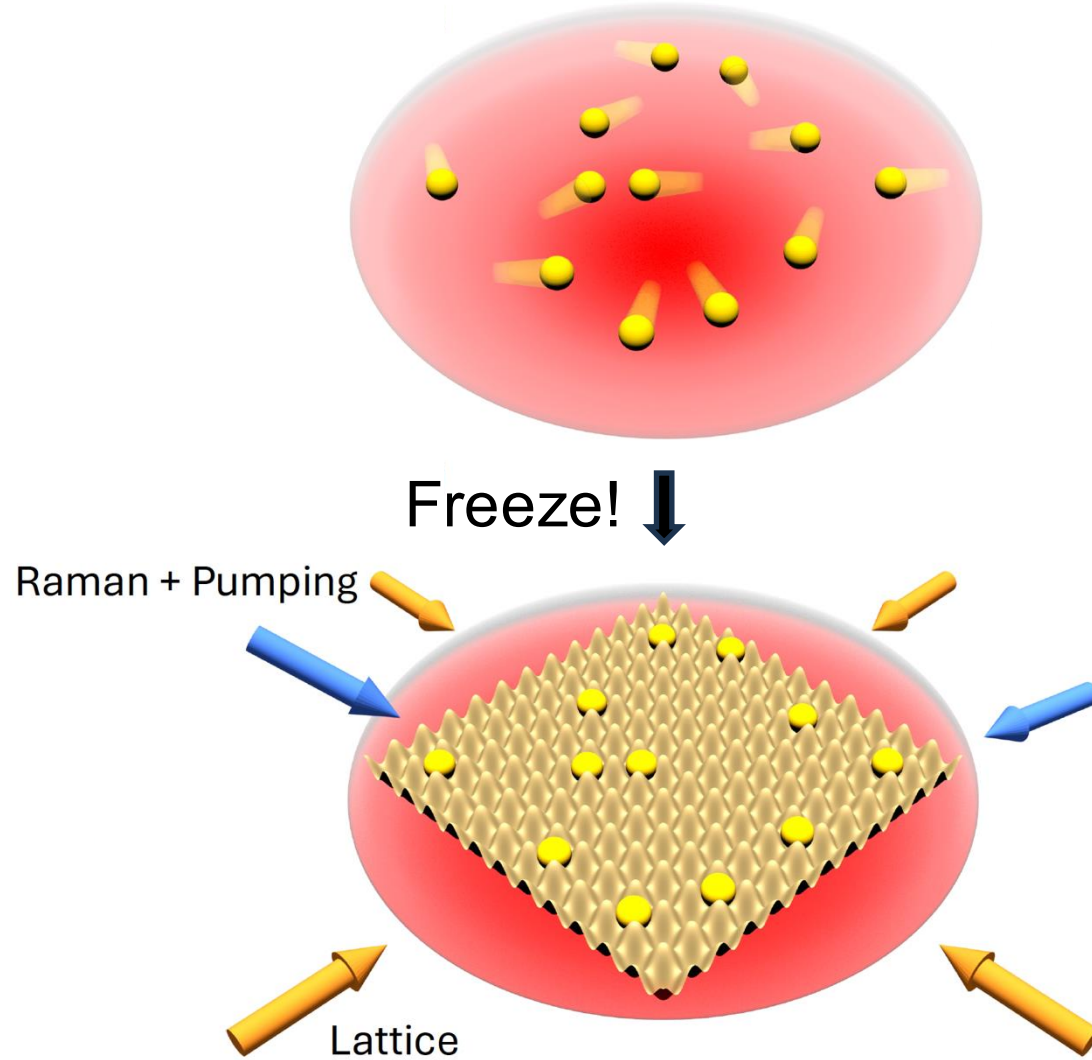
R Yao, S Chi, M Wang, RJ Fletcher, MZ, PRL **134**, 183402 (2025);
see also: Fermions: Yefsah group, PRL **134**, 183403 (2025), Bosons: Ketterle group, PRL **134**, 183401 (2025)

Build a “continuum” quantum gas microscope

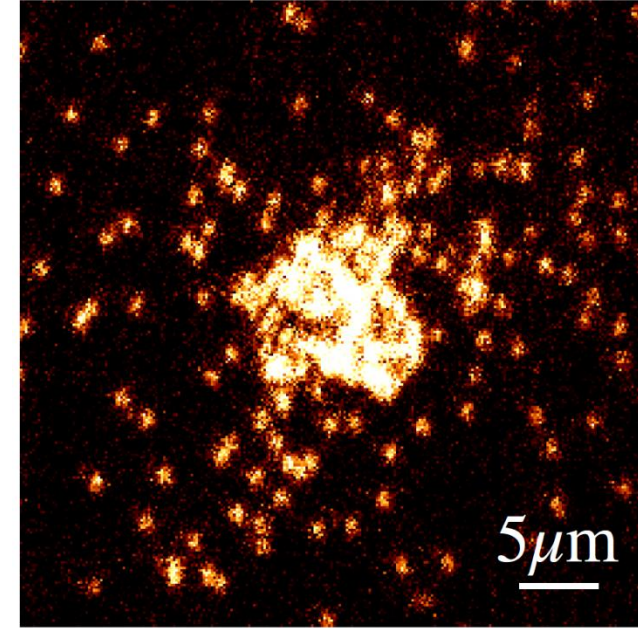


R Yao, S Chi, M Wang, RJ Fletcher, MZ, PRL **134**, 183402 (2025);
see also: Fermions: Yefsah group, PRL **134**, 183403 (2025), Bosons: Ketterle group, PRL **134**, 183401 (2025)

Build a “continuum” quantum gas microscope

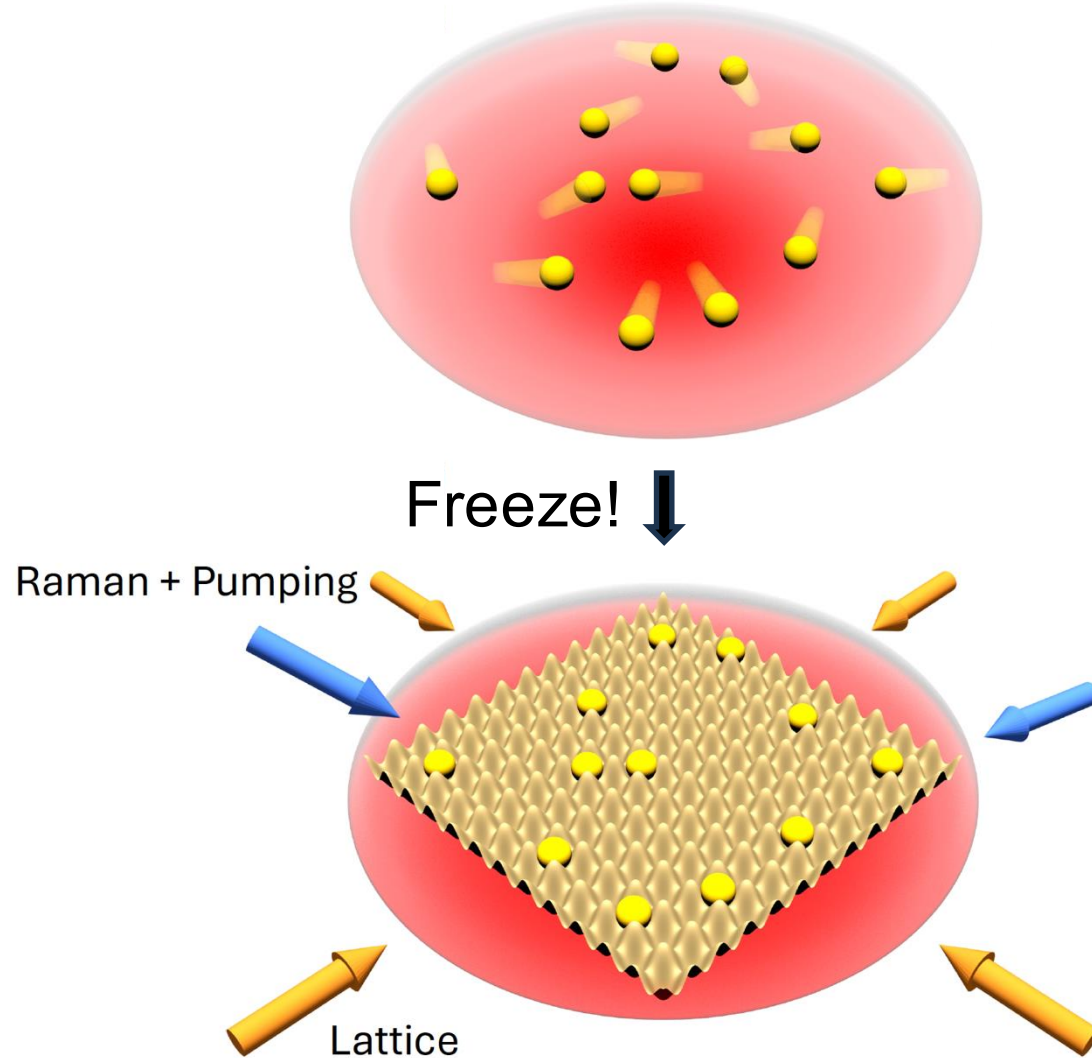


Bosons

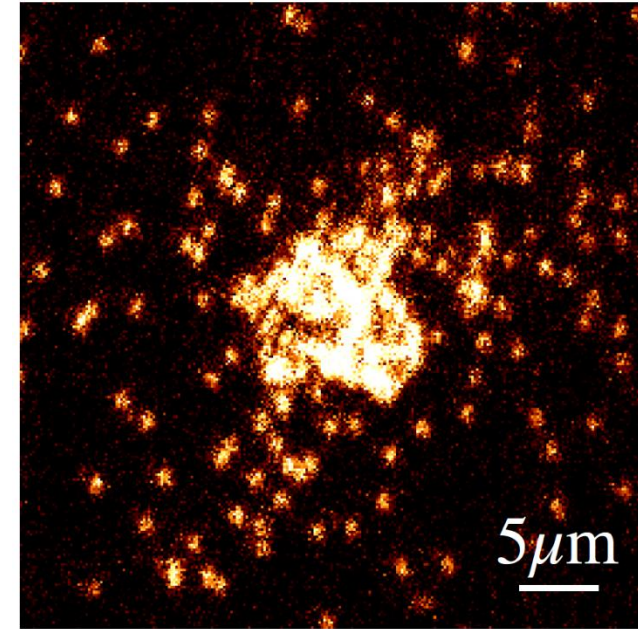


R Yao, S Chi, M Wang, RJ Fletcher, MZ, PRL **134**, 183402 (2025);
see also: Fermions: Yefsah group, PRL **134**, 183403 (2025), Bosons: Ketterle group, PRL **134**, 183401 (2025)

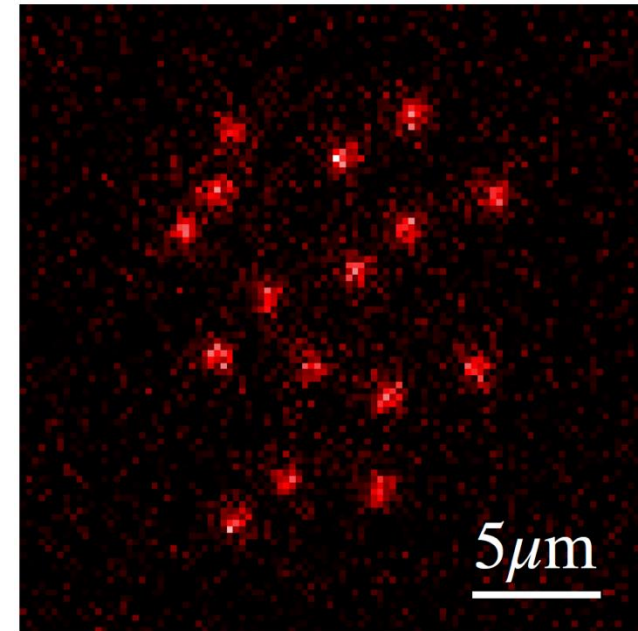
Build a “continuum” quantum gas microscope



Bosons



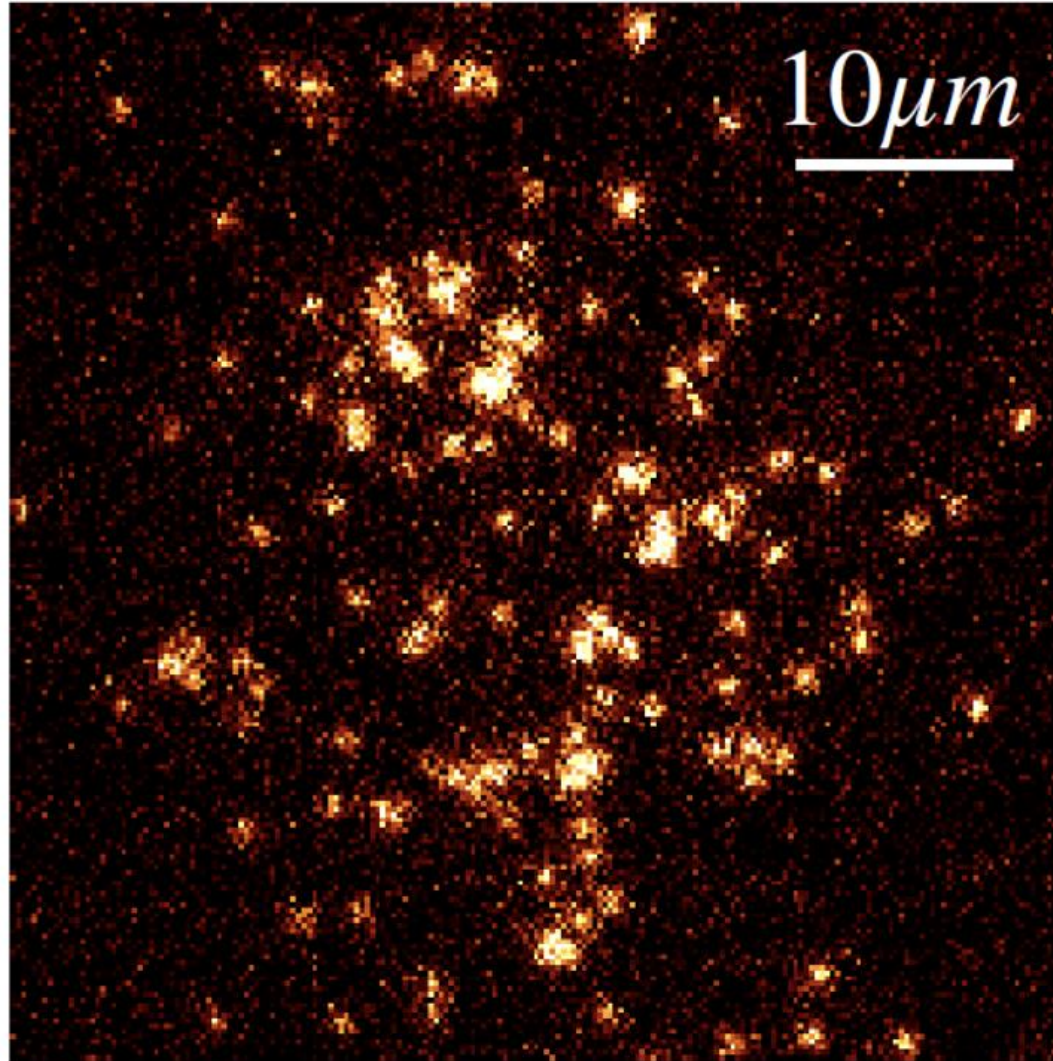
Fermions



R Yao, S Chi, M Wang, RJ Fletcher, MZ, PRL **134**, 183402 (2025); see also:
Fermions: Yefsah group, PRL **134**, 183403, Bosons: Ketterle group, PRL **134**, 183401

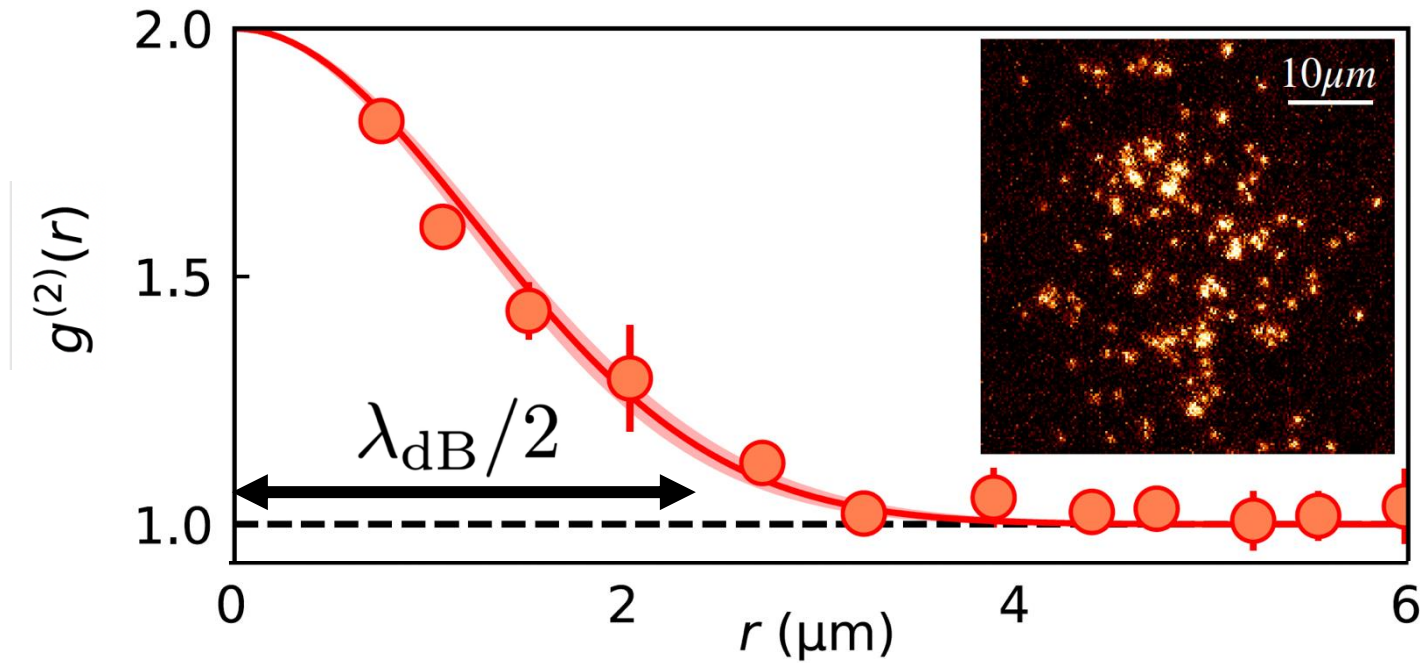
Pair correlation function $g^{(2)}(r)$

Thermal
Bosons



Pair correlation function $g^{(2)}(r)$

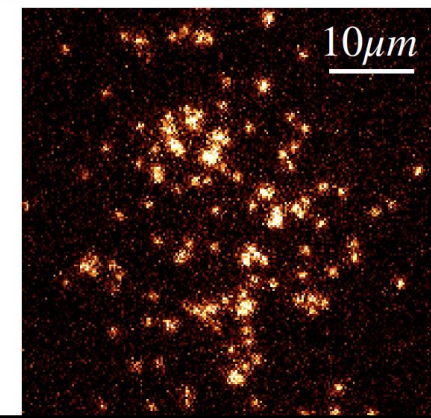
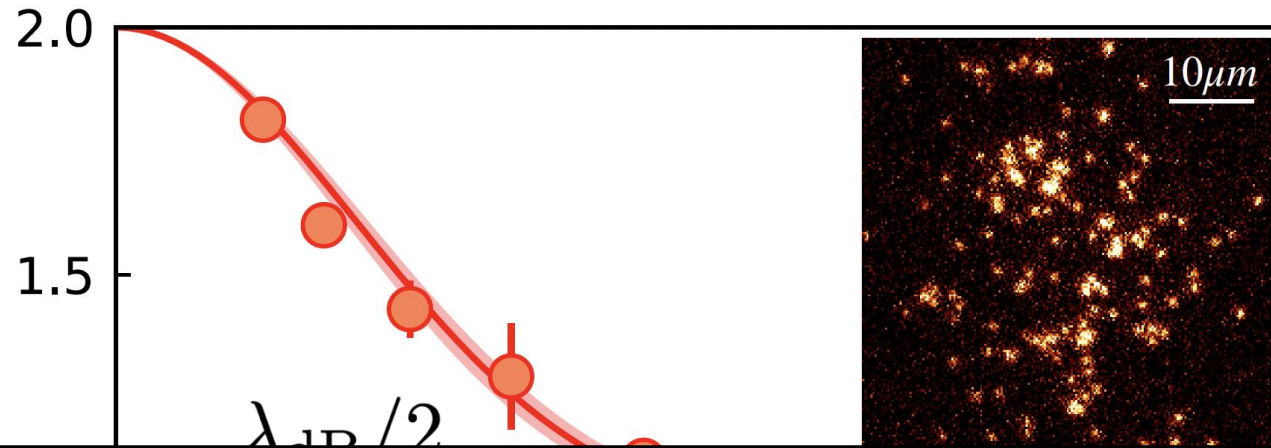
Thermal
Bosons



$T = 6.9(3)$ nK

Pair correlation function $g^{(2)}(r)$

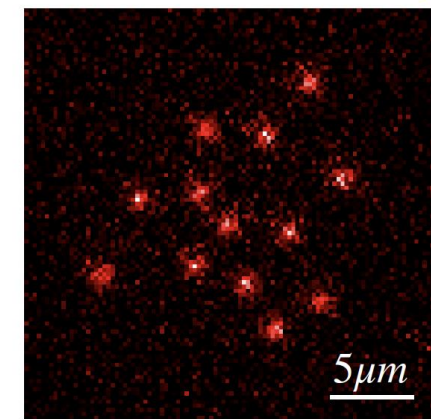
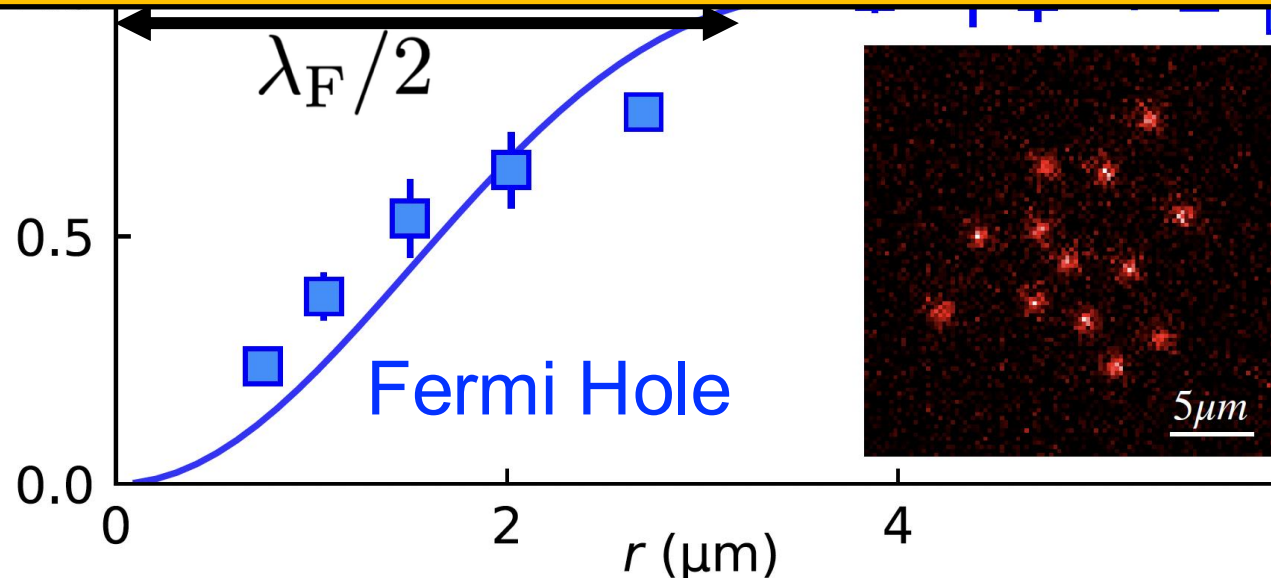
Thermal
Bosons



$T = 6.9(3) \text{ nK}$

A microscopic view of correlated quantum matter

Fermions

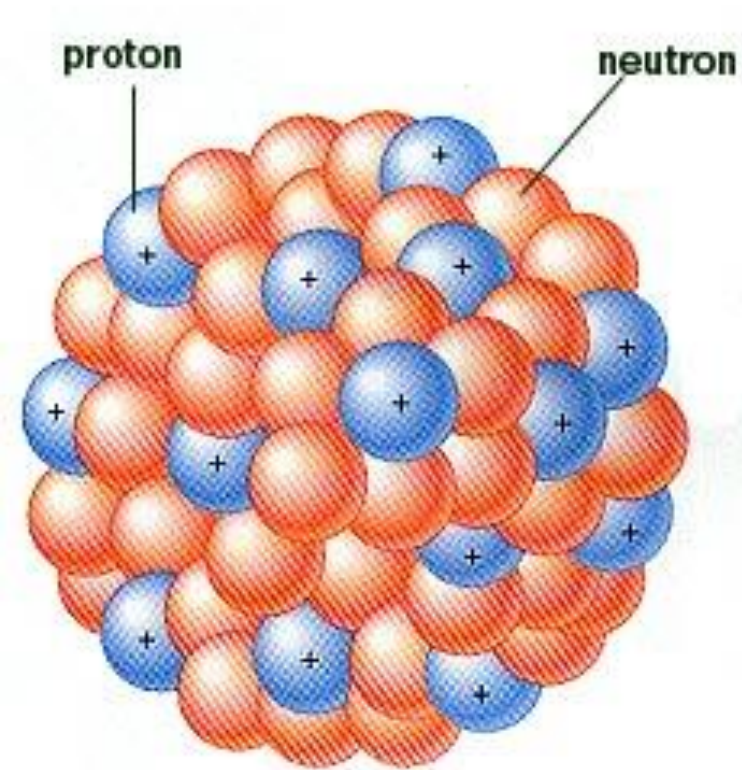


$T = 3.7(3) \text{ nK}$

$T_{\text{F}} = 45 \text{ nK}$

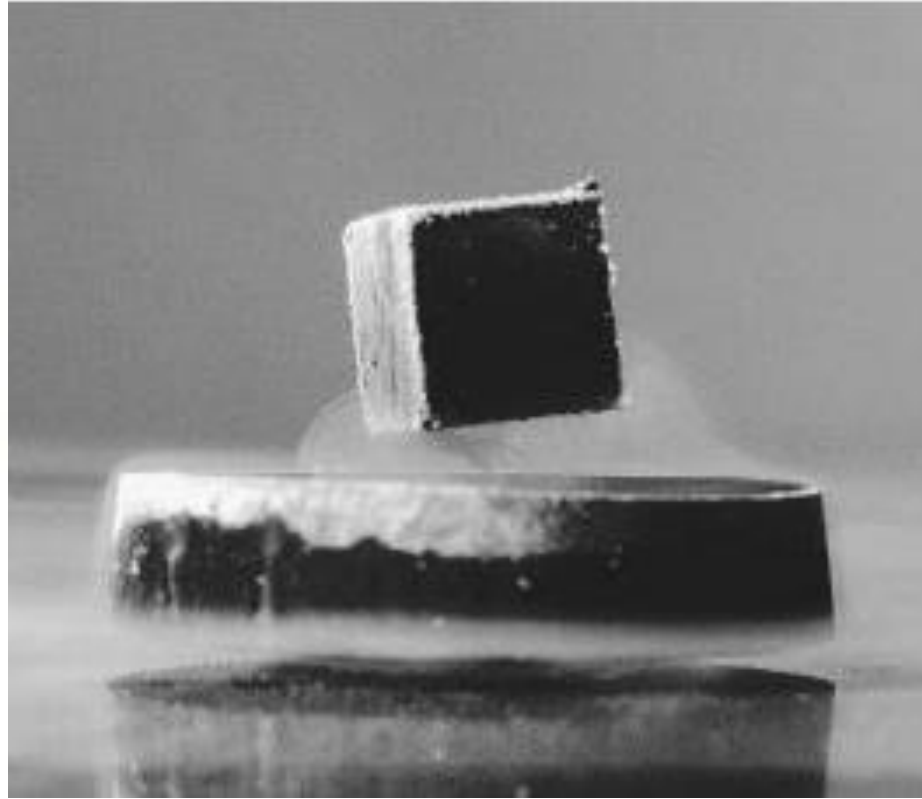
$T/T_{\text{F}} = 0.08$

Matter is made of Fermions



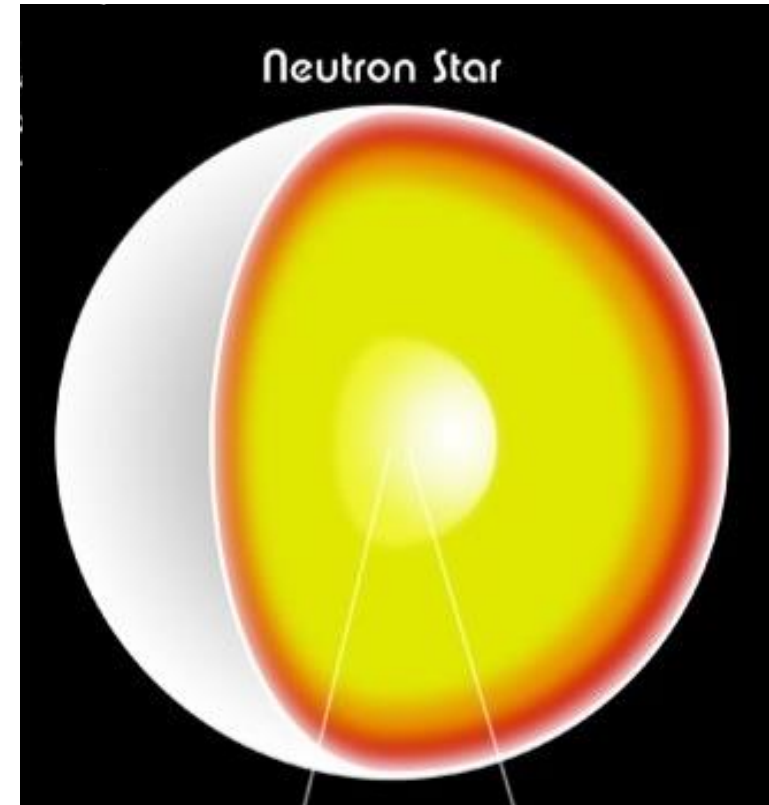
Nuclei

10^{-15} m



Superconductors

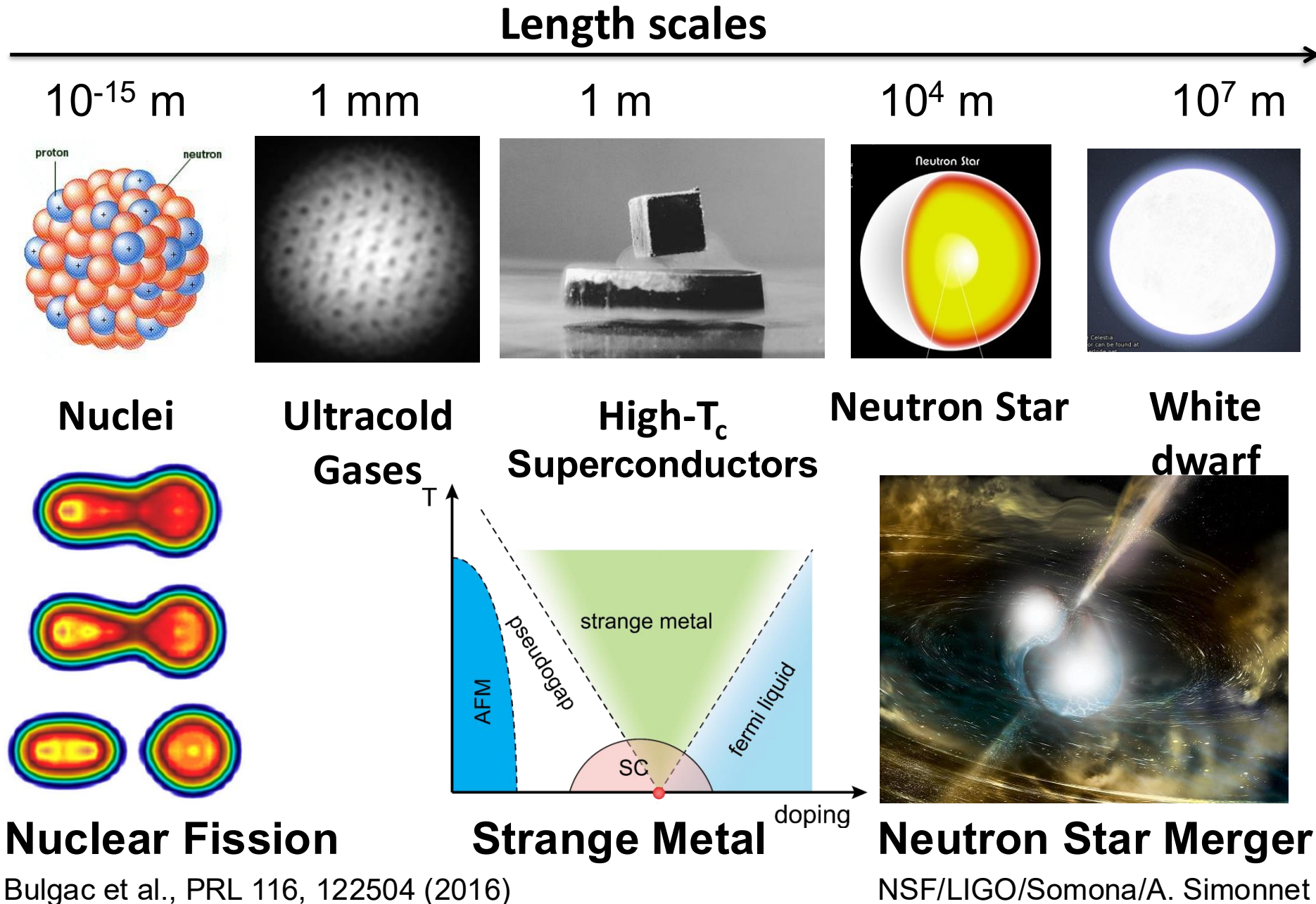
1 cm



Neutron Stars

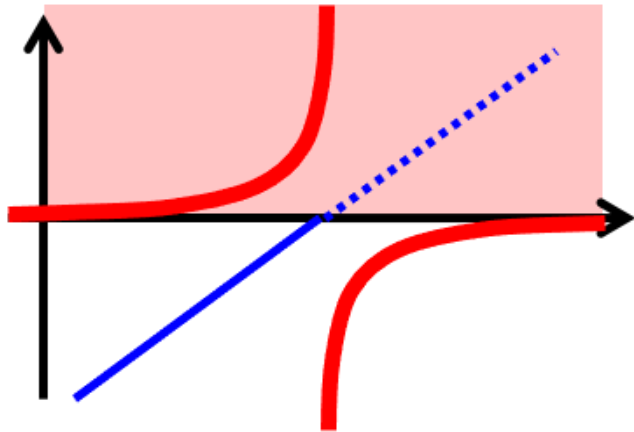
20 km

Strongly Correlated Fermi Systems

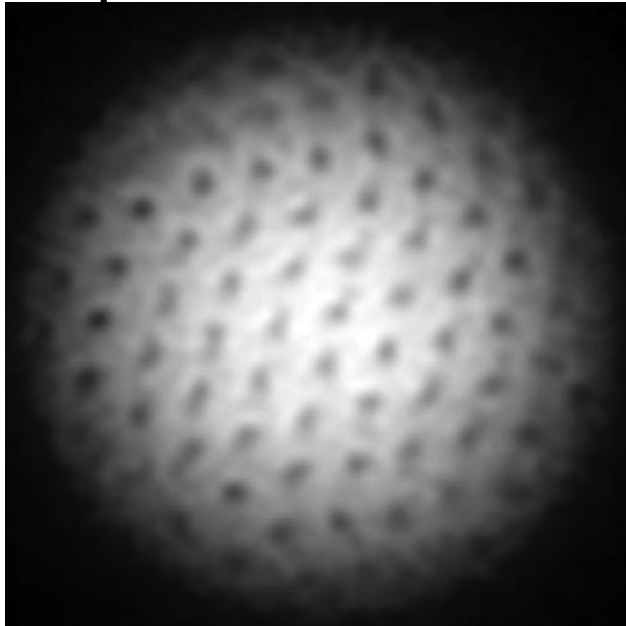


Realizing strongly interacting Fermi systems

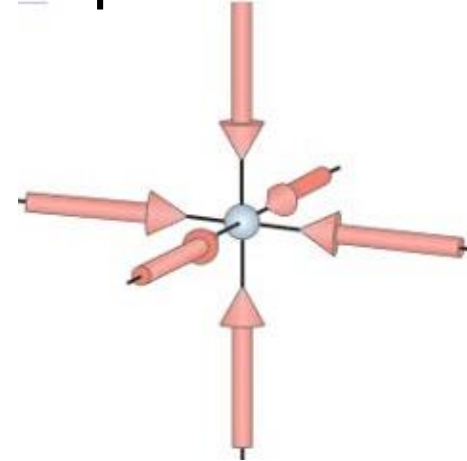
Feshbach Resonances



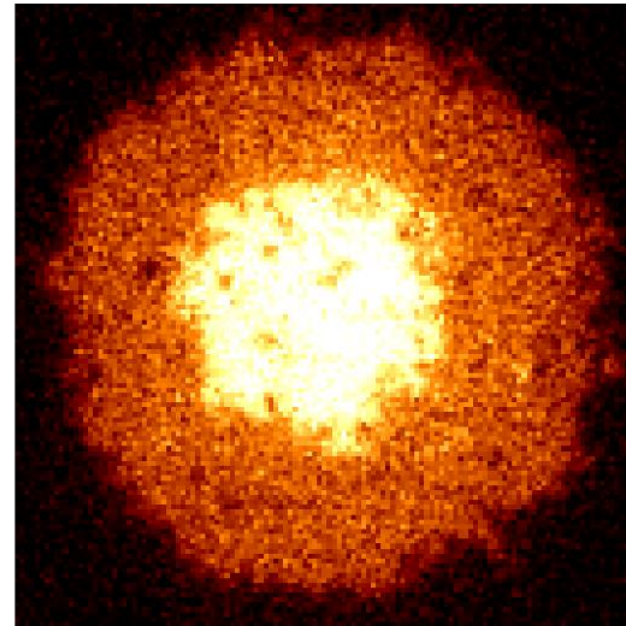
Superfluid Fermi Gas



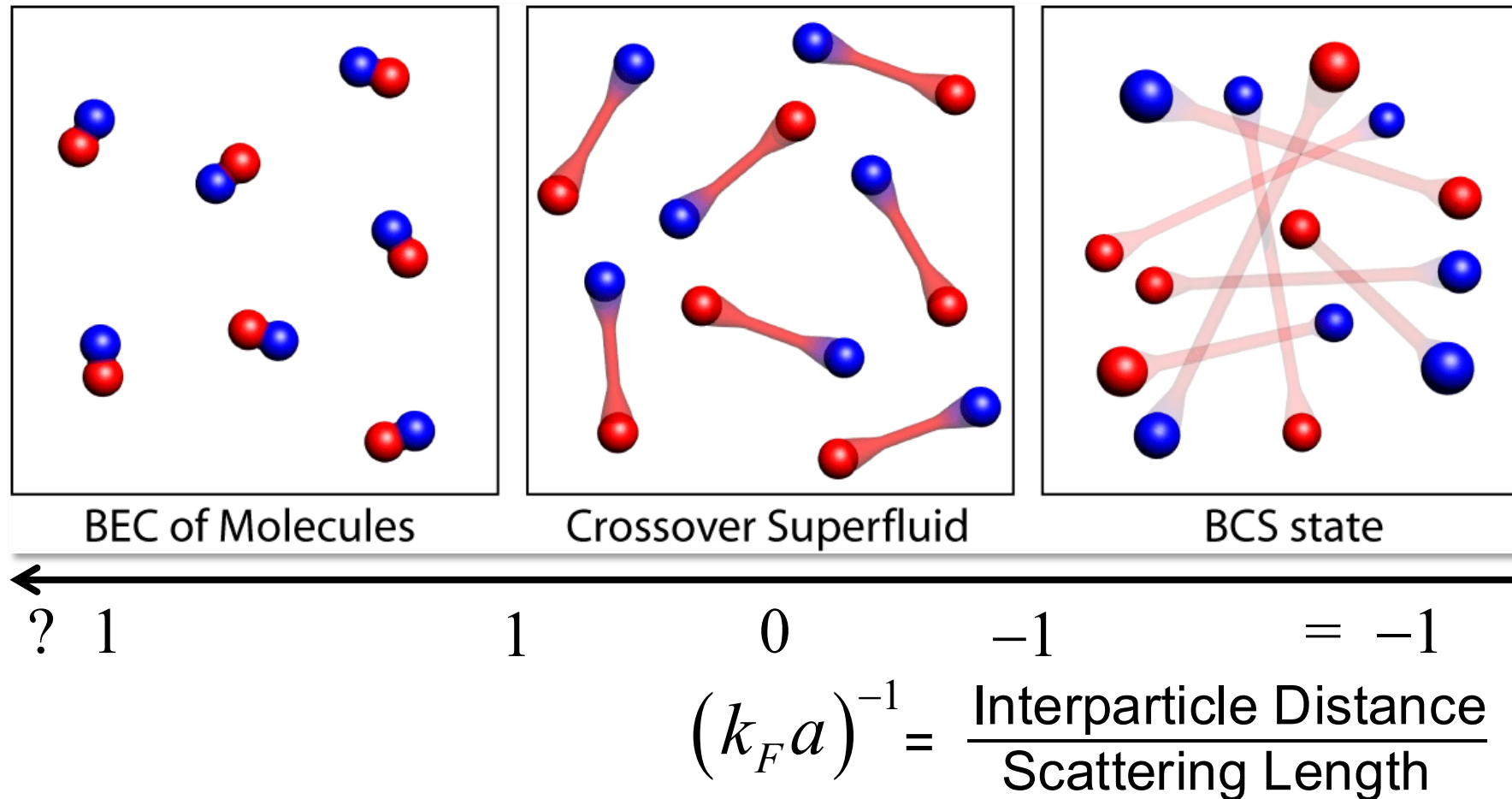
Optical Lattices



Fermi-Hubbard Gas



Superfluid Fermi Gas – From BEC to BCS

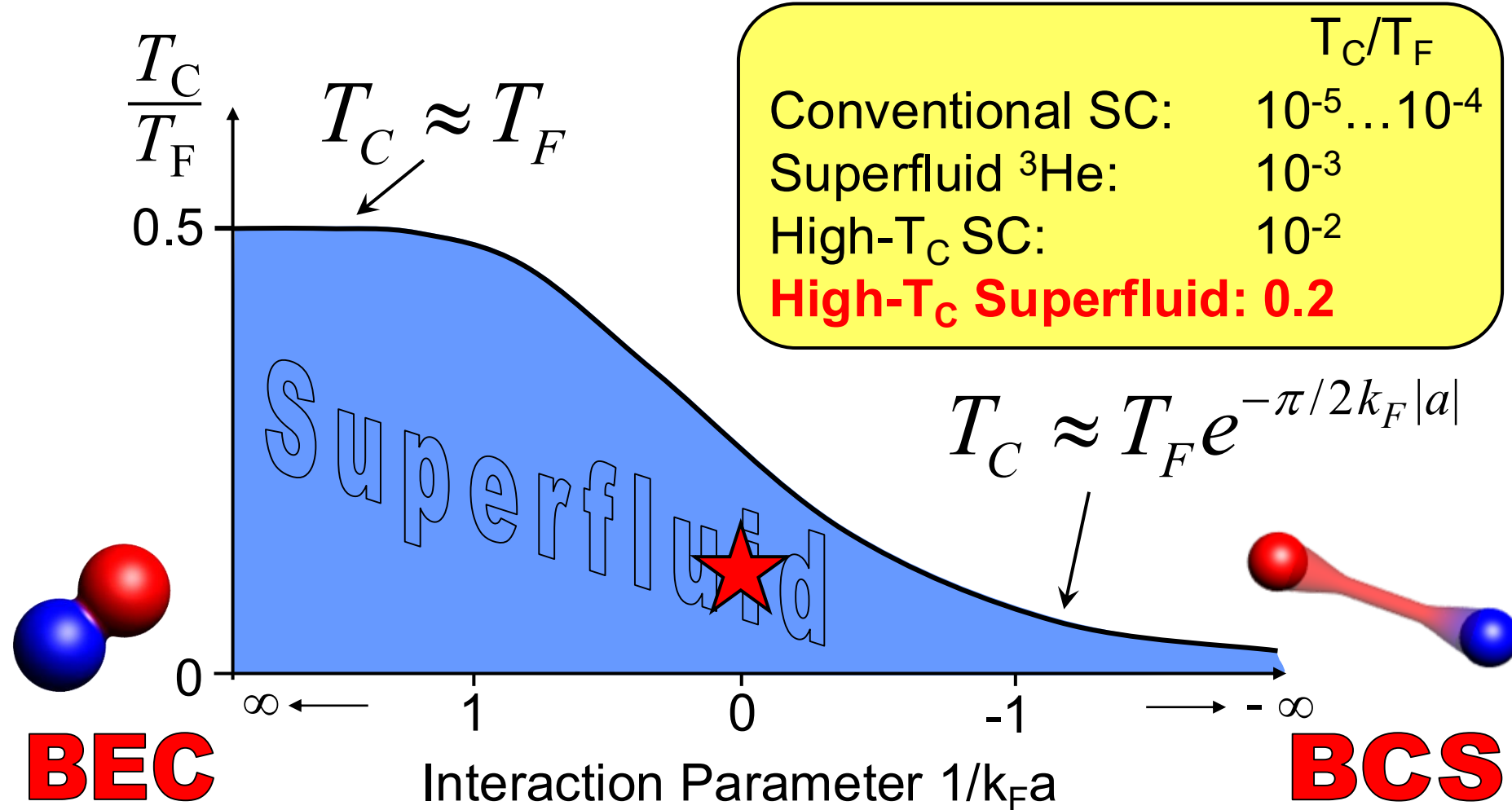


Theory: Eagles, Leggett, Nozières, Schmitt-Rink, Zwerger, Randeria, Timmermans, Holland, Strinati, Combescot, Griffin, Stoof, Levin, Giorgini, Stringari, Pitaevskii,

Early Experiments (2002-2006): Duke, JILA, MIT, Rice, Innsbruck, ENS

Reviews: Randeria, Zwerger, MZ, ed. W. Zwerger, (2012), MZ, in Novel Superfluids eds. Bennemann, Keterson (2015), Giorgini, Pitaevskii, Stringari, RMP 2008, Ketterle, MZ, Varenna 2008, Bloch, Dalibard, Zwerger, RMP 2008

Critical Temperature for Fermionic Superfluidity

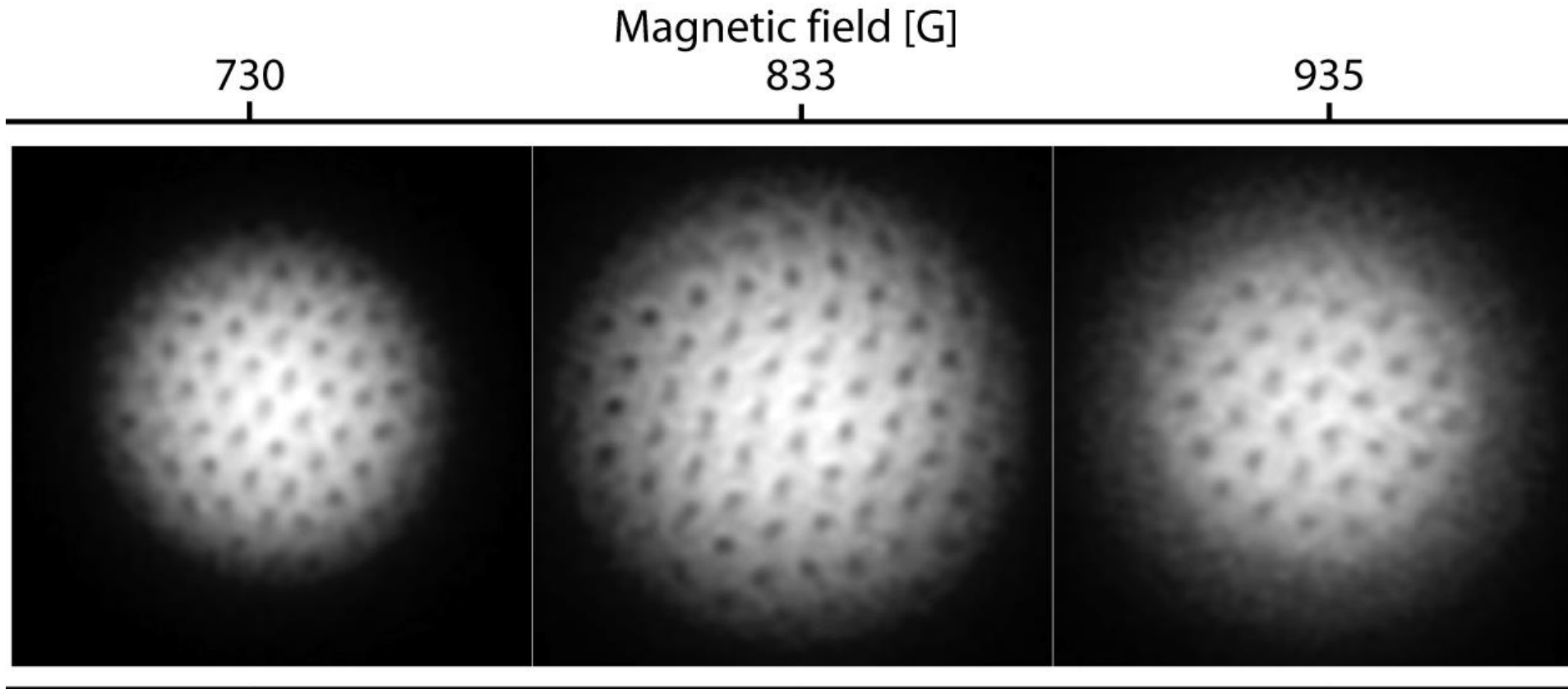


Theory: Eagles, Leggett, Nozières, Schmitt-Rink, Zwerger, Randeria, Timmermans, Holland, Strinati, Combescot, Griffin, Stoof, Levin, Giorgini, Stringari, Pitaevskii,

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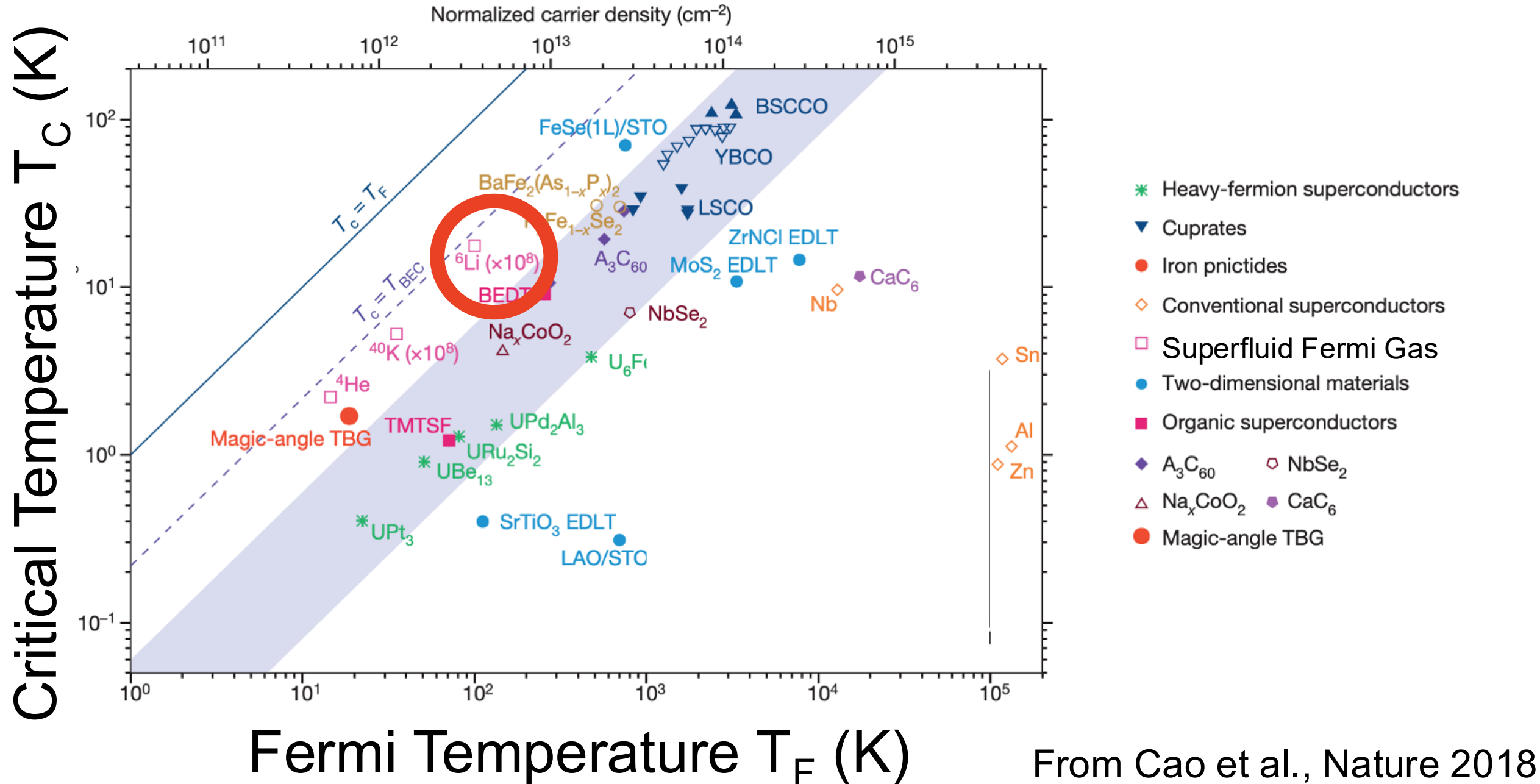
Vortex lattices in the BEC-BCS Crossover



**Scaled to the density of electrons,
superfluidity would occur far above room temperature**

M.W. Zwierlein, J.R. Abo-Shaeer, A. Schirotzek, C.H. Schunck, W. Ketterle,
Nature 435, 1047-1051 (2005)

Fermi Gas on the Uemura Plot



The Unitary Fermi Gas

Only two length scales:

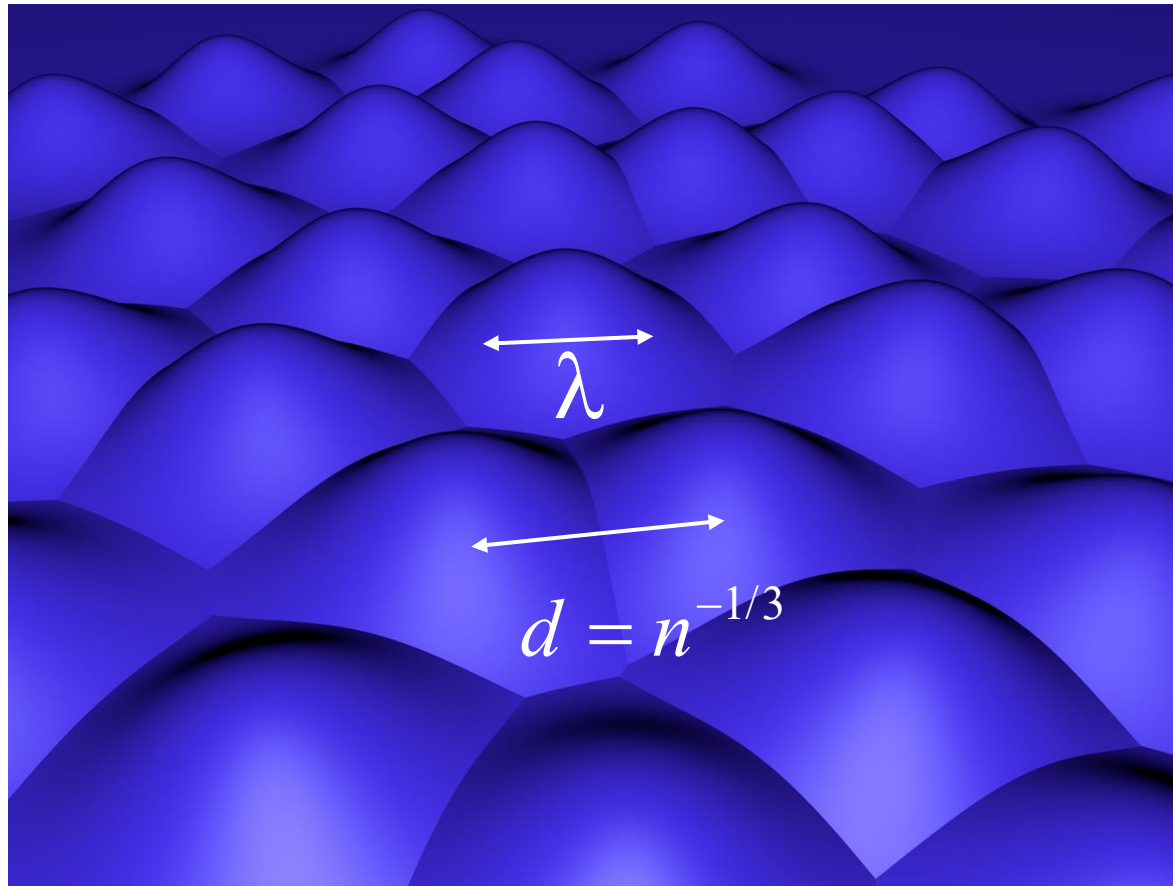
Interparticle spacing $n^{-1/3}$

De Broglie wavelength λ_{dB}

Only two energy scales:

Fermi energy E_F

Temperature $k_B T$



The Unitary Fermi Gas

Only two length scales:

Interparticle spacing $n^{-1/3}$

De Broglie wavelength λ_{dB}

Only two energy scales:

Fermi energy E_F

Temperature $k_B T$

→ Universal equation of state

Duke/NC State, ENS, JILA, Innsbruck, Swinburne, MIT... also see: 2D Bose Gas: ENS, Cambridge

Again just like Fermi wrote:
--- only with different $P(x)$

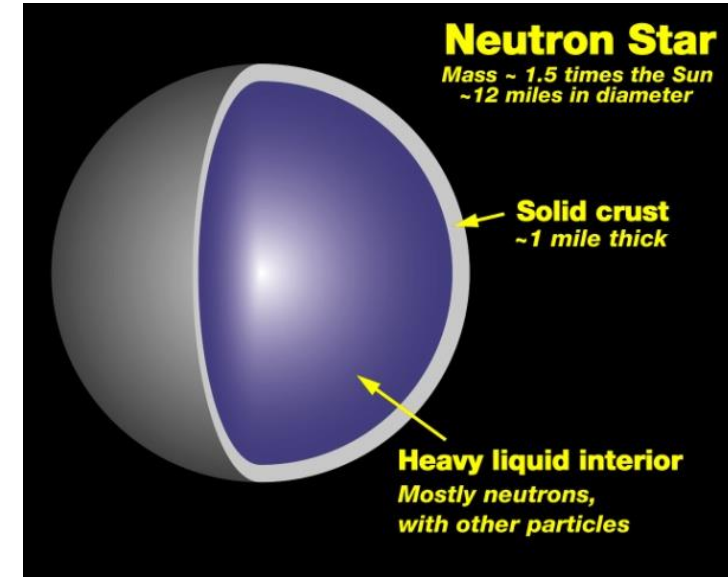
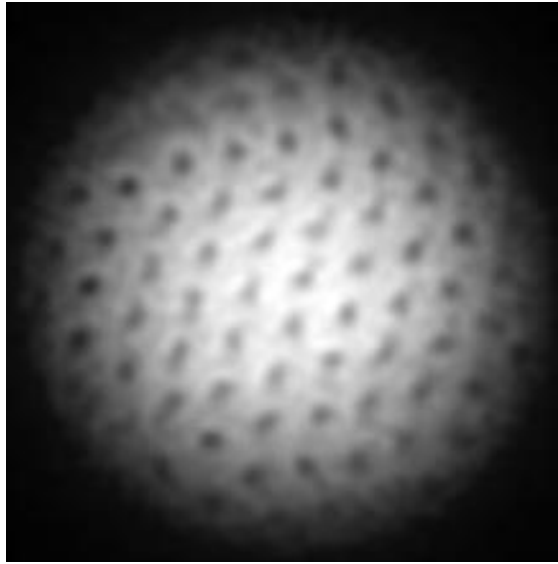
$$p = \frac{2}{3} n \bar{\epsilon} = \frac{\hbar^2 n^{5/3}}{2 \pi m} P\left(\frac{2 \pi m k T}{\hbar^2 n^{2/3}}\right)$$

Ground-State Energy:

$$E = \xi \frac{3}{5} N E_F \quad \xi_{\text{Exp}} = 0.37(1)$$

Critical Temperature: $T_c = 0.17(1) T_F$

Relation to equation of state of neutron matter



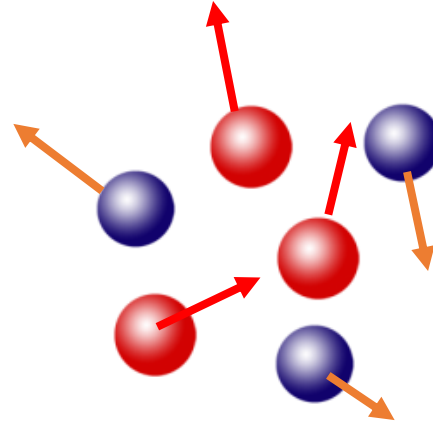
Property	Atoms	Neutrons
Spin	Pseudospin $\frac{1}{2}$	Spin $\frac{1}{2}$
Interparticle distance $n^{-1/3}$	$1 \mu\text{m}$	1 fm
Density	10^{13} cm^{-3}	10^{38} cm^{-3}
Fermi Energy	$1 \mu\text{K} = 10^{-10} \text{ eV}$	$10^{12} \text{ K} = 150 \text{ MeV}$
Scattering length a	freely tunable	-19 fm

Both systems lie in universal regime: $a ? n^{-1/3}$

small print: neglecting effective range

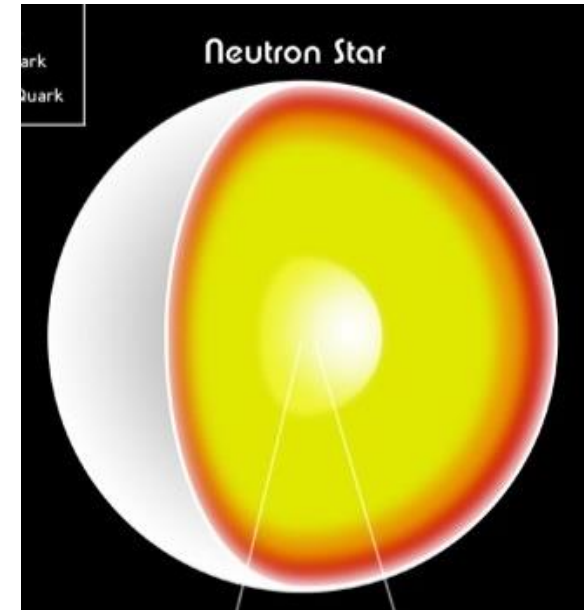
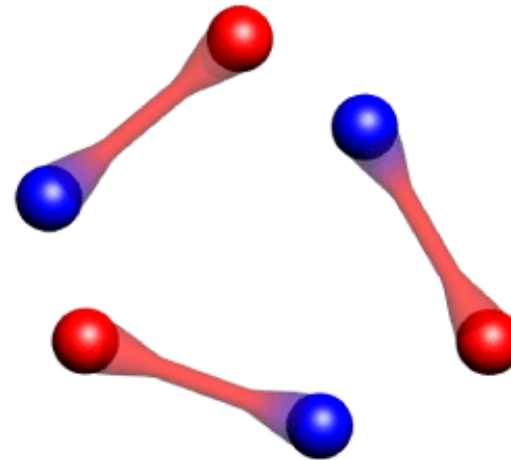
Classical gas Equation of State:

$$P = nk_B T$$

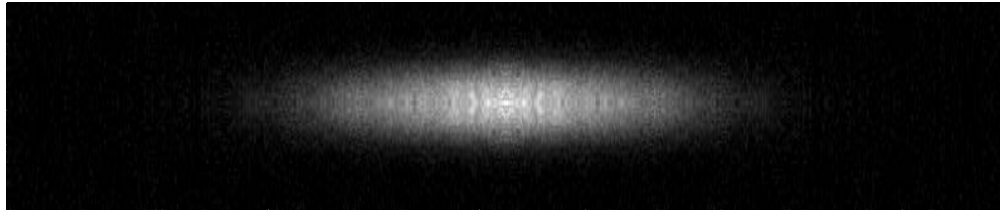


And for a strongly interacting Fermi gas?

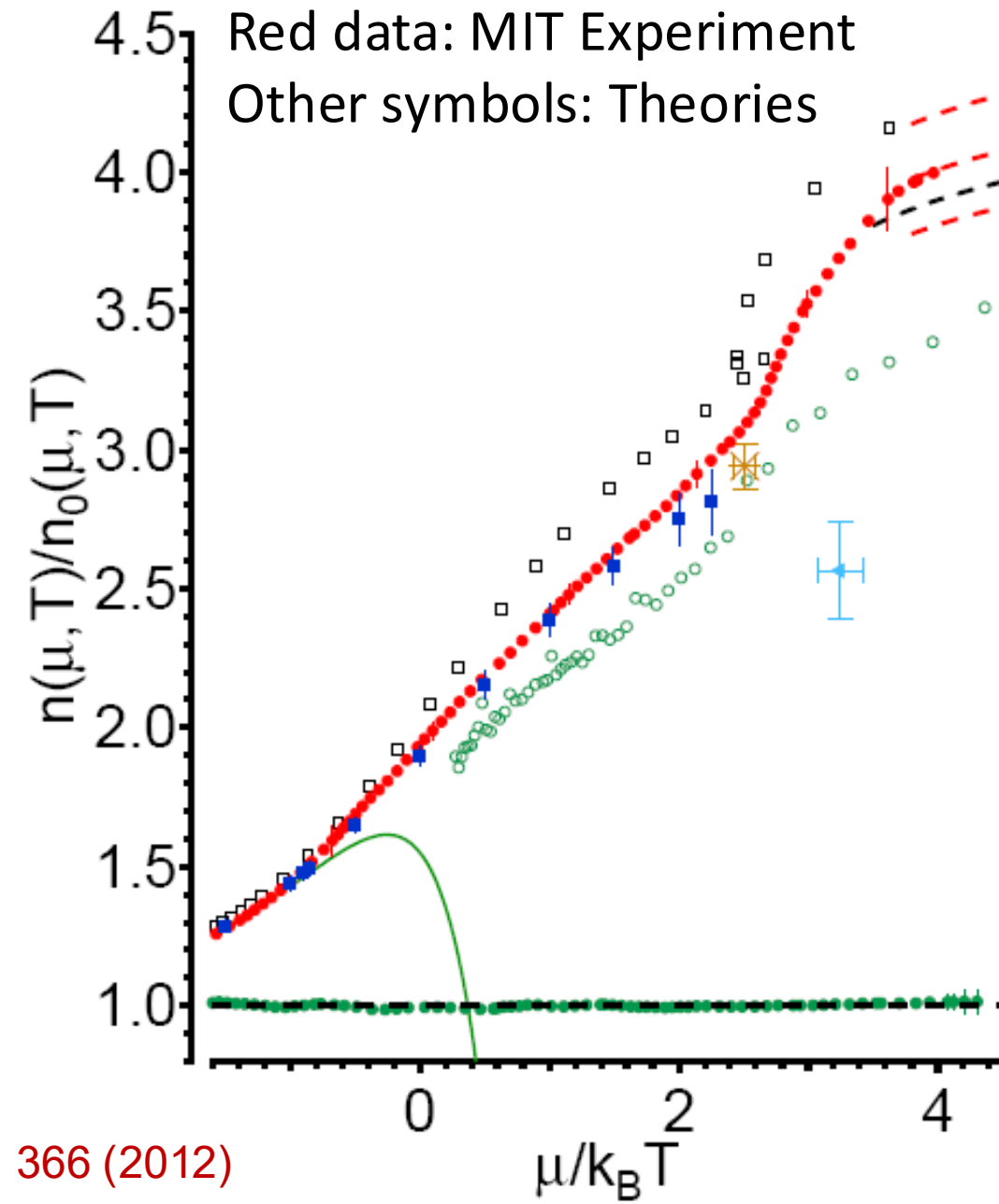
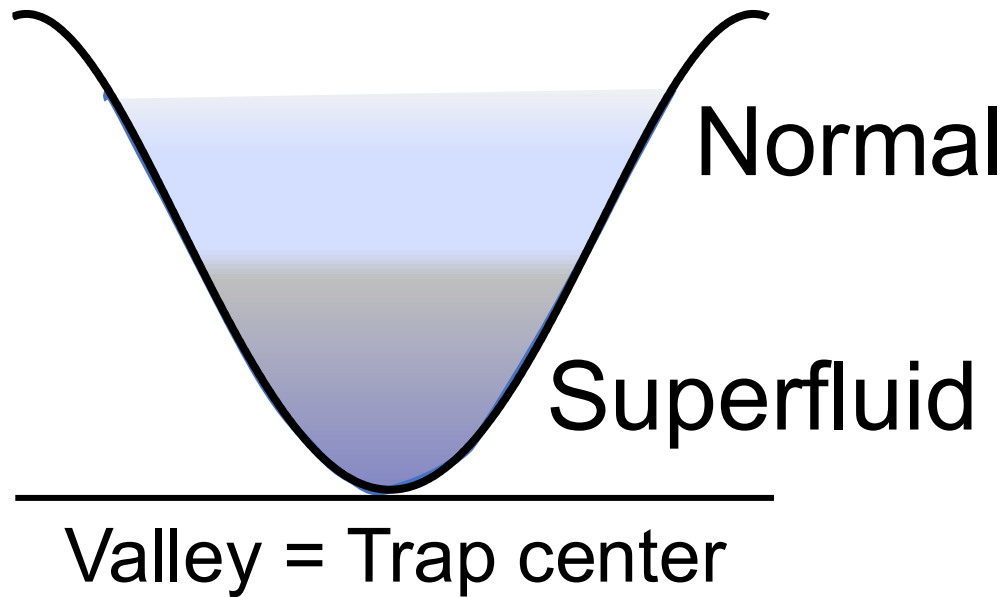
$$P(n, T) = ?$$



Measurement of the Equation of State



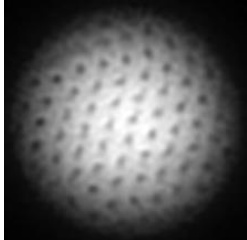
Hill = Trap edge



Mark Ku, Ariel Sommer, Lawrence Cheuk, MZ,
Science **335**, 563-567 (2012)

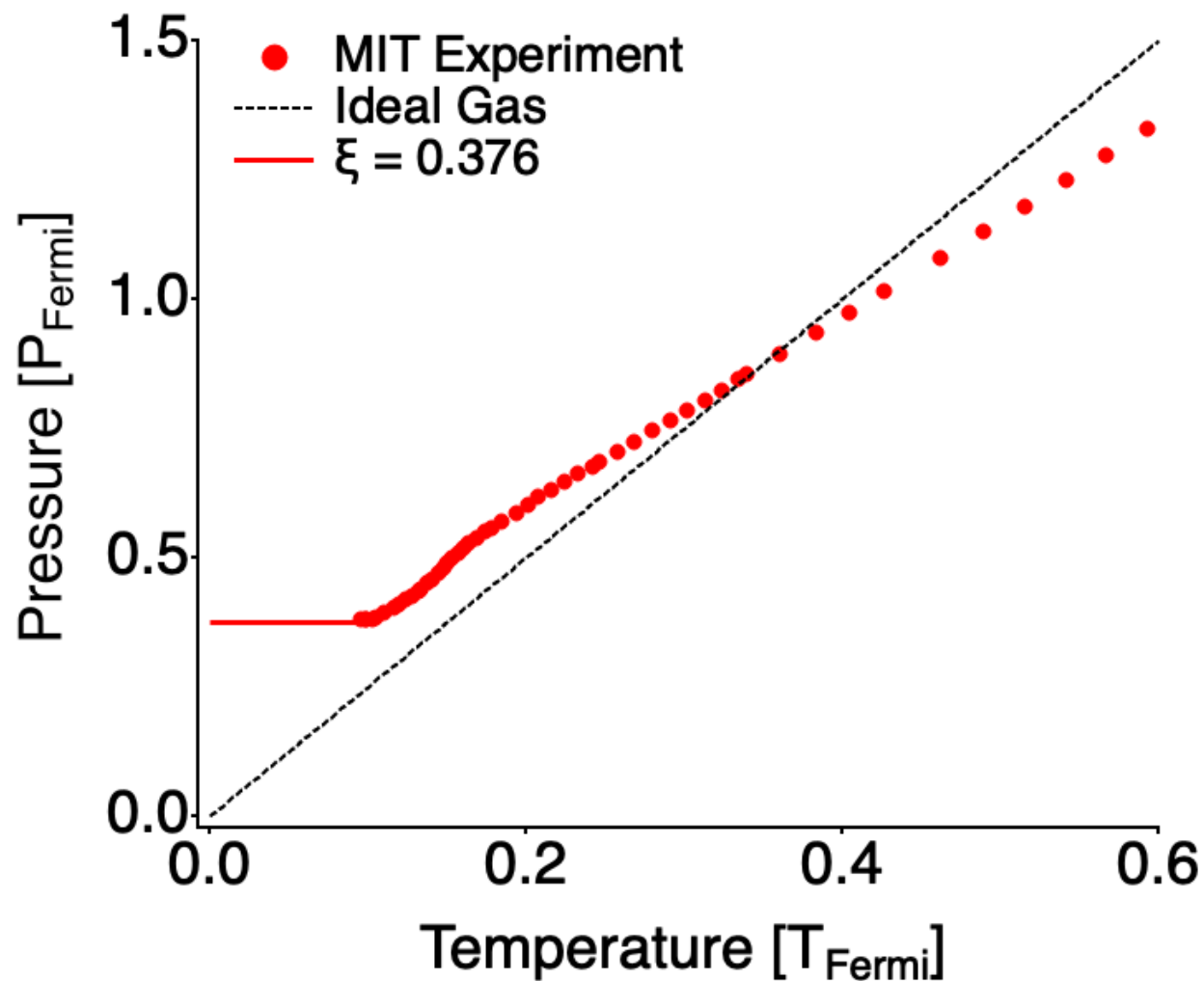
K. Van Houcke, F. Werner, E. Kozik, N. Prokofev, B. Svistunov,

M. Ku, A. Sommer, L. Cheuk, A. Schirotzek, MZ, *Nature Physics* **8**, 366 (2012)



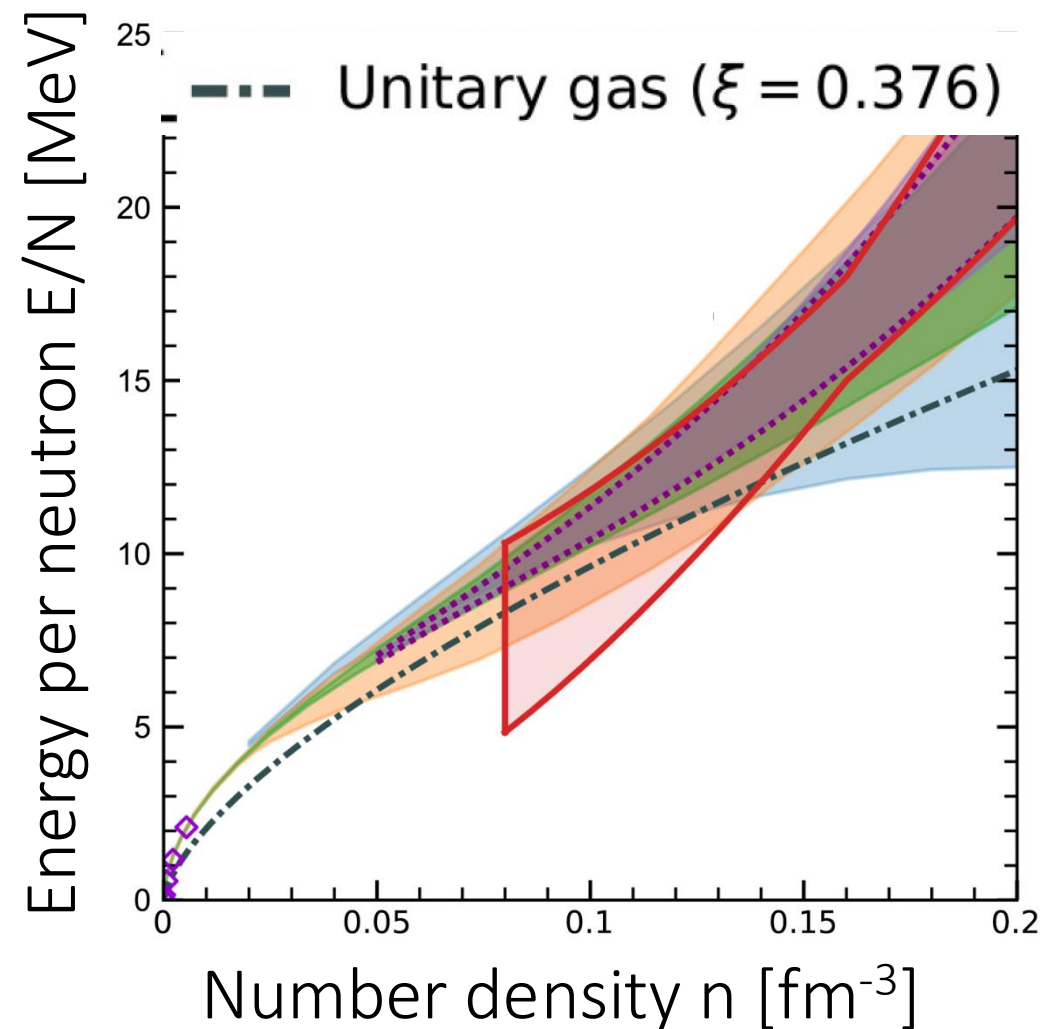
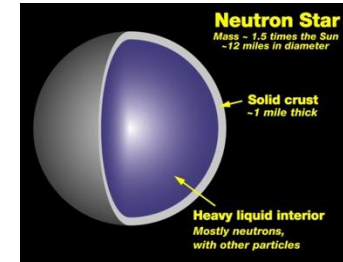
Ultracold Fermi Gas

Ku, Sommer, Cheuk, Zwierlein, Science (2012)



Neutron Star

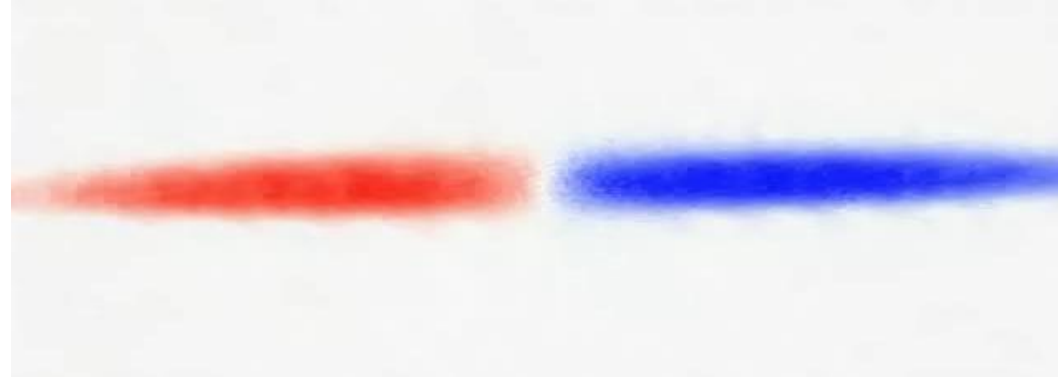
Huth et al., Nature (2021)



→ Ultracold Fermi gas constrains neutron star equation of state

Transport in the Unitary Fermi gas

e.g. Spin transport:



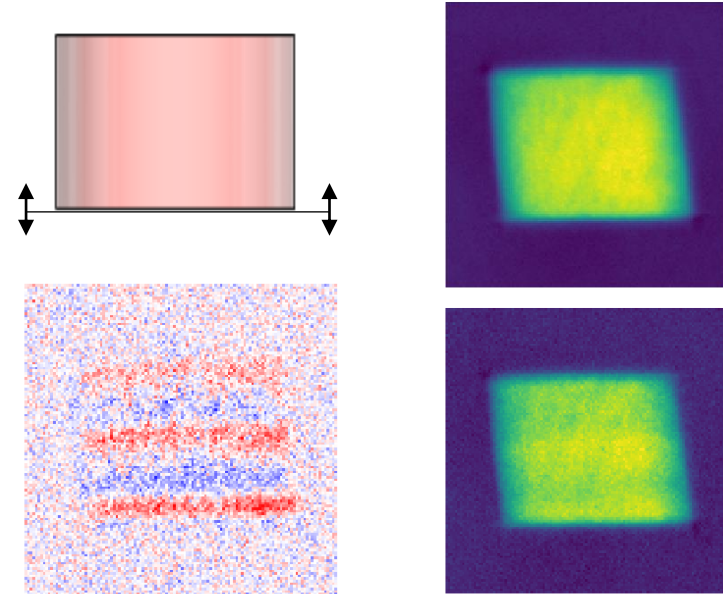
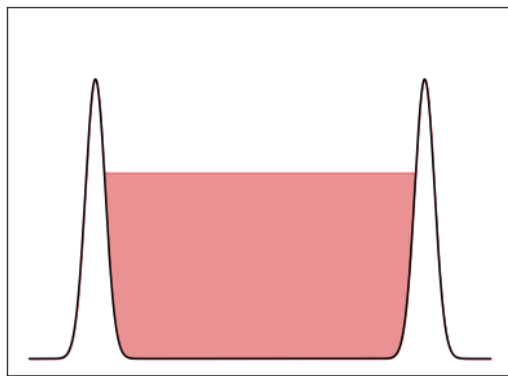
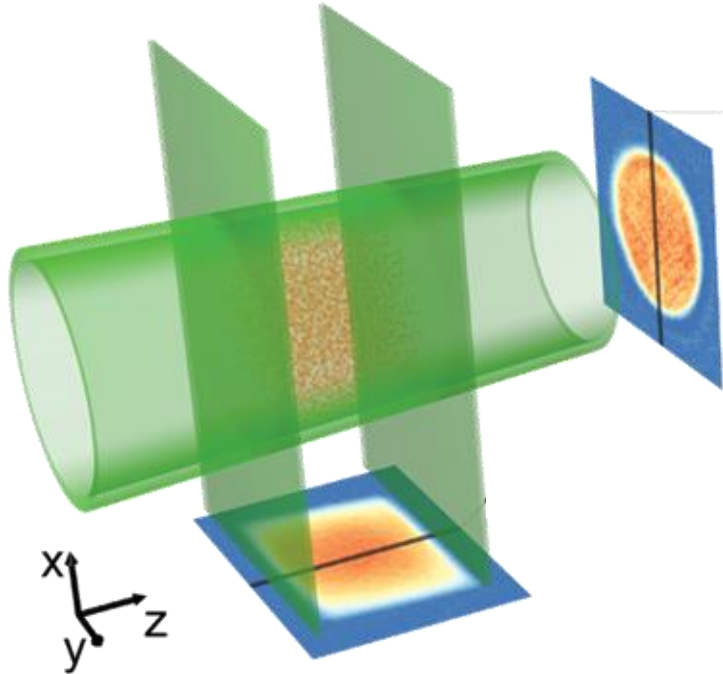
Mean free path $\sim$ Interparticle spacing d

Average velocity $v : \frac{h}{md}$ Collision time $\tau : \frac{h}{E_F}$

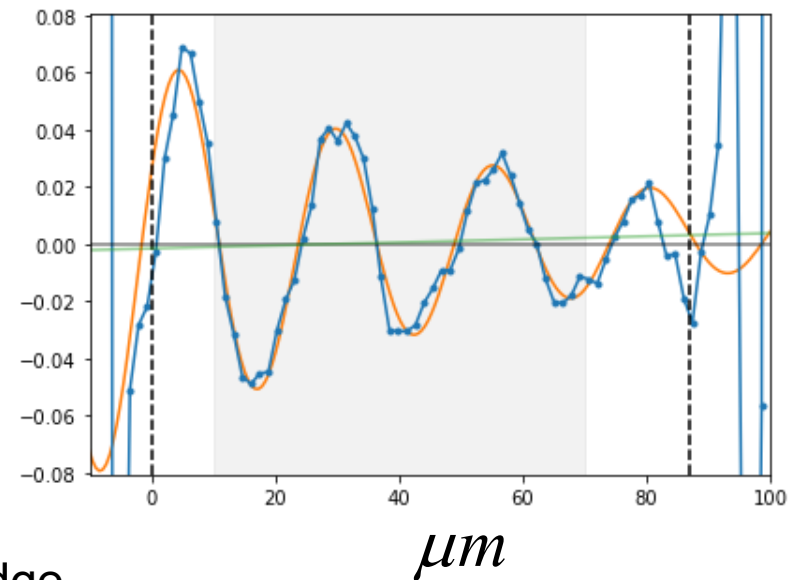
Diffusivity (charge, spin, momentum, thermal)

$$D : vl : \frac{h}{m}$$

Creating Sound Waves in a Box of Fermions



$$\frac{\delta\rho}{\rho}$$



See also: 2D Bose gas, ENS, 2D+3D Cambridge
Fermi gas: NCState, Hamburg, Yale

Viscosity of Superfluid Helium-4

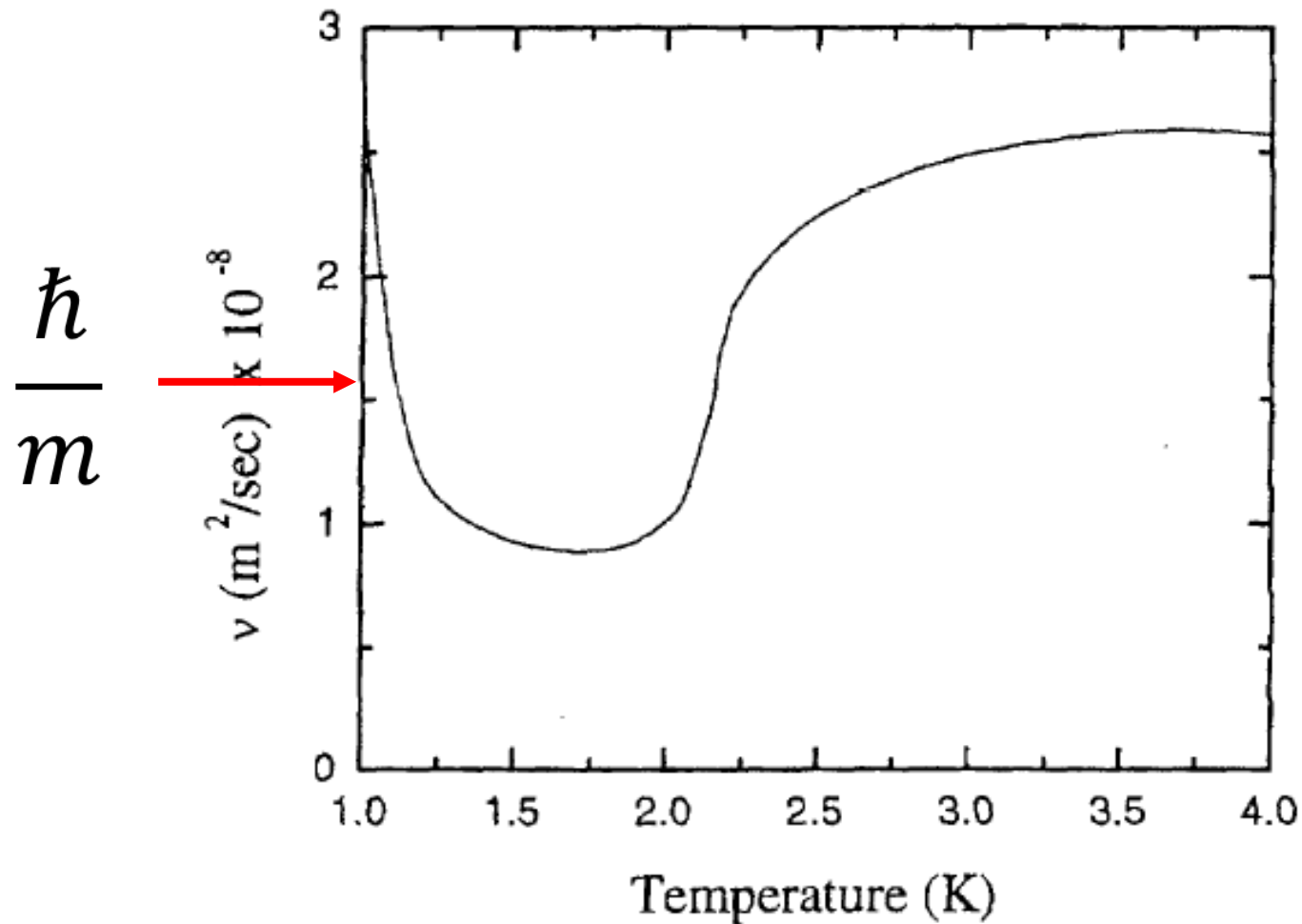
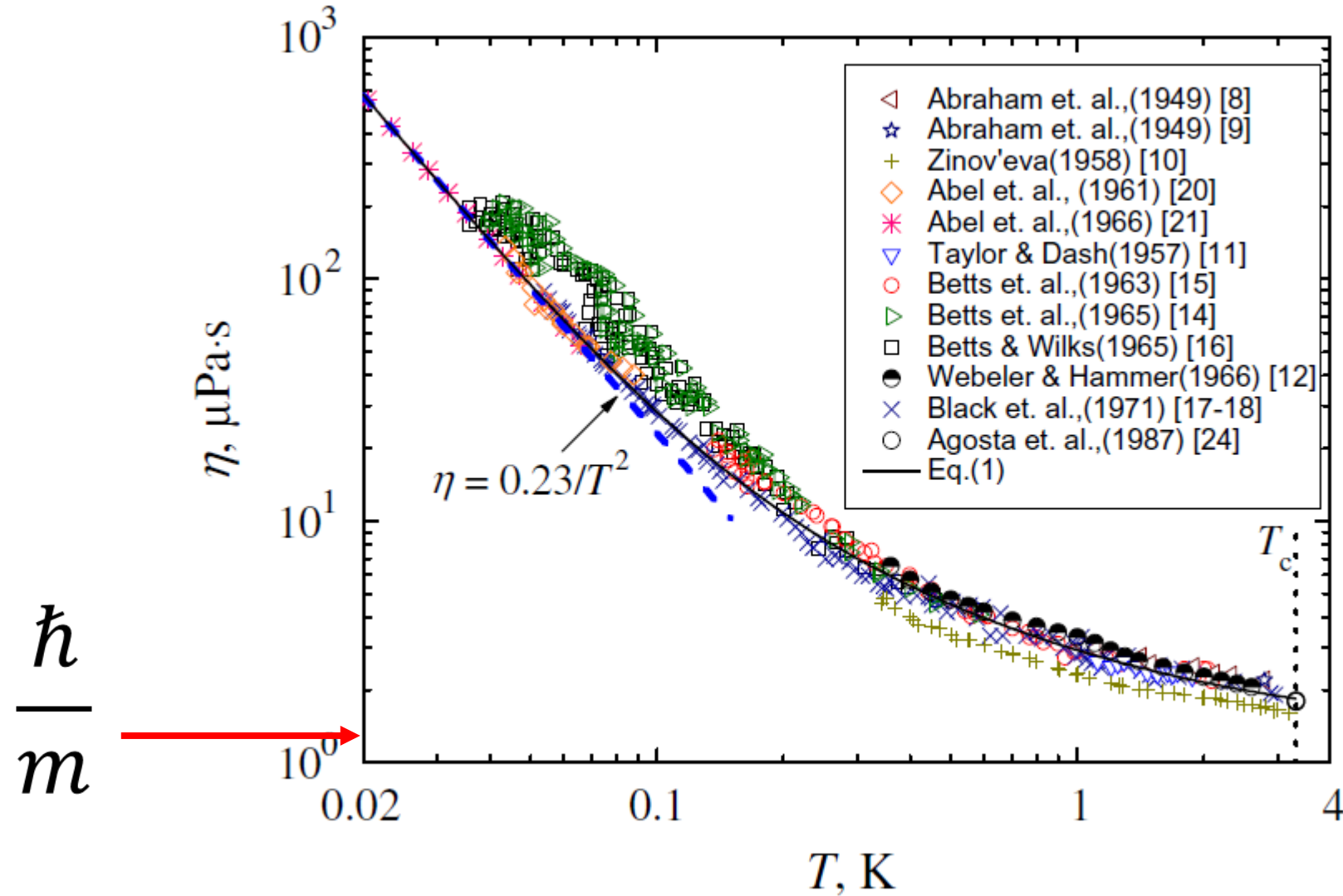


FIG. 12.3. The recommended values of the kinematic viscosity of liquid ^4He , $\nu = \eta/\rho$, as a function of temperature at the saturated vapor pressure.

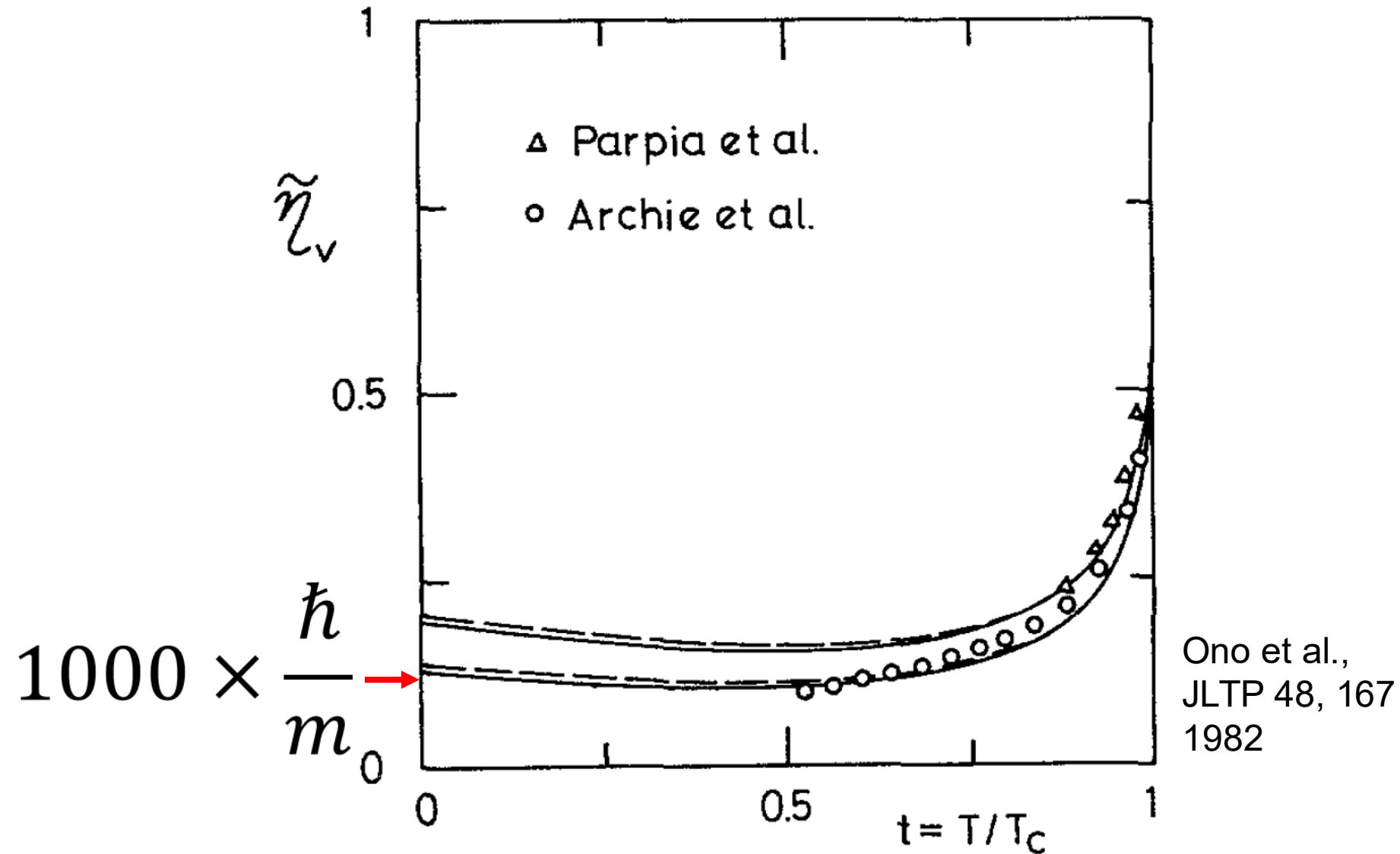
Viscosity of Superfluid Helium-3

Helium-3 above T_c : Fermi Liquid behavior

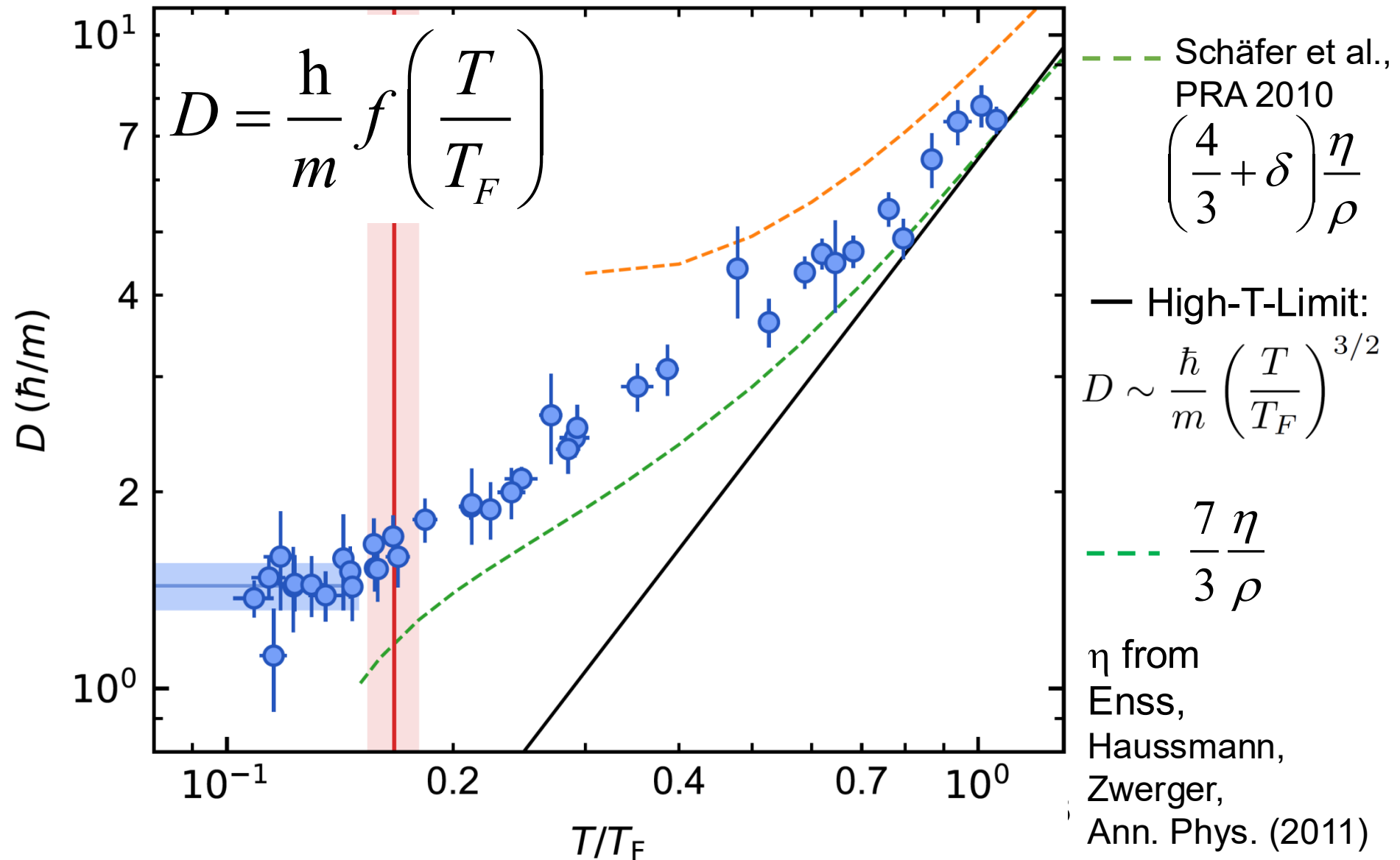


Viscosity of Superfluid Helium-3

Helium-3 below T_c : Sudden decrease, then constant



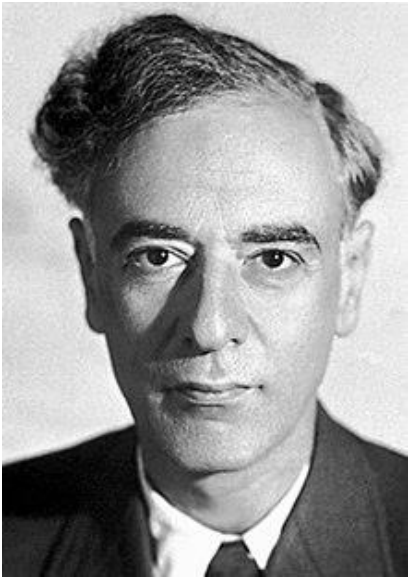
Quantum Limited Sound Diffusion



Two-Fluid Model for Superfluids

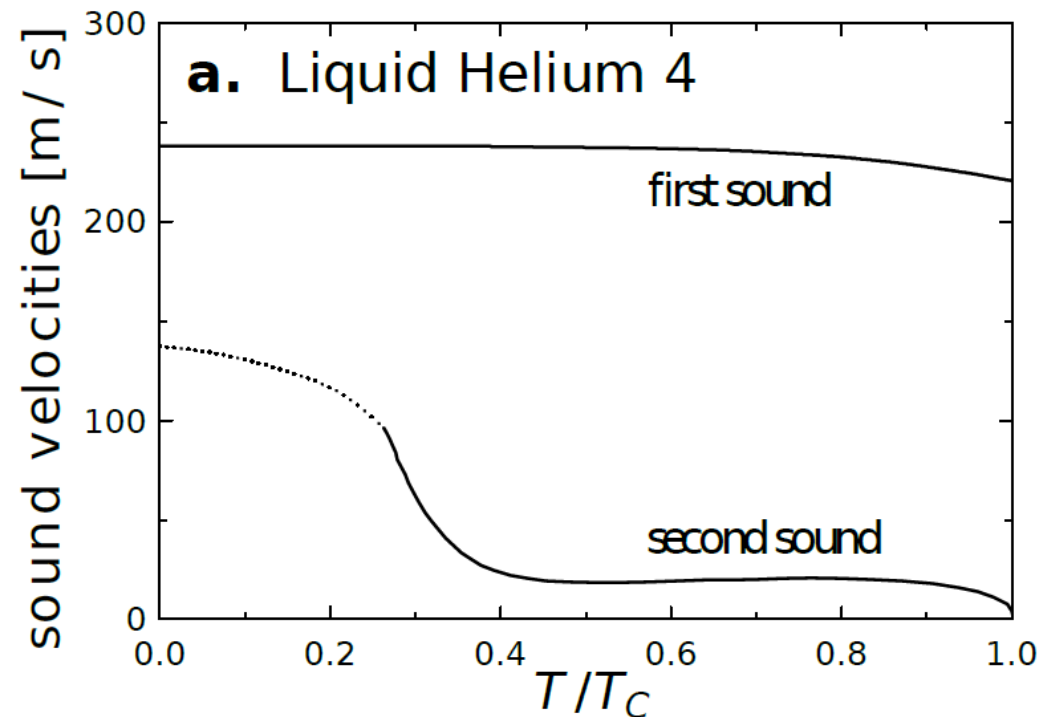


Laszlo Tisza



Lev Landau

Superfluid + Fluid of excitations
→ Two Sound Modes!



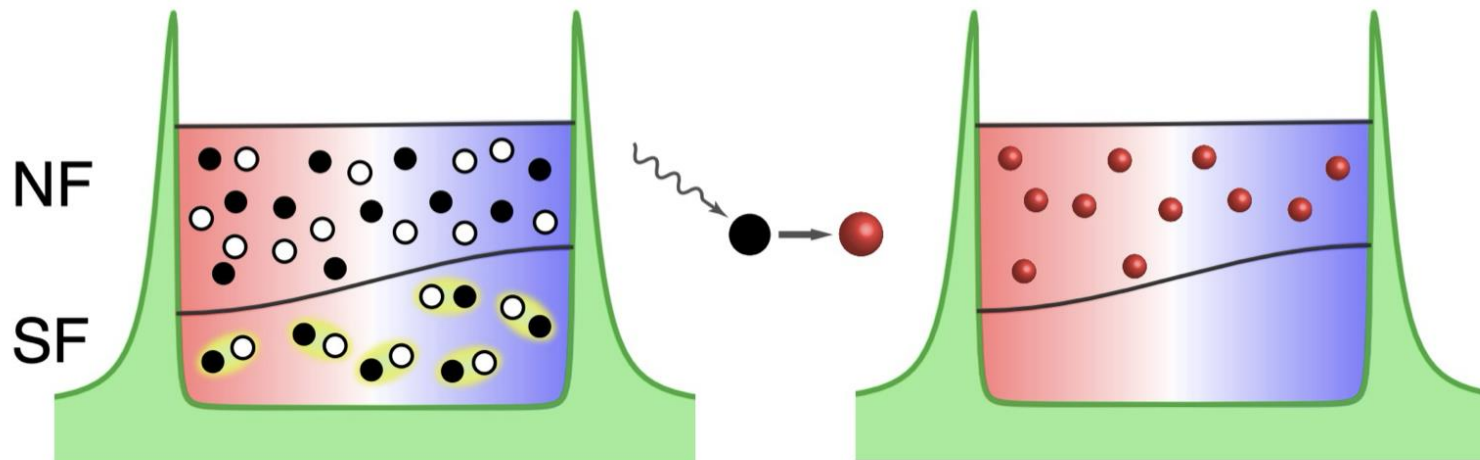
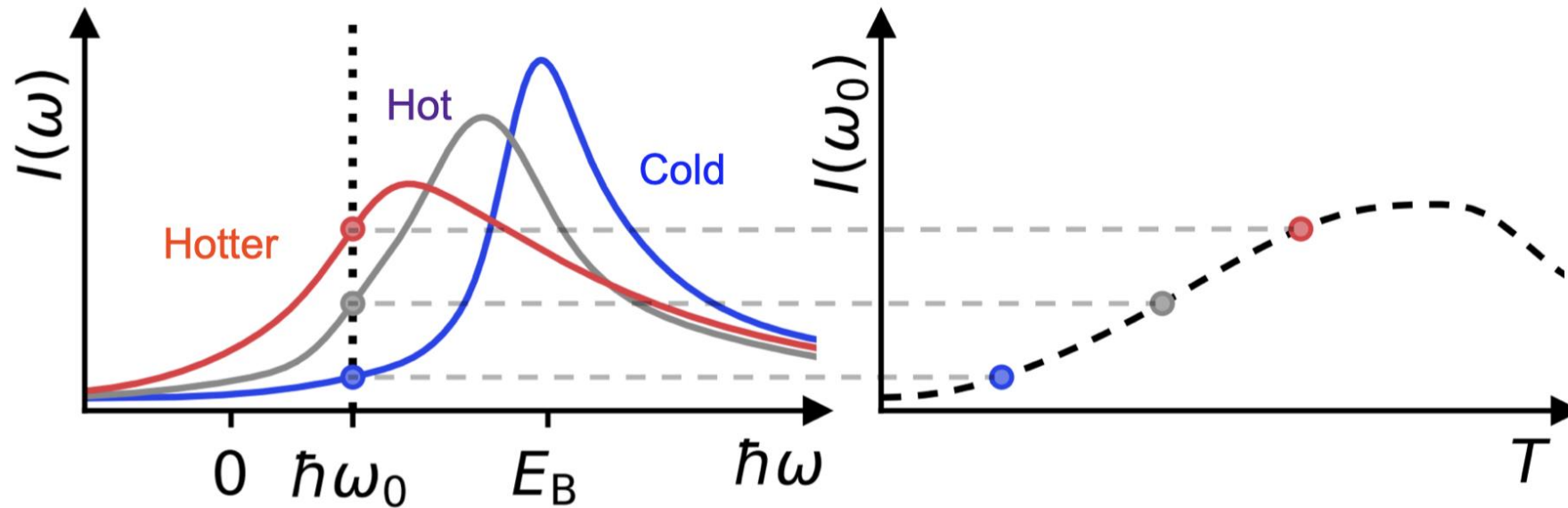
Observation of Second Sound in Quasi-1D geometry:
Sidorenkov et al., Grimm, Stringari&Piatevskii, Nature 2013

On bosons: weakly interacting situation: JILA, MIT

Hydrodynamic gas: van der Straten, 2D Bose Gas: Cambridge, ENS

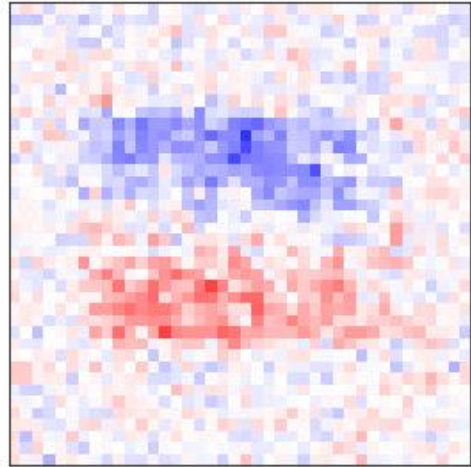
A local thermometer for heat transport

RF absorption spectra are temperature dependent

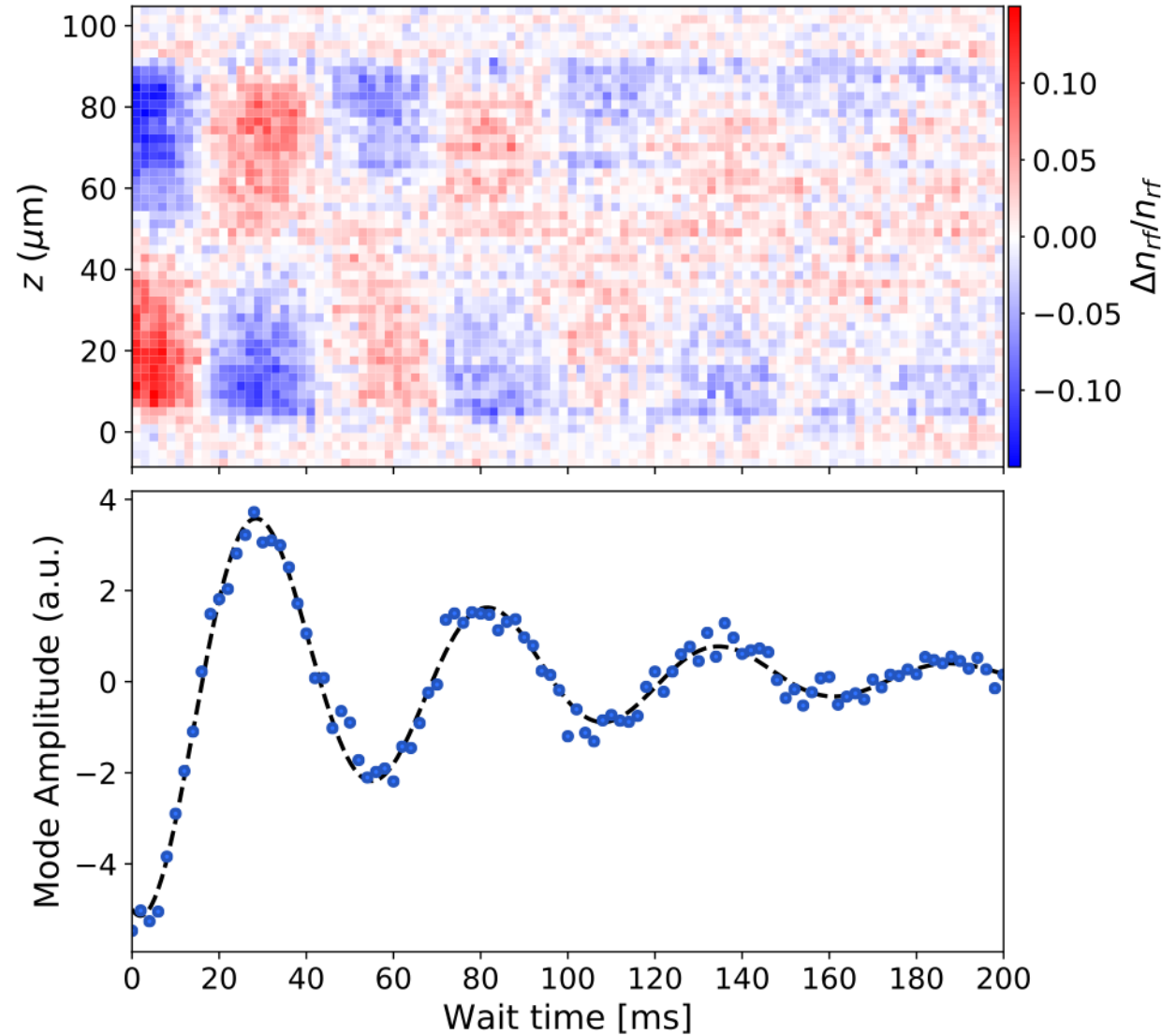
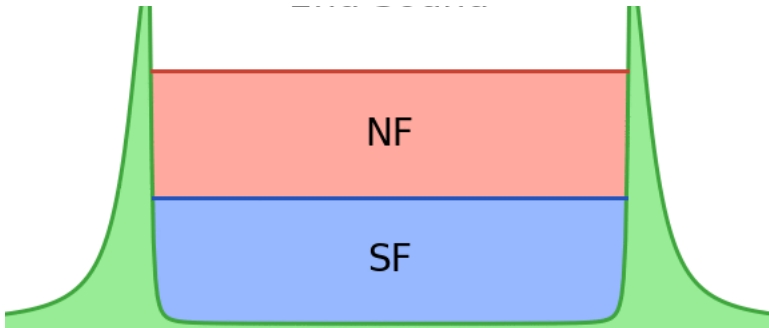


Second Sound

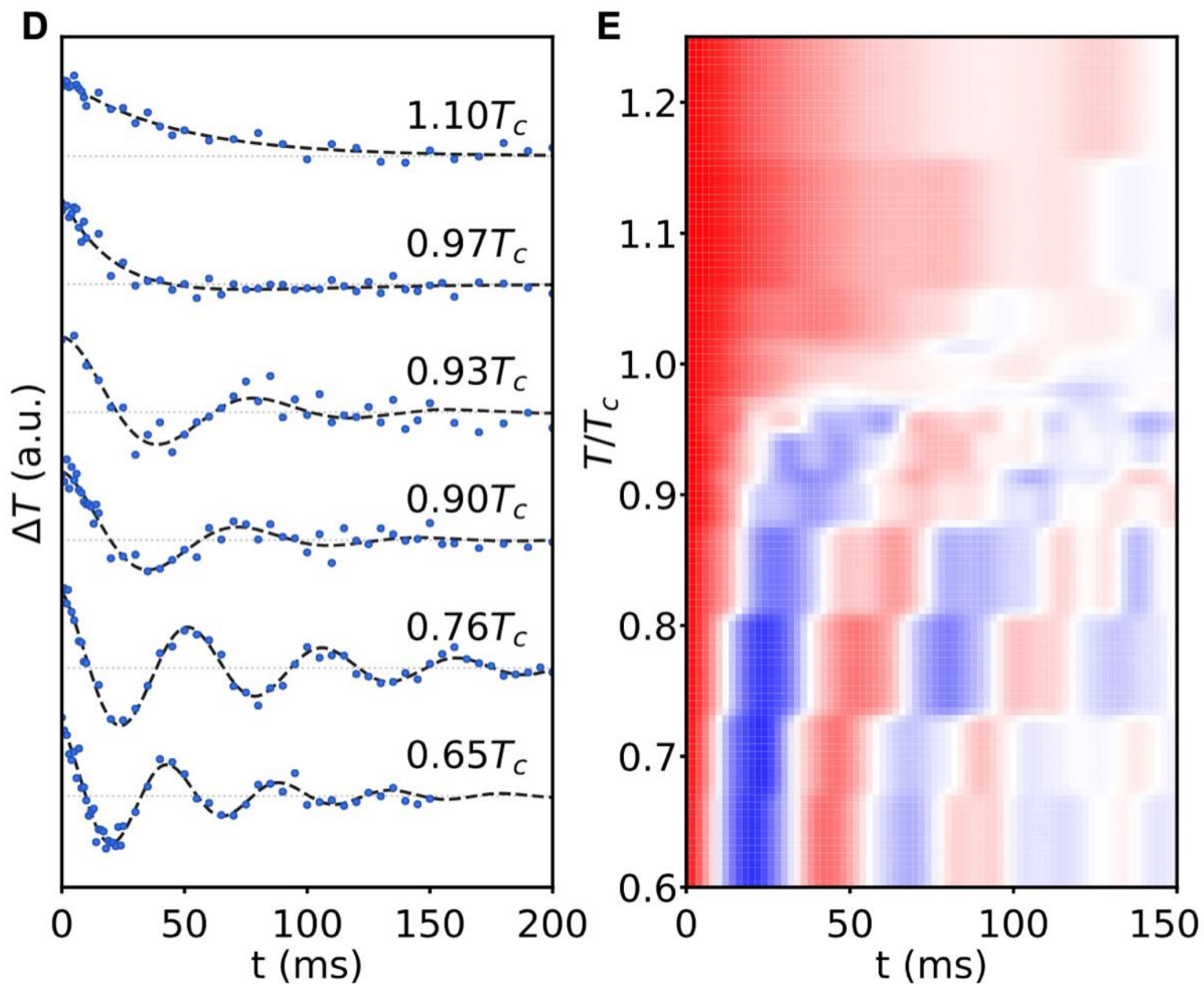
Temperature Probe



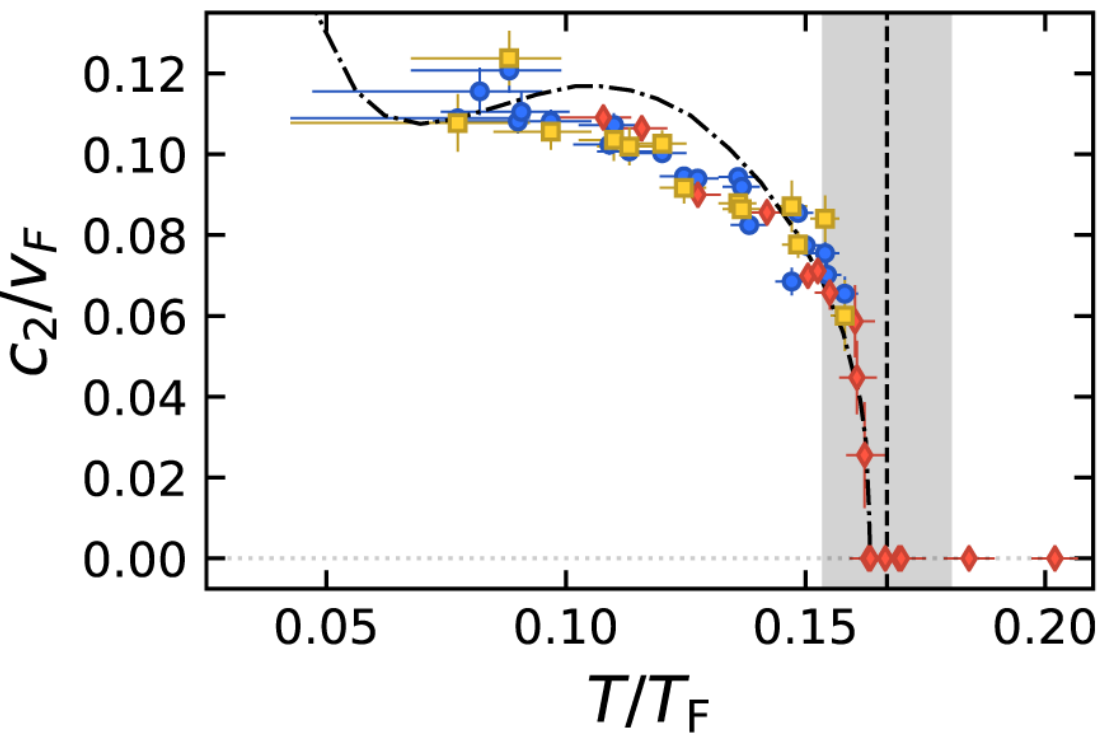
$t = 0$ ms



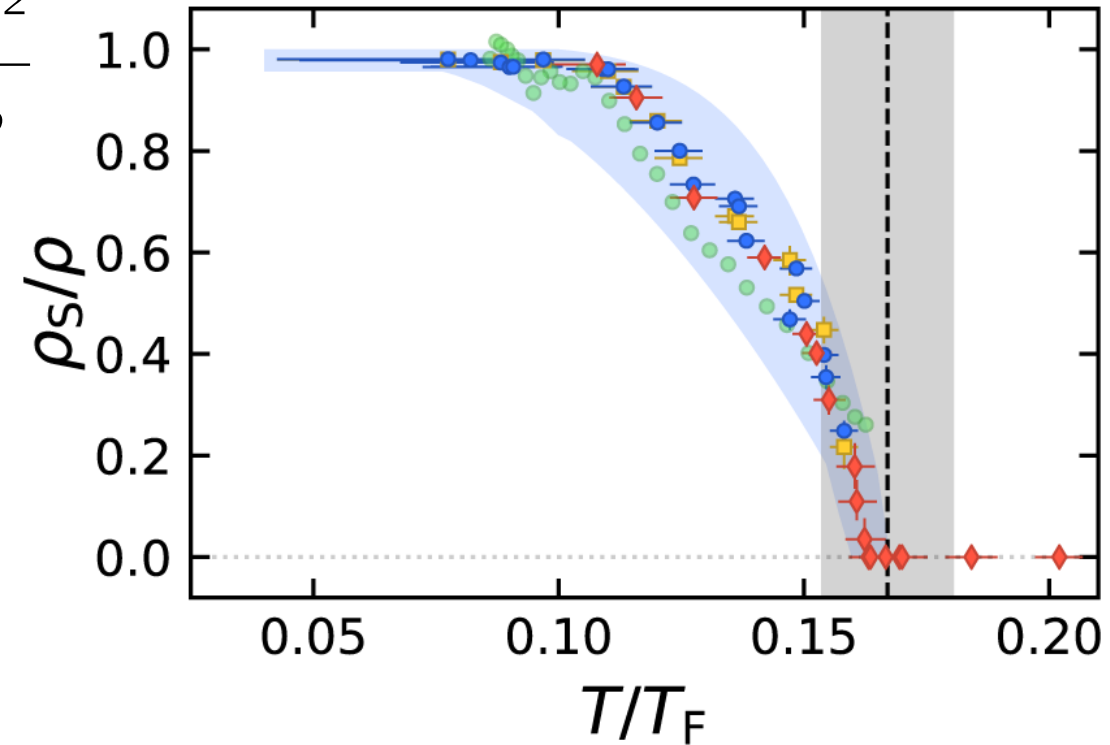
Transition from normal to superfluid



Superfluid Fraction from Second Sound



$$C_2^2 = \frac{\rho_S}{\rho_N} \frac{T S^2}{C_p}$$



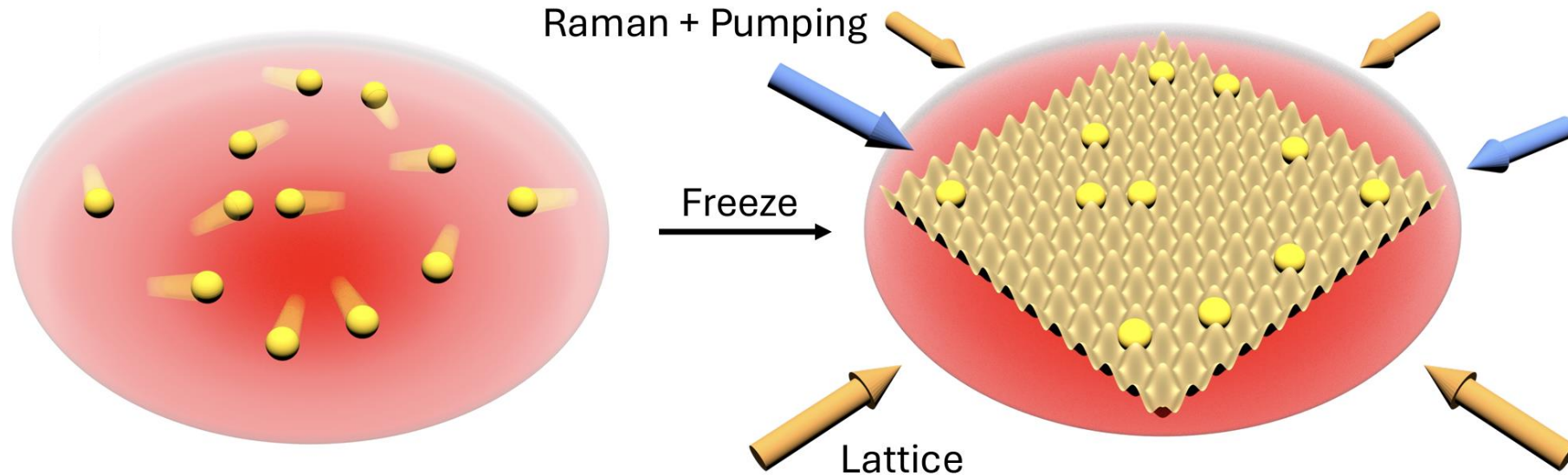
- Oscillation Frequency Scan
- Free evolution, resonant shaking
- ◆ Free evolution, local heater

--- E. Taylor, et.al., Phys. Rev. A **80**, 053601 (2009).

● Second sound in quasi-1D Fermi gas
L. Sidorenkov, *et al.*, *Nature* **498**, 78–81 (2013).

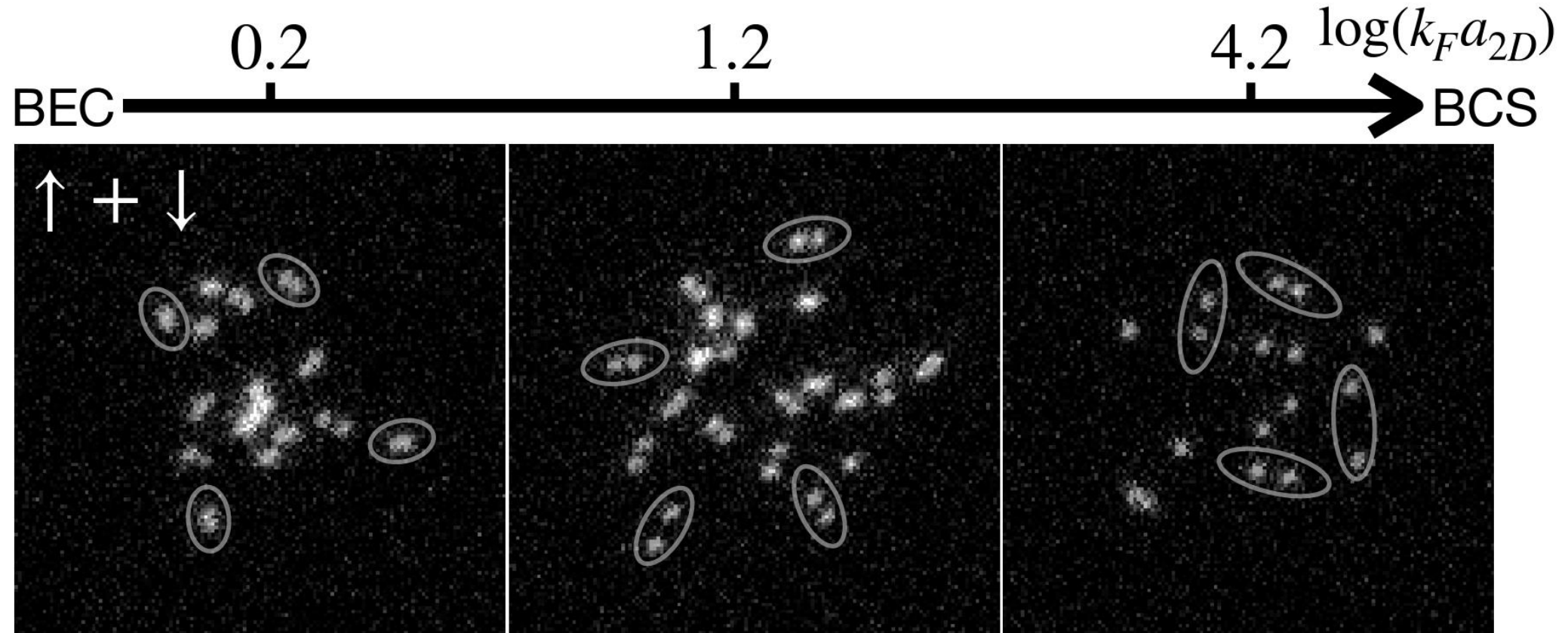
A microscopic view of Fermion Pairs

Apply the continuum gas microscope on a 2D Fermi gas in the BEC-BCS Crossover

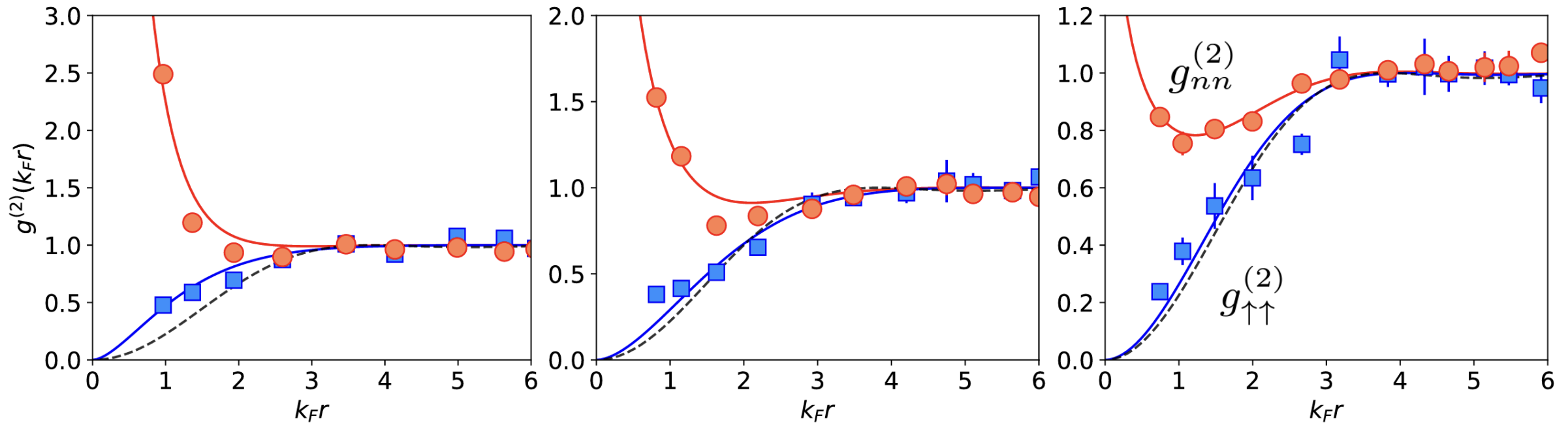


A microscopic view of Fermion Pairs

Apply the continuum gas microscope on a 2D Fermi gas in the BEC-BCS Crossover



Pair correlation function $g^{(2)}(r)$



- Like fermions repel $\rightarrow$ Fermi Hole
- Unlike fermions attract $\rightarrow$ Contact+Pairing

Access to microscopic correlations
in strongly interacting quantum gases

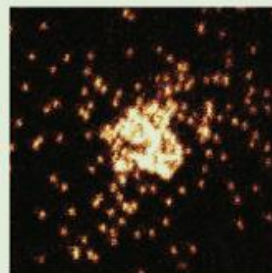
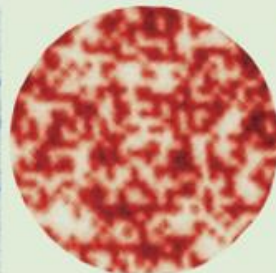
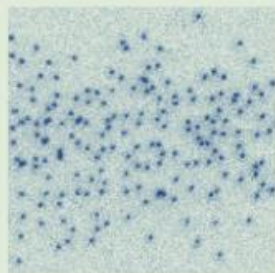
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PHYSICAL REVIEW LETTERS

Published week ending

9 MAY 2025

PRL 134 (18), 180201–189901, 9 May 2025 (406 total pages)



Yefsah group, PRL **134**, 183403 (2025) (free Fermions)
Ketterle group, PRL **134**, 183401 (2025) (Bosons)

R Yao, S Chi, M Wang, RJ Fletcher, MZ,
PRL **134**, 183402 (2025) (Bosons + Fermions)

18

Published by
American Physical Society

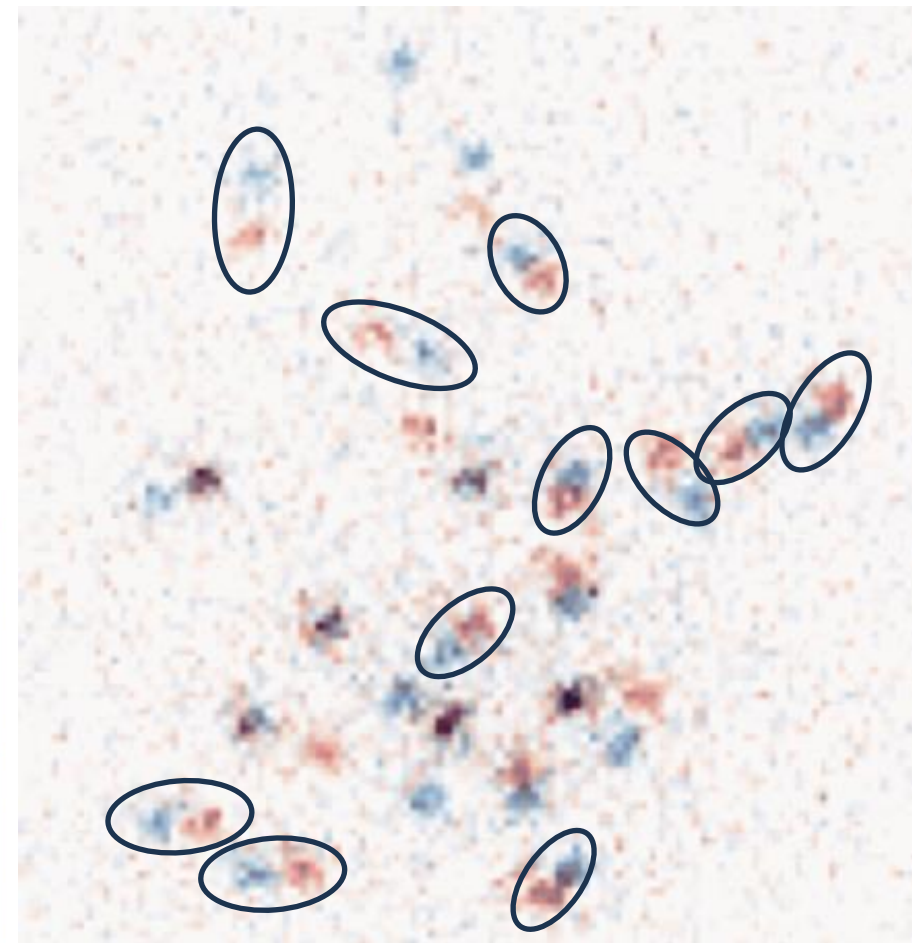
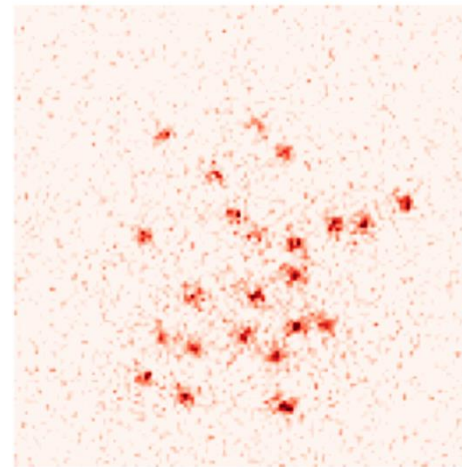
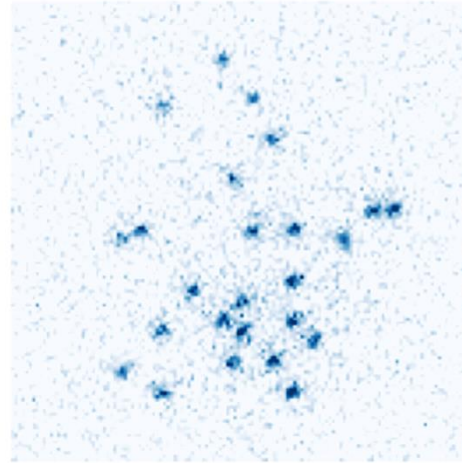
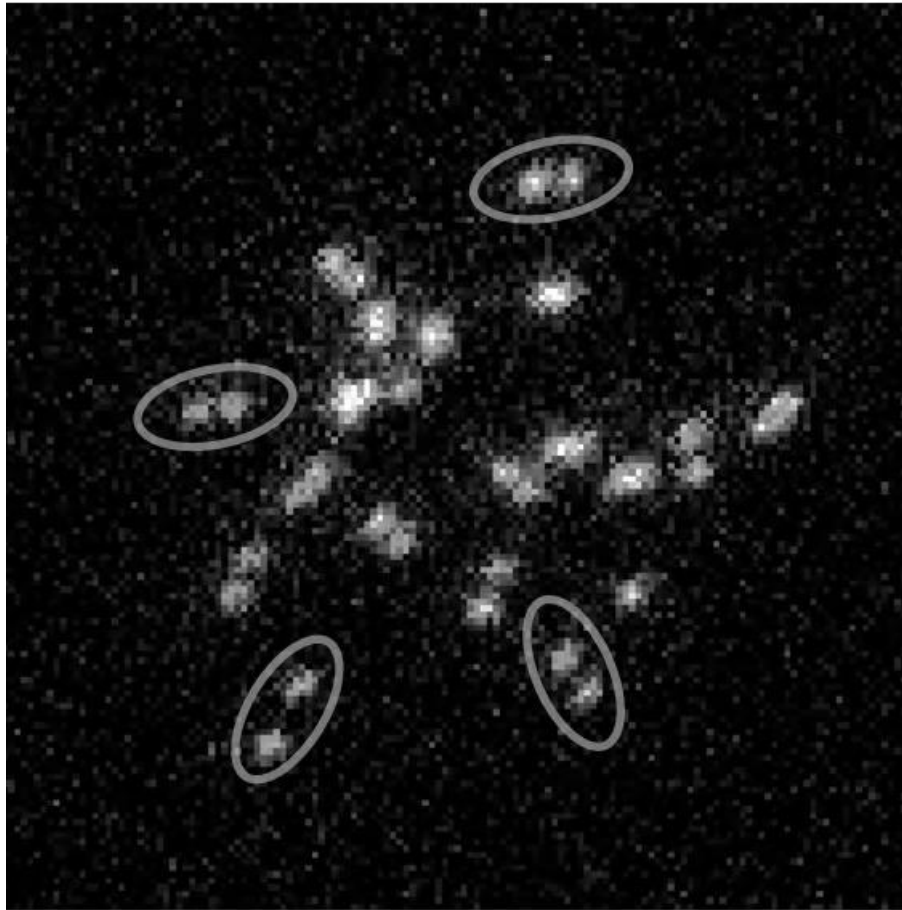


Volume 134, Number 18

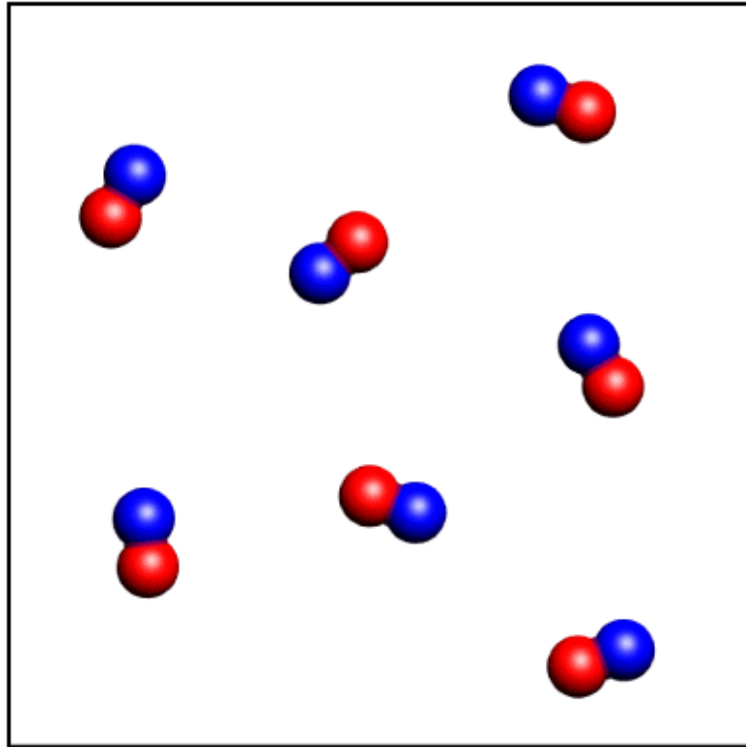
From black & white to color TV



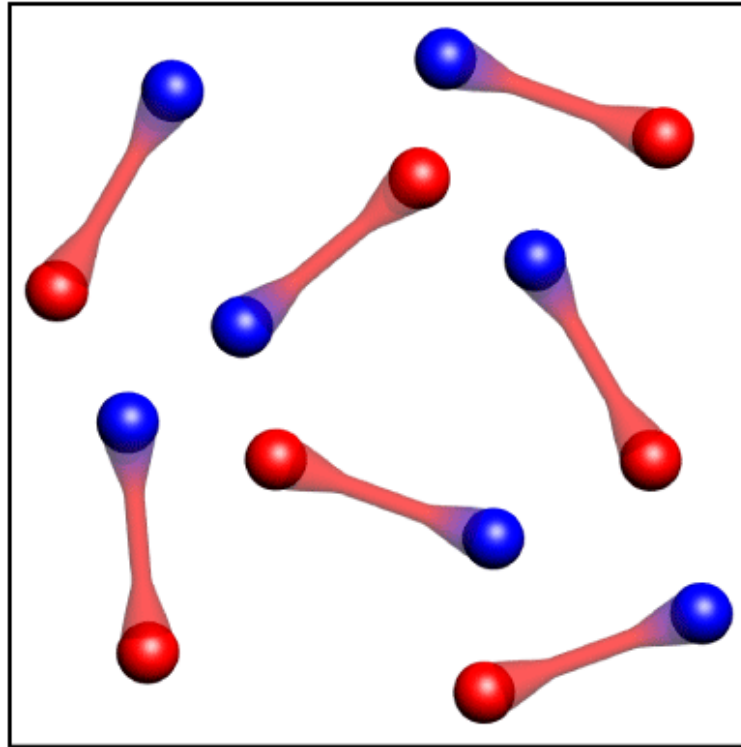
A microscopic view of Fermion Pairs



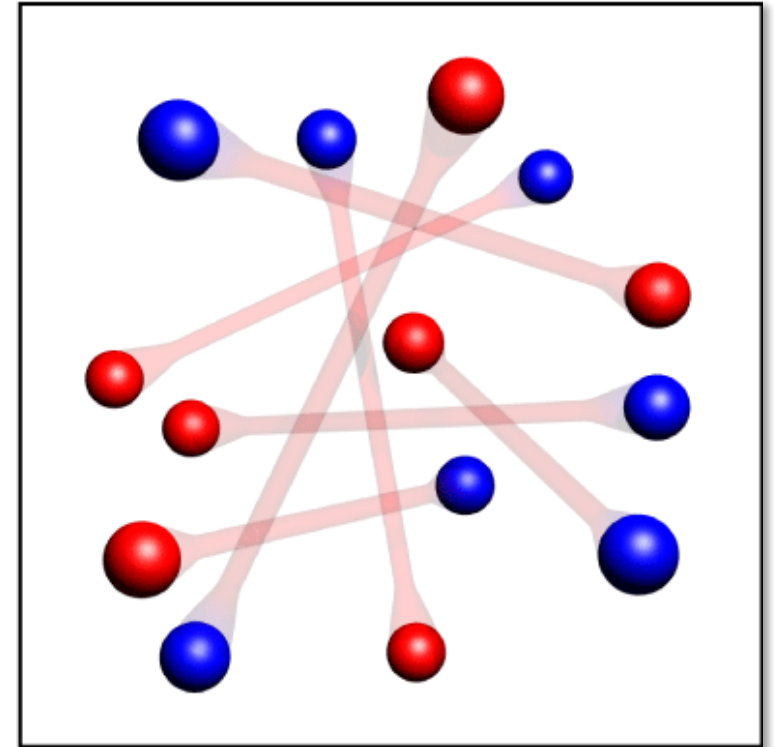
Fermion pairs in the BEC-BCS Crossover



BEC of Molecules

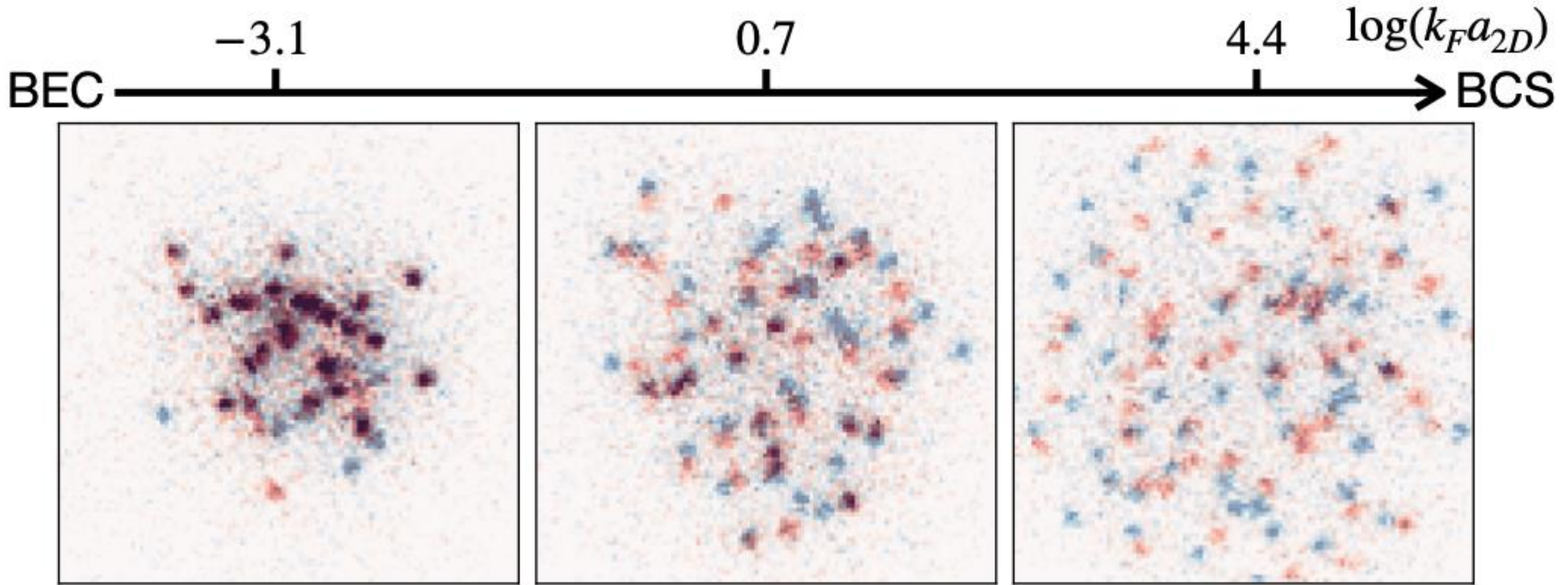


Crossover Superfluid



BCS state

Fermion pairs in the BEC-BCS Crossover



The contact

- Is a thermodynamic variable:
measures change of energy with respect to interaction strength:

$$C = \frac{\Delta \text{ Energy}}{\Delta \text{ Interaction Strength}}$$

- measures the strength of short-range correlations:

$$C \sim \langle \psi_{\uparrow}^{\dagger}(0) \psi_{\downarrow}^{\dagger}(0) \psi_{\downarrow}(0) \psi_{\uparrow}(0) \rangle$$

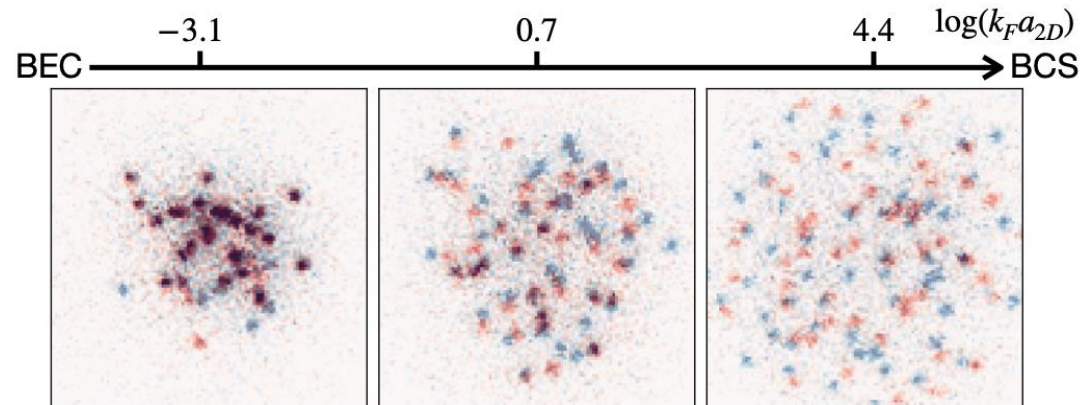
- governs the tail of the momentum distribution:

$$C = n(k) k^4 \quad \text{at large } k$$

- Controls e.g. the wings of RF spectra, photoassociation strength
- etc... (see Shina Tan 2005/2008, Baym et al. 2007, Werner Castin 2012, et al. mult.)

The contact

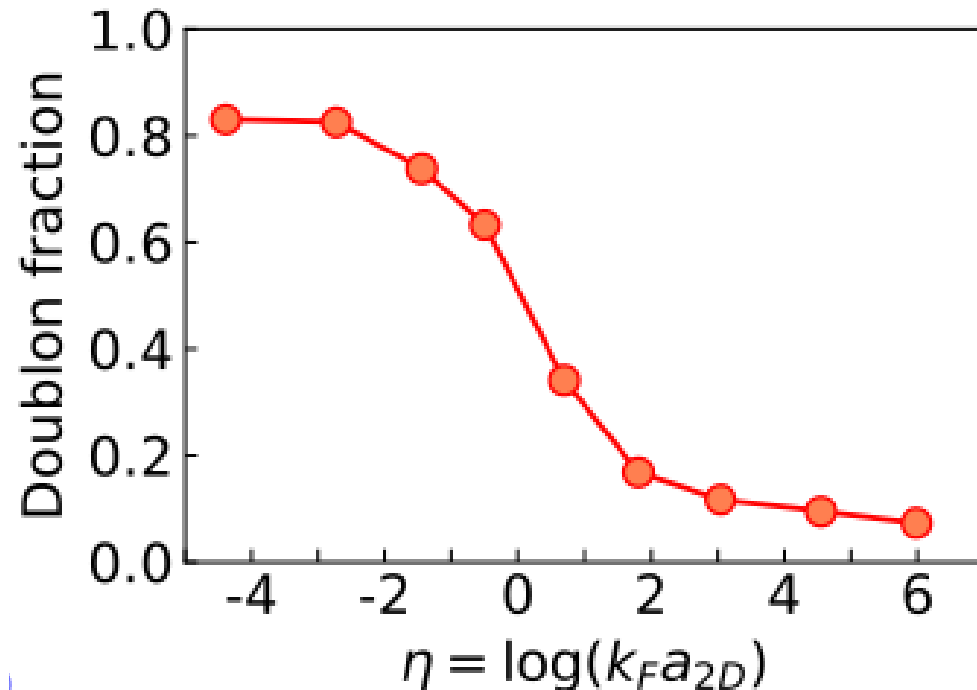
... can now just be read off from the number of doublons in the images!



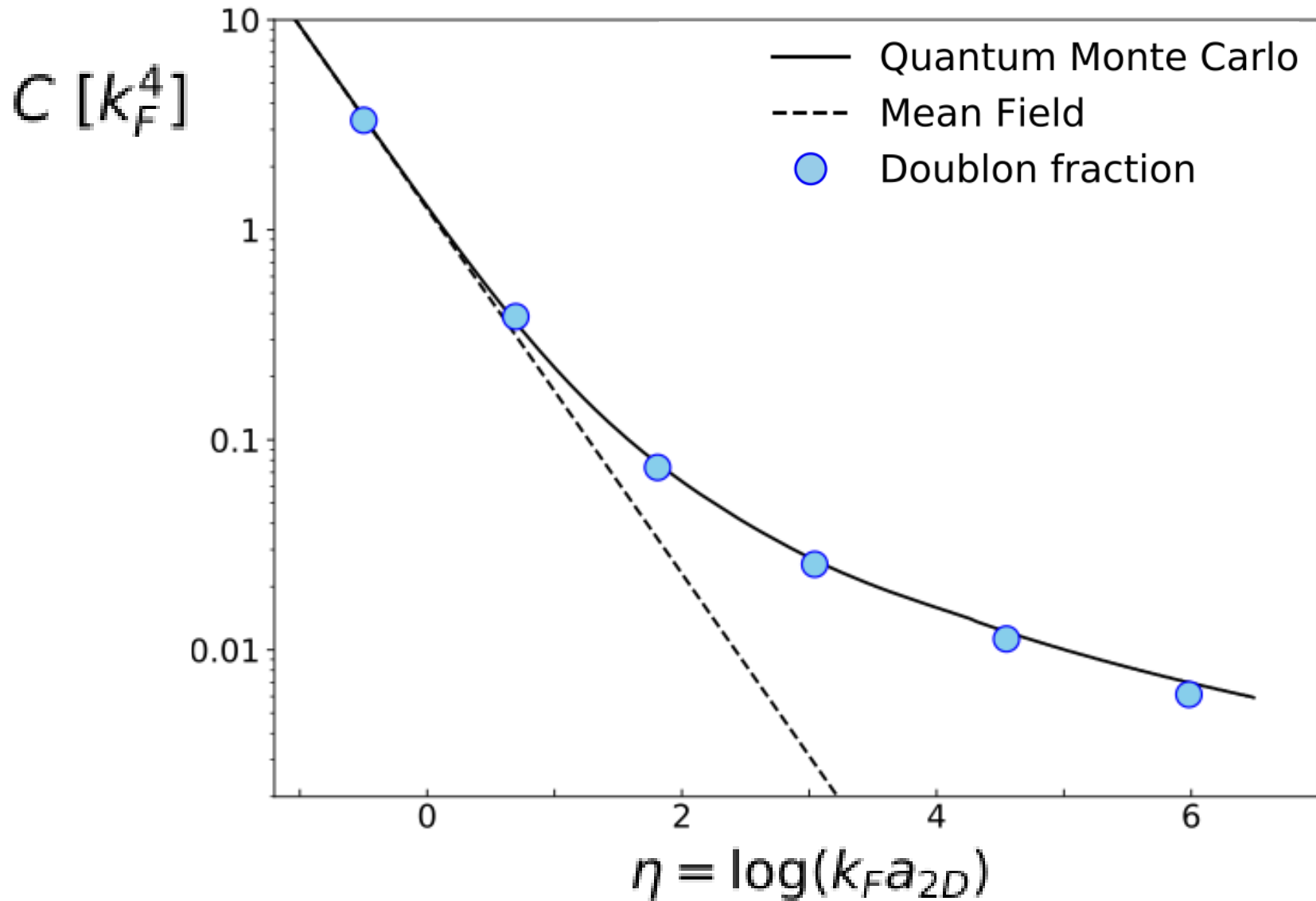
$$g_{\uparrow\downarrow}(r) \sim C * \text{two-body piece}$$

→ By integration, the contact measures the number of pairs in a given region

→ Doublon fraction in a lattice cell gives contact



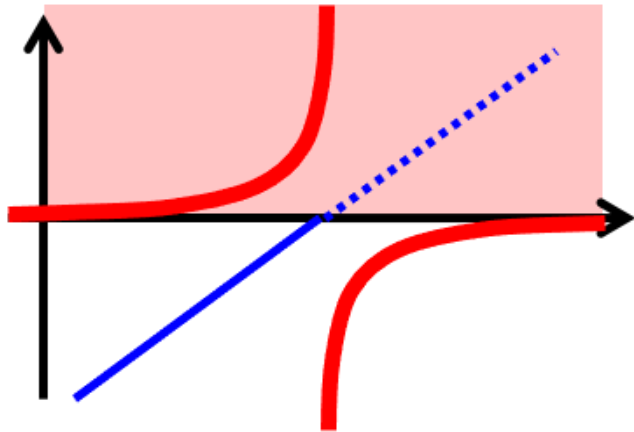
The contact



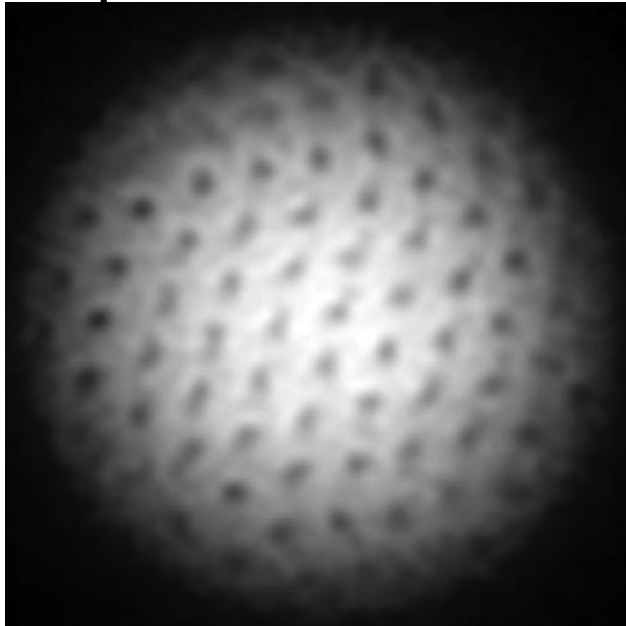
Alternatively: Use behavior of $g_2(r)$, Yao, Chi, Wang, Fletcher, MZ, PRL **134**, 183402 (2025)
Or use difference in total and spin-projected numbers: Yefsah group: arXiv:2504.01885

Realizing strongly interacting Fermi systems

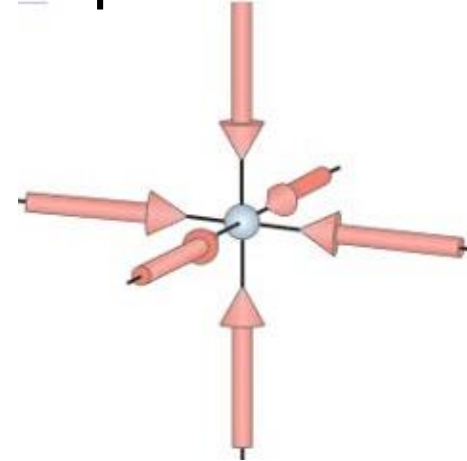
Feshbach Resonances



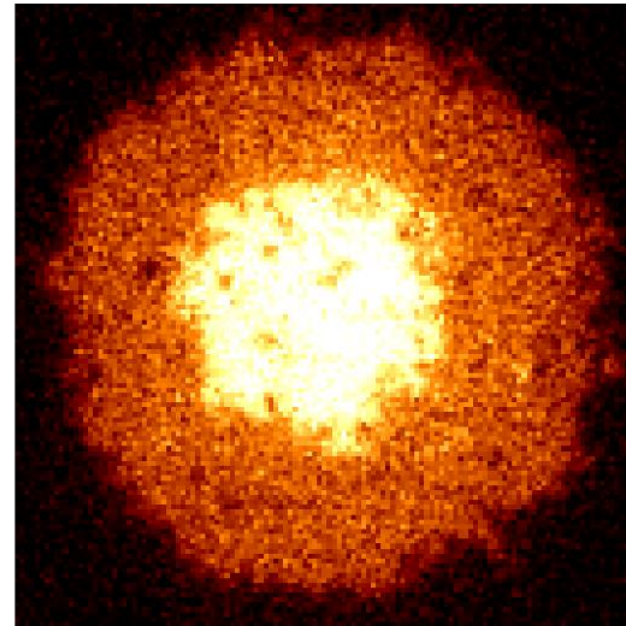
Superfluid Fermi Gas



Optical Lattices



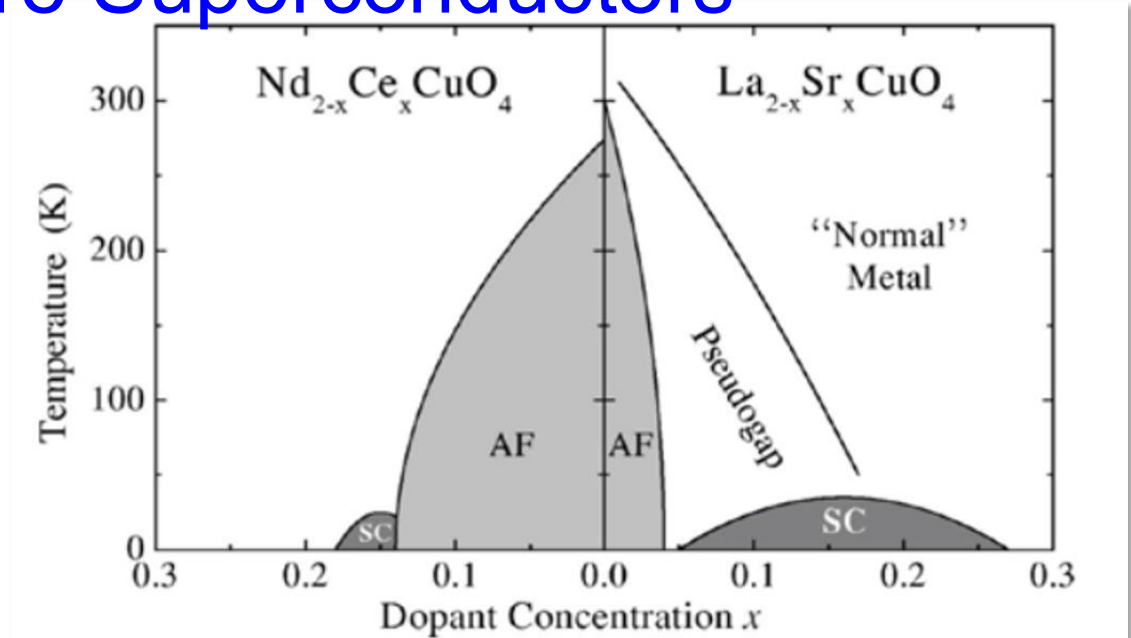
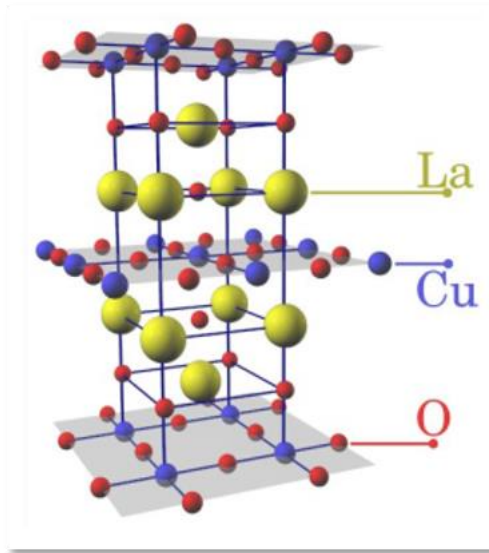
Fermi-Hubbard Gas



Quantum Simulation of the Fermi-Hubbard Model

$$H = -t \sum_{\langle i,j \rangle, \sigma} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow}$$

Introduced to describe strongly correlated electrons
May contain essential features of
High-Temperature Superconductors

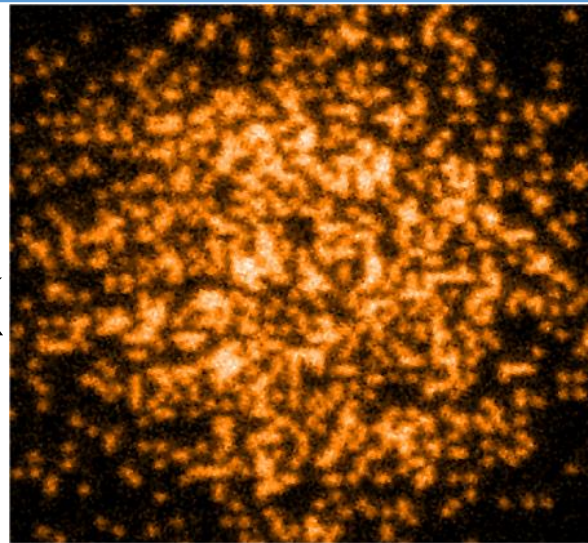


Damascelli et al., RMP 2003

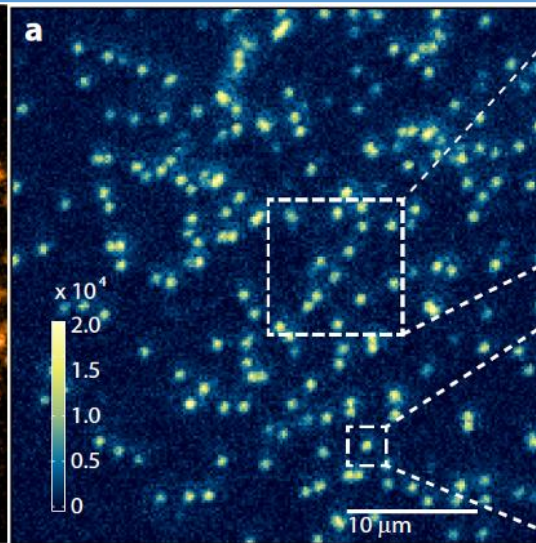
Many cold atom experiments: ETH, MPQ, Rice, MIT, Harvard, Florence, Bonn, Princeton, USTC, ...

Fermi Lattice Gas Microscopes

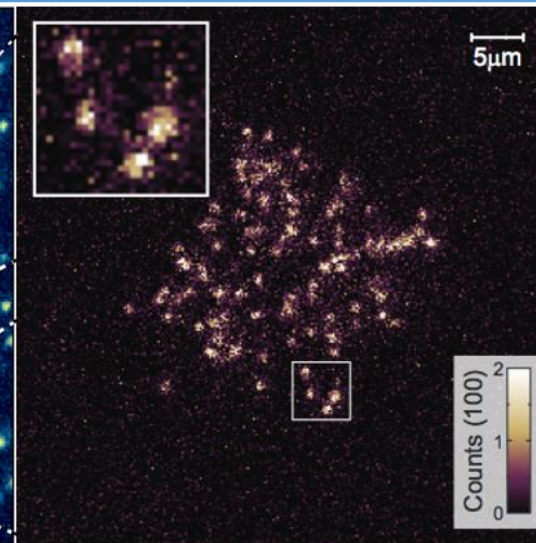
^{40}K



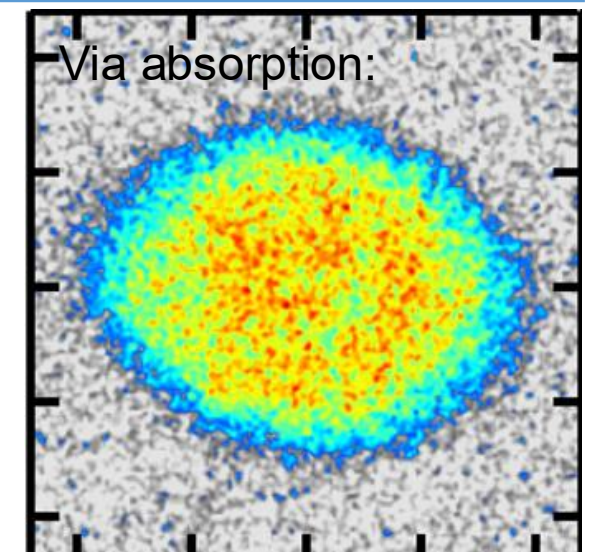
Zwierlein group, MIT
PRL **114**, 193001 (2015)



Kuhr group, Strathclyde
Nat. Phys. **11**, 738 (2015)

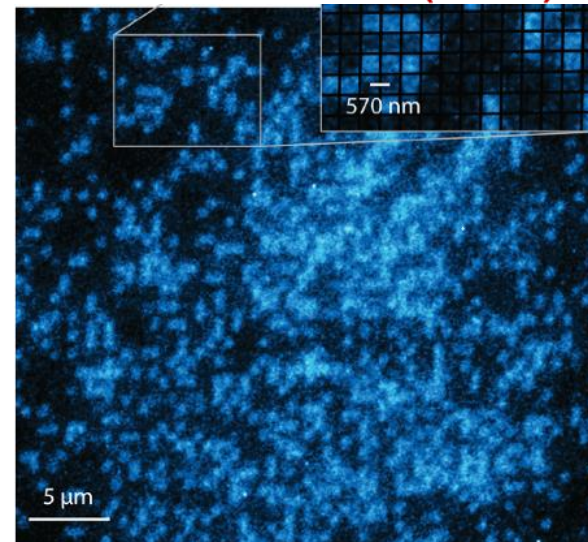


Thywissen group, Toronto,
PRA **92**, 063406 (2015)

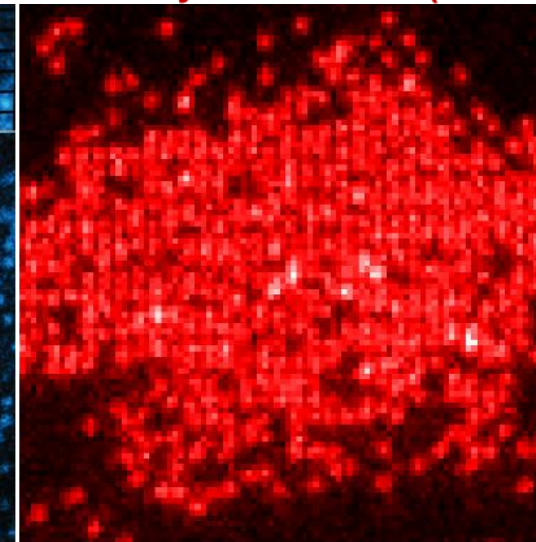


Köhl group, Bonn,
PRL **116**, 175301 (2016)

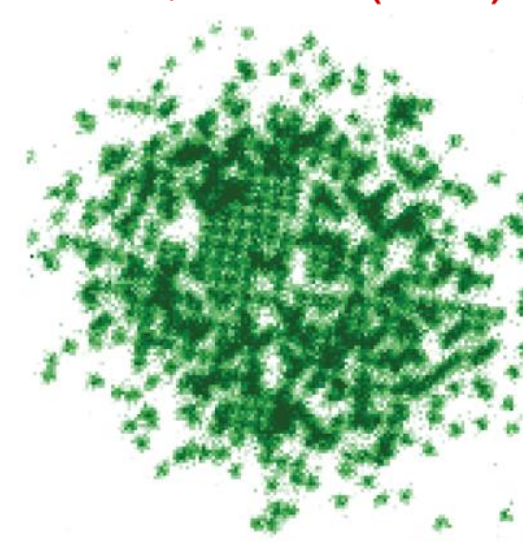
^6Li



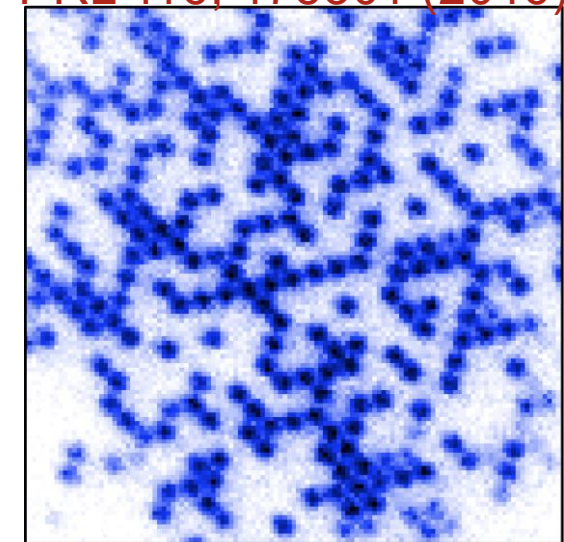
Greiner group, Harvard
PRL **114**, 213002 (2015)



Gross/Bloch group, MPQ
PRL **115**, 263001 (2015)



Bakr group, Princeton
Science, **357**, 1385-1388 (2017)



Schauß group, Virginia
PRX Q **2**, 020344 (2021)



Thomas Hartke



Botond Oreg



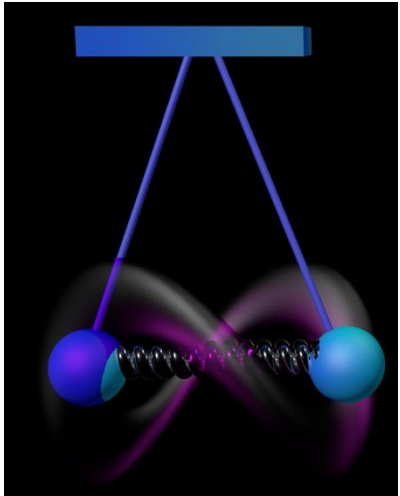
Carter Turnbaugh



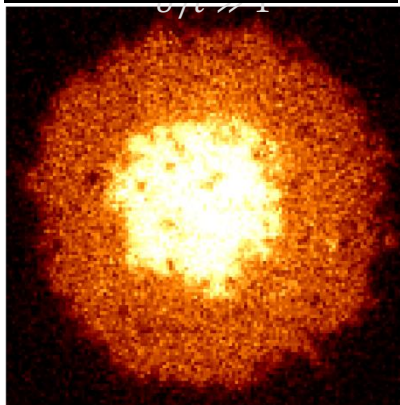
Ningyuan Jia



Jens Hertkorn

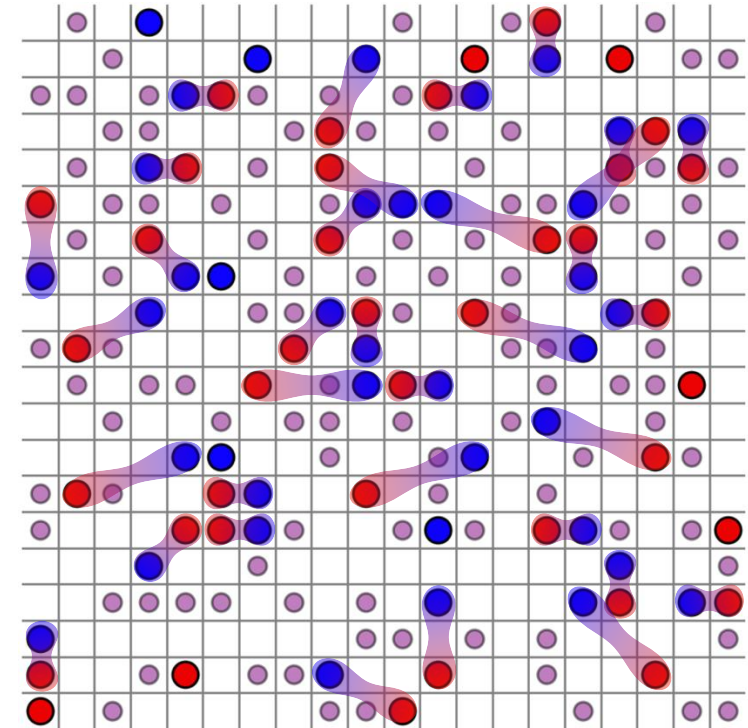


Quantum Register:
Nature 601, 537 (2022)



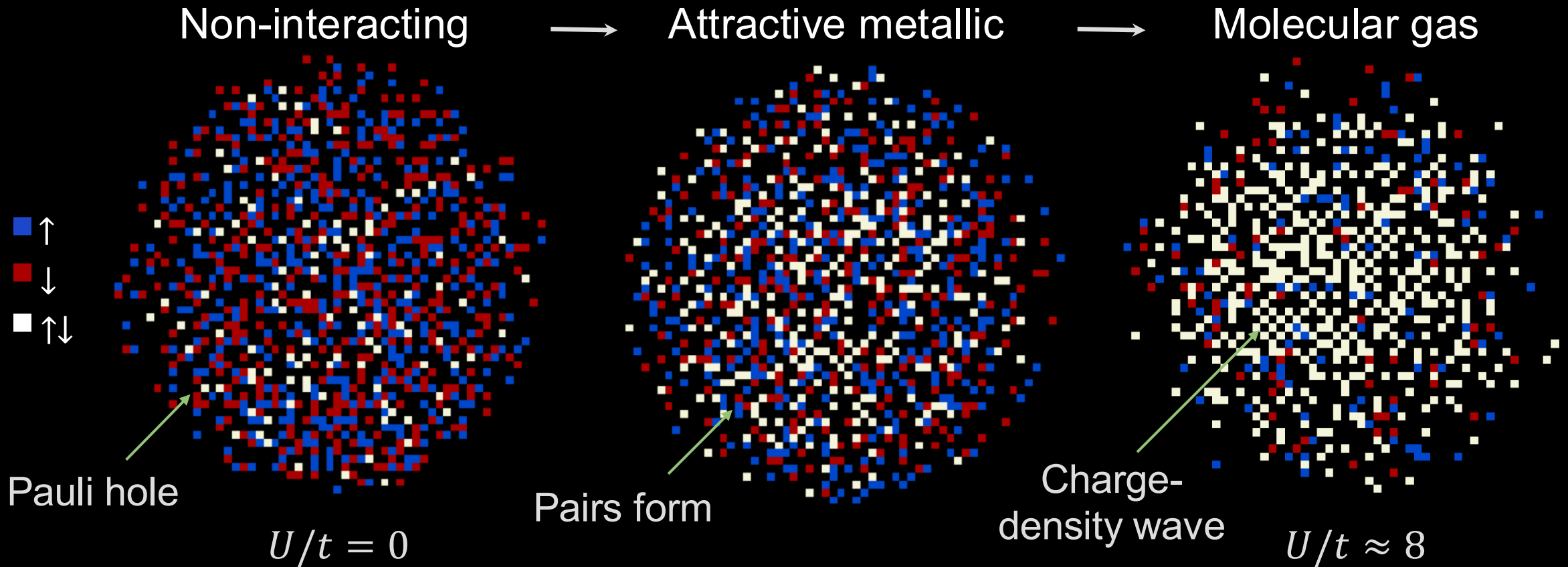
Doublon-Hole Correlations:
PRL 125, 113601 (2020)

Fermion Pairing:
Science 381, 82 (2023)

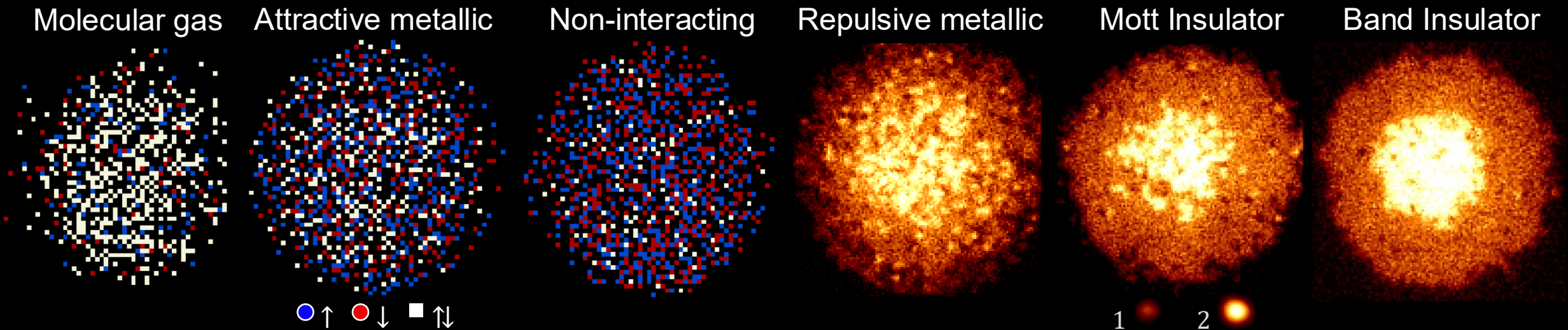
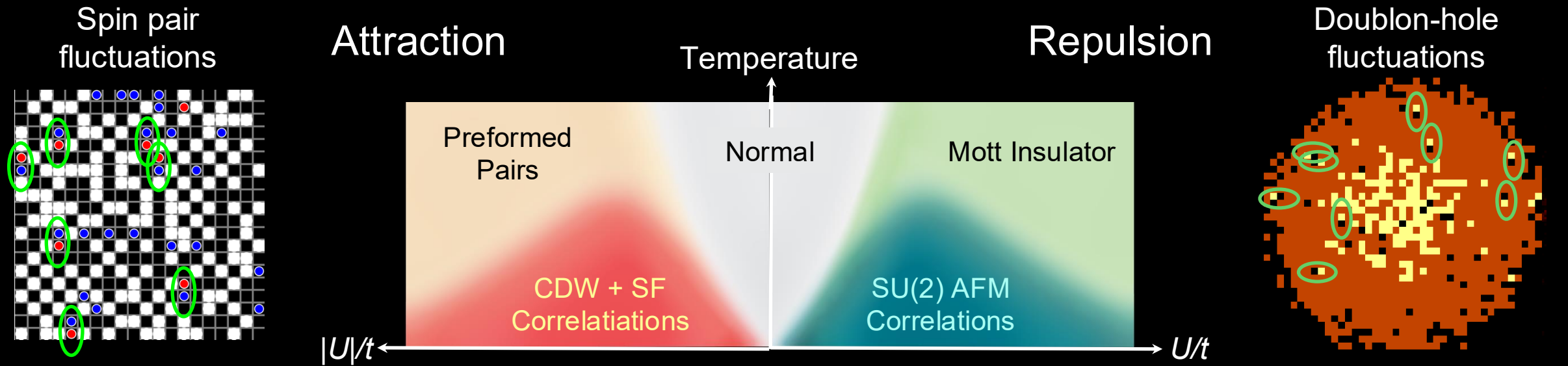


Spin Doping: arXiv:2511.10605 (2025)

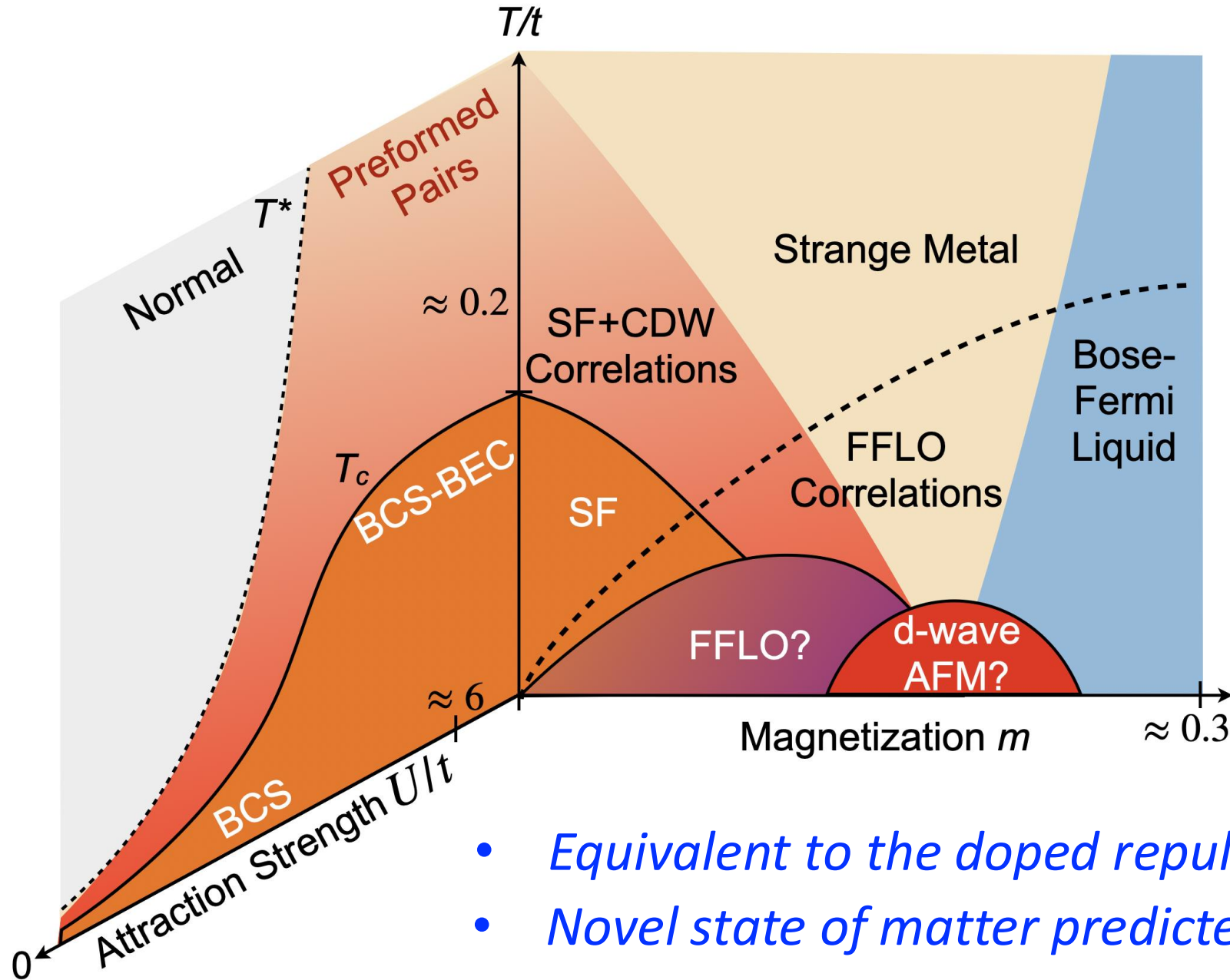
Attractive Fermi-Hubbard



Phases of the Fermi-Hubbard Model

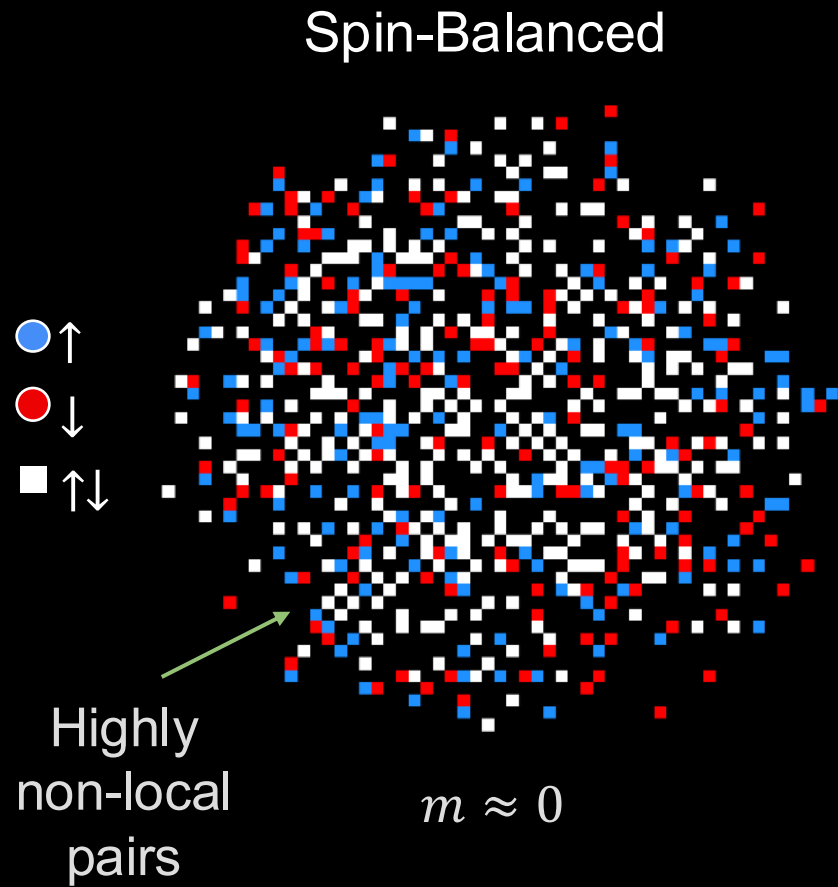


Adding loners: Spin-doped attractive Hubbard gas

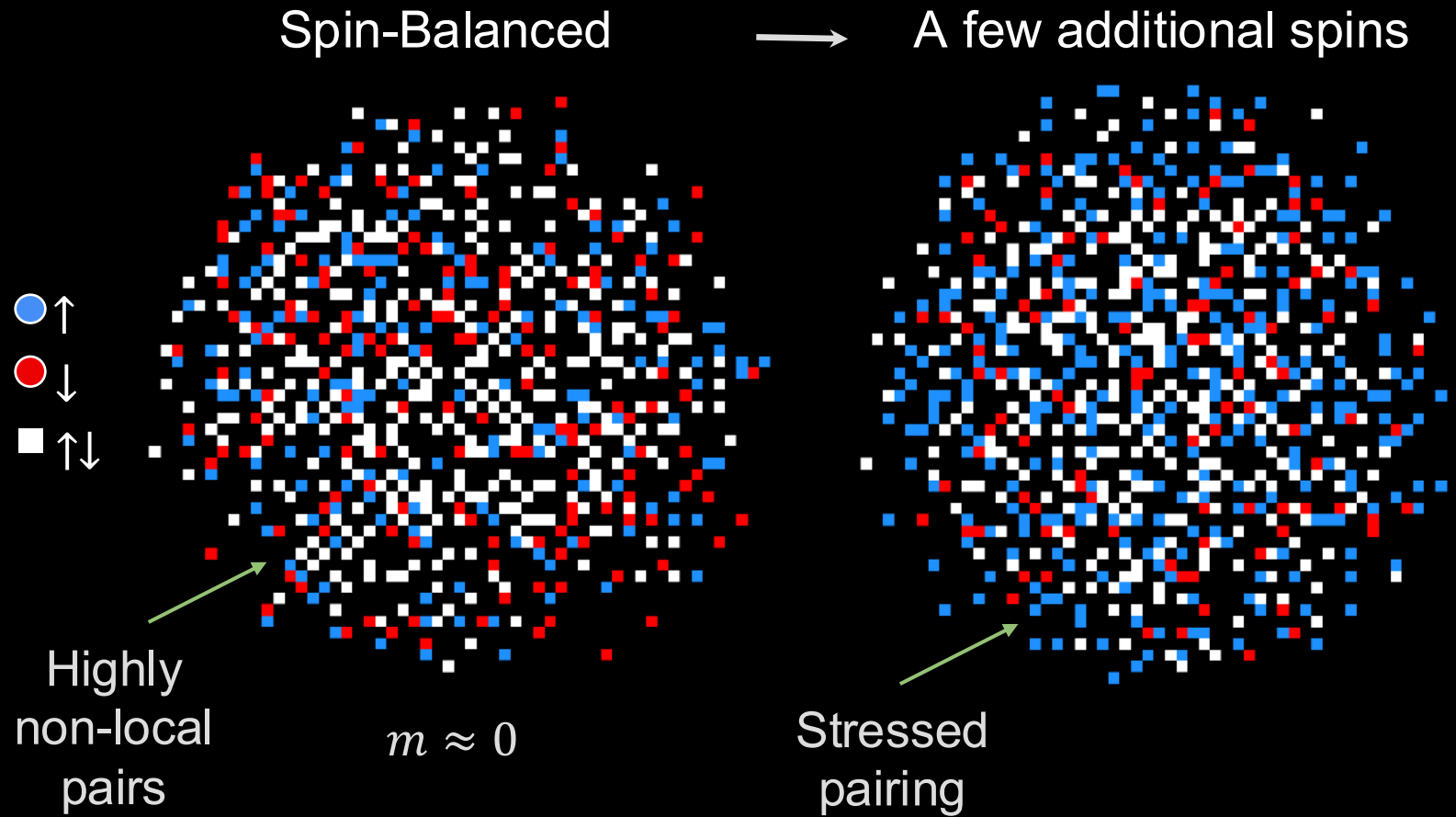


- *Equivalent to the doped repulsive Hubbard gas*
- *Novel state of matter predicted: FFLO state*

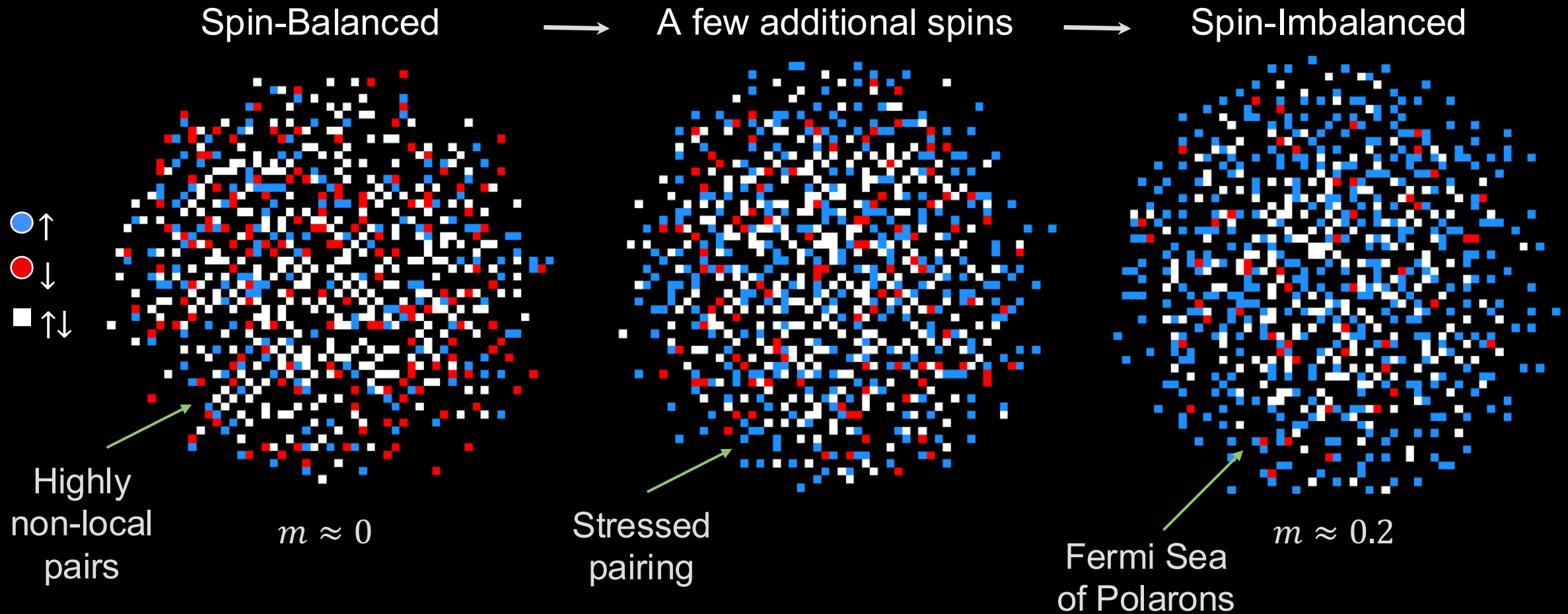
Attractive Fermi-Hubbard



Attractive Fermi-Hubbard + Spin Imbalance



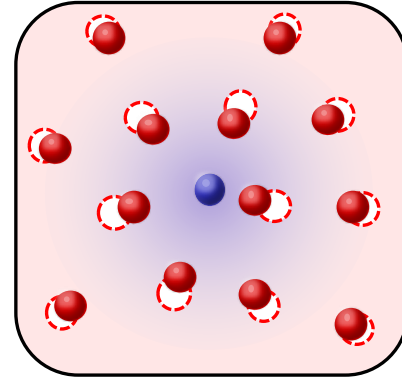
Attractive Fermi-Hubbard + Spin Imbalance



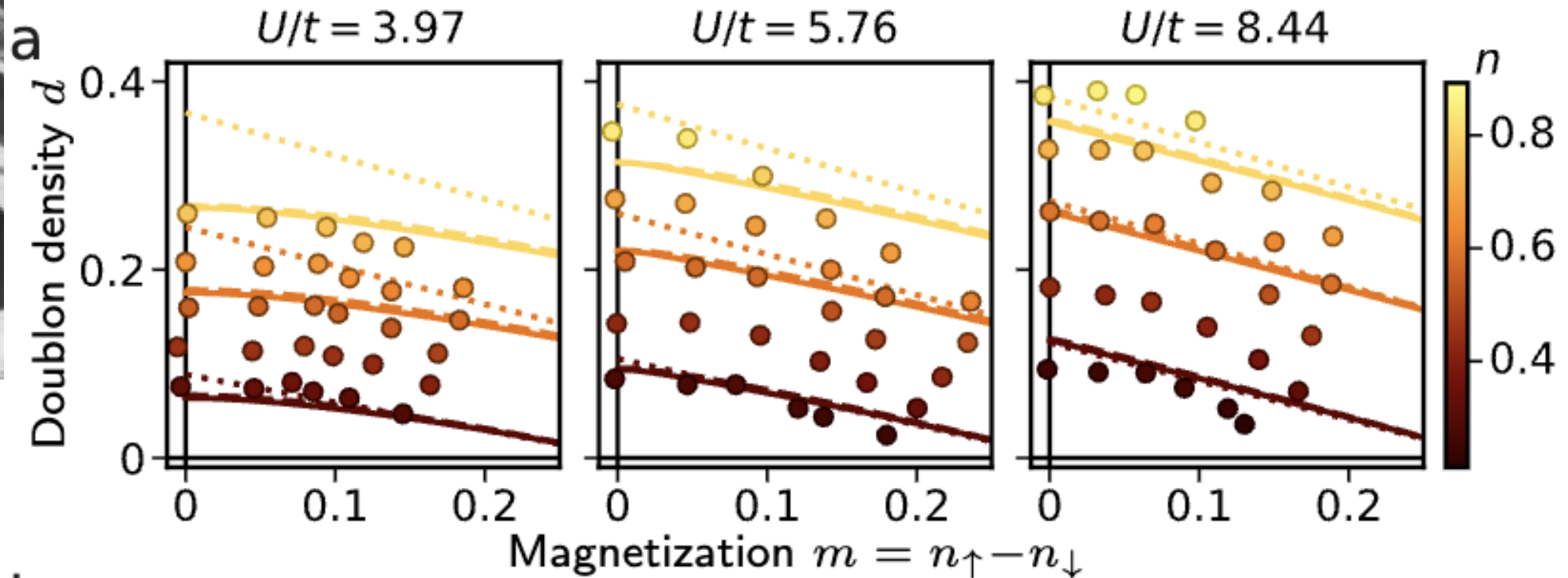
Fermi Polarons at large magnetization:

Fermi Polarons: Swimming in the Fermi Sea

Review: [arXiv:2501.09618](https://arxiv.org/abs/2501.09618) (2025)

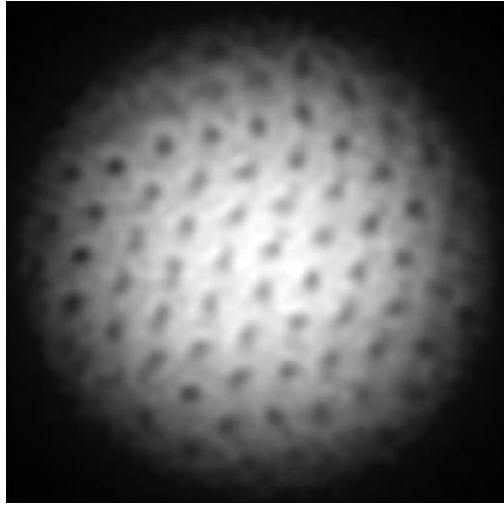


Doublon density:

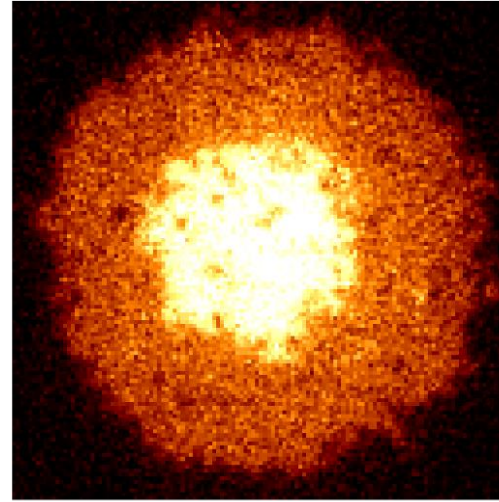


Dashed line: Fermi gas of polarons

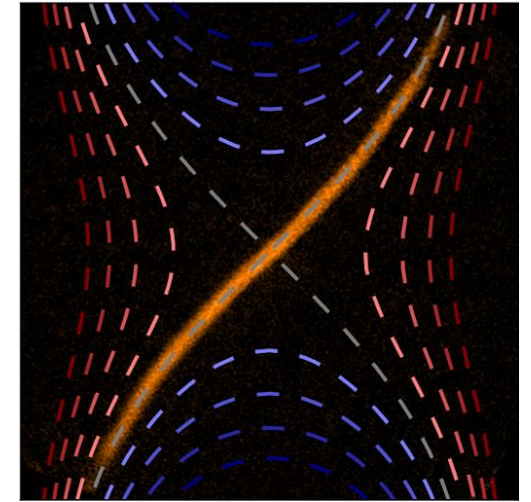
Quantum Many-Body Physics under the Microscope



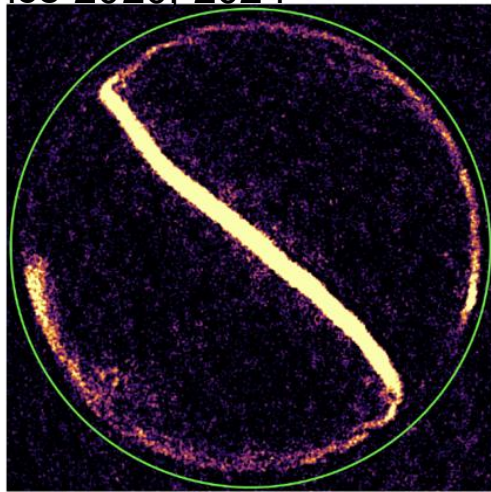
Superfluid Fermi Gas
Science 2020, 2024



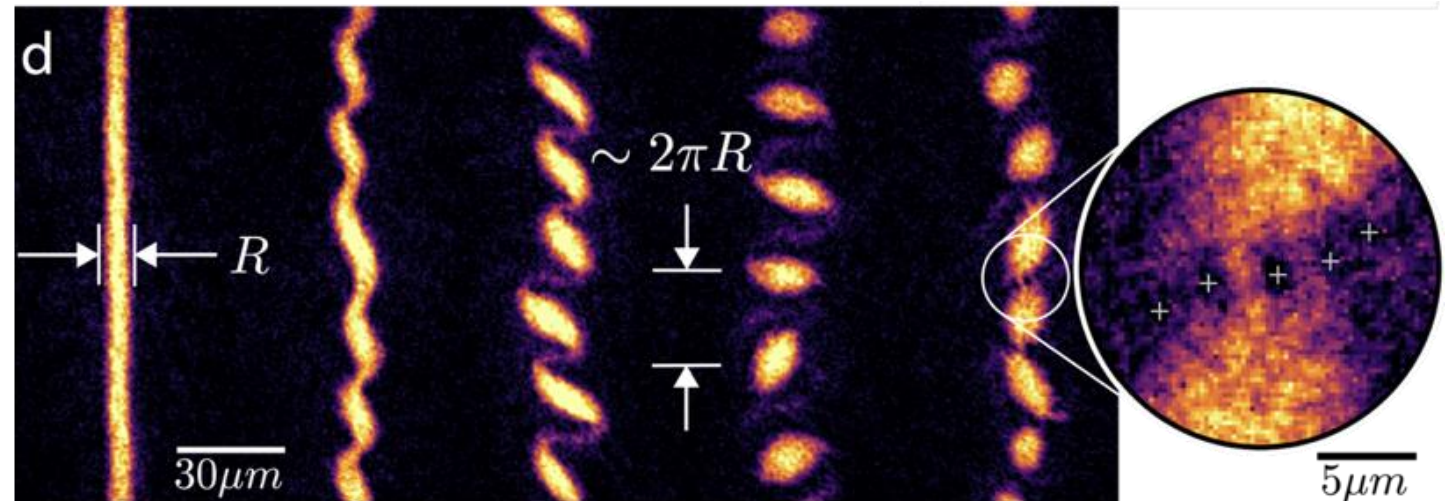
Fermi-Hubbard systems
Science 2023



Quantum Hall states
Science 2021

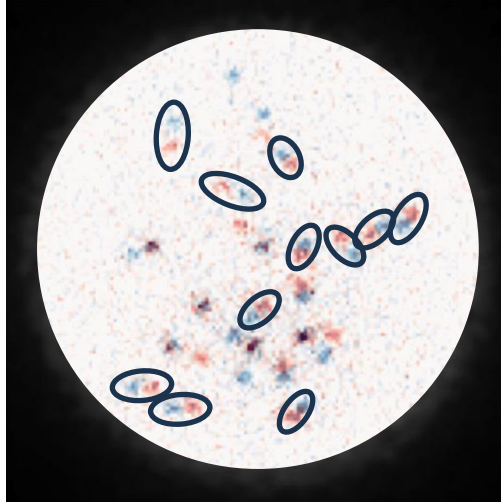


Topologically protected
Edge states Nat. Phys. 2024



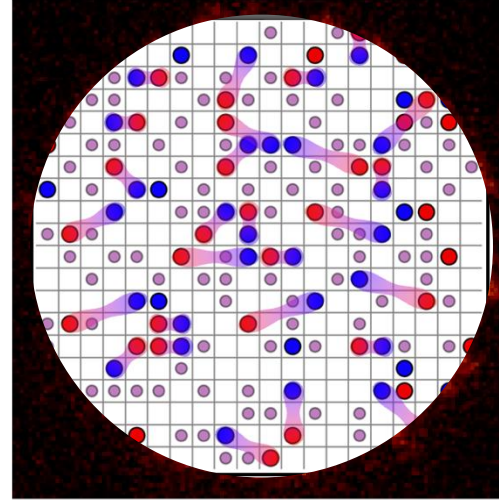
Non-Equilibrium Dynamics,
Quantum + Classical Turbulence Nature 2022

Quantum Many-Body Physics under the Microscope



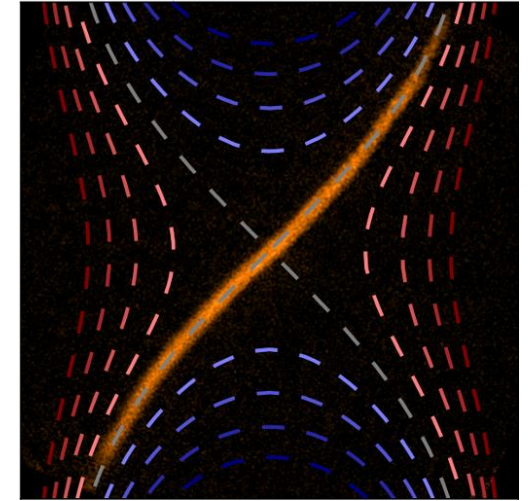
Superfluid Fermi Gas

Science 2020, 2024



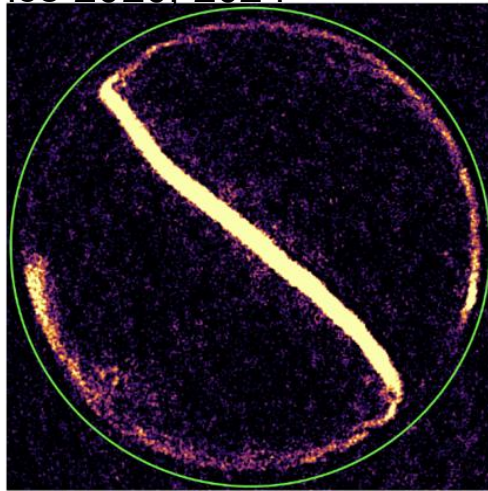
Fermi-Hubbard systems

Science 2023



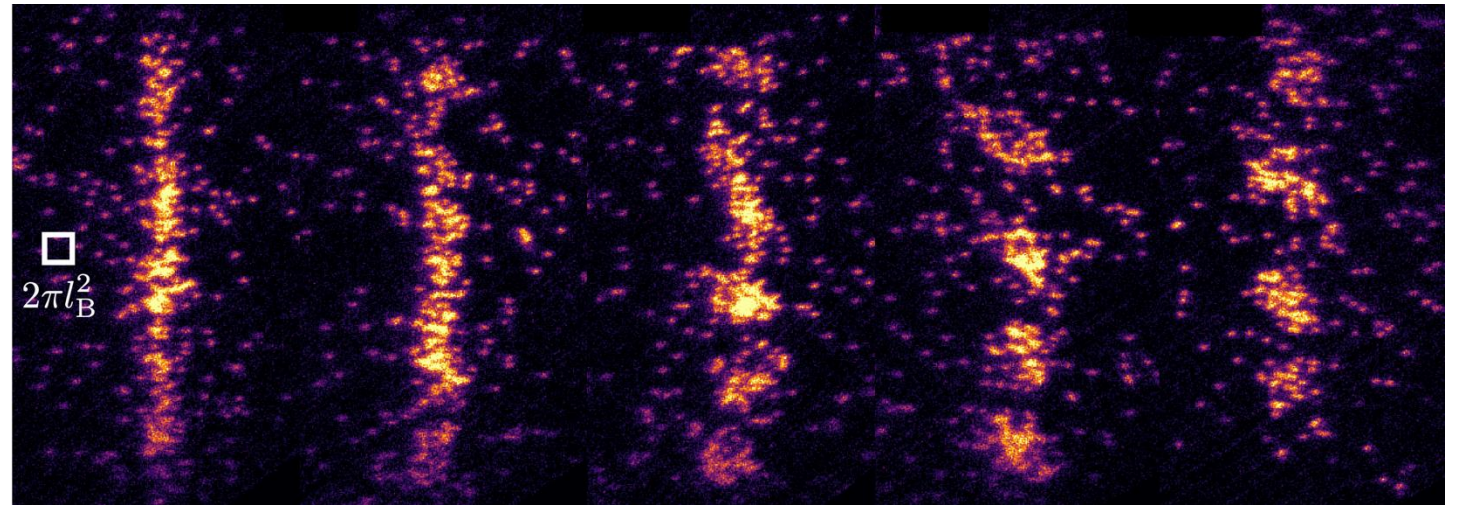
Quantum Hall states

Science 2021



Topologically protected
Edge states

Nat. Phys. 2024



Non-Equilibrium Dynamics,
Quantum + Classical Turbulence

Nature 2022

Fermi's Legacy for the Future

- Think outside the box!
- Look for universality – transfer insights from low-energy (e.g. atomic) physics to high-energy (nuclei, quarks...)
- Combine Theory and Experiment – Complete Physics!

Some Future topics:

- Fulde-Ferrell-Larkin-Ovchinnikov phase
- Topological superfluids (e.g. p-wave with molecules?)
- Modeling the Standard Model: Gauge Theories
- Quantum Computation with Fermions

Bosons and Fermions



Bosons and Fermions

BEC 1/Fermi3

Fermionic Superfluids

Eric Wolf

Recent Members:

Biswaroop Mukherjee

Parth Patel

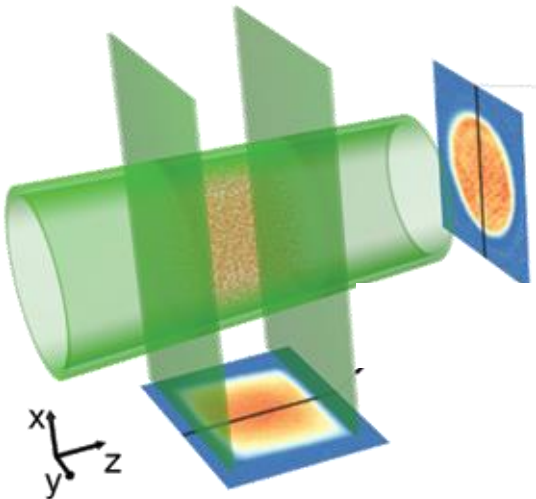
Zhenjie Yan

Richard Fletcher

(→ MIT)

Recent Visitor:

Chris Vale



Fermi 1

NaK Dipolar Molecules

Huan Bui

Yiming Zhang

Jacob VanArsdale

Recent Members:

Alex Yu Chuang

Carsten Robens

Zoe Yan (→ Chicago)

Yiqi Ni

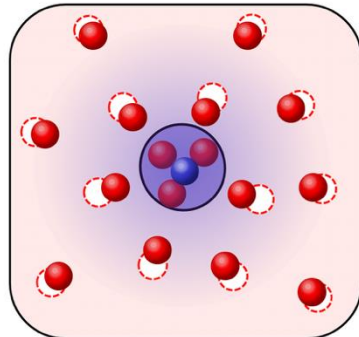
Theory collaborations:

Eugene Demler

Pavel Dolgirev

Kushal Seetharam

Arthur Christianen



Fermi 2

Fermi Gas Microscope

Botond Oreg

Carter Turnbaugh

Zihan Xu

Jens Hertkorn

Recent Members:

Ningyuan Jia

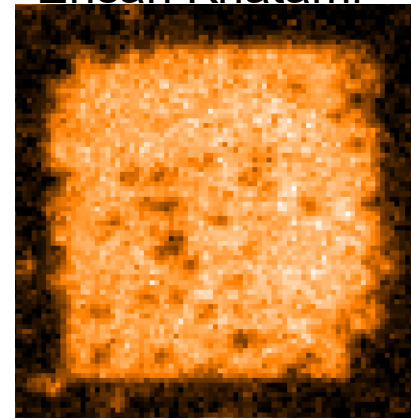
Thomas Hartke

Theory collaborations:

Shiwei Zhang

Chunhan Feng

Ehsan Khatami



Fermi3

Rotating Superfluids

Ruixiao Yao

Sungjae Chi

Kevin Wang

in collaboration with

Richard Fletcher

Recent Members:

Airlia Shaffer-Moag

Cedric Wilson

Theory collaborations:

Valentin Crépel

Tapio Simula

